February 5, 2021

PHYSICS 12b – Midterm

This exam covers all of the readings in Griffiths and the Lectures for the first four weeks of class, through HW 4.

This is an **OPEN BOOK** exam with the following limitations: You may consult only your textbook (Griffiths), notes that you have taken in recitation or lecture, online lecture notes, and HW solns (including your own). No other references are allowed. You may use a calculator and symbolic manipulation programs (eg. Mathematica) for integrals or algebra only - but not for graphing. In general they are not required.

In your studies, do not look at problems or exams from previous years of Phys 2 or 12.

The time limit is **4 HOURS**, in one continuous sitting. A half-hour break beyond the 4 hours is permitted during this period, as long as you are not working on the exam during this time. No credit for overtime work.

The MT is **DUE** Wednesday, Feb. 10 at 5:00PM and should be turned in via Gradescope. Late MTs will not be accepted for credit except by prior arrangement with the TAs.

There are four problems on pages 2–4 for a total of 50 points.

The exam counts for 15% of your grade. To get partial credit, show as much work as you can!

Problem 1 (Short-ish Problems)

(a) (3 points) Measurements of the energy spectrum of photons left over from the Big Bang indicate that it is an excellent Black-Body with a present temperature of ~ 2.7 K. Use the formula derived in lecture for the spectral energy distribution $u(\nu, T)$ of Black Body Radiation to find the frequency where this distribution is maximized. Express your answer in MHz.

(b) (2 points) Use what you've learned about the property of delta functions to show that $\delta(\sqrt{x}) = 0$.

(c) (2 points) Estimate the deBroglie wavelength of a so-called "thermal" neutron ($v \sim 2200$ m/s). Note: $m_n \sim 1.67 \times 10^{-27}$ kg

(d) Answer the following TRUE or FALSE questions. If the answer is false, briefly explain why it is false.

(i) (2 points)
$$\frac{\partial \langle T \rangle}{\partial t} = \left\langle \frac{\partial T}{\partial t} \right\rangle .$$

(ii) (2 points) If a particle is in an energy eigenstate, then I can predict with certainty the result of a measurement of the particle's momentum.

(iii) (2 points) For a particle in an infinite potential box, you can make an arbitrarily precise (a.k.a. an ideal) measurement of it's position.

(iv) (2 points) For a particle in an infinite potential box, you can make an arbitrarily precise (a.k.a. an ideal) measurement of it's momentum.

(v) (2 points) For a particle in an infinite potential box, you can make an arbitrarily precise (a.k.a. an ideal) measurement of it's energy.

(e) (4 points) A particle of mass m in a 1D harmonic oscillator potential, $V(x) = m\omega_0^2 x^2/2$,cannot be in a zero energy state because we would then, simultaneously, know both its position and momentum; a violation of Heisenberg's Uncertainty Principle (HUP). Use the HUP to approximately estimate the ground state energy for the 1D oscillator by assuming that the particle's momentum and position are related by the HUP and then minimizing the total energy:

$$E = \frac{p_x^2}{2m} + m\omega_0^2 x^2/2$$

(f) (4 points) Use commutators to evaluate $d\langle x^2 \rangle/dt$ for a 1D free particle along x.

Problem 2 (Phun with Phree Particles)

Consider a free particle wave packet of the form

$$\psi(x,0) = Ae^{ik_0 x} \quad , \quad \frac{-a}{2} \le x \le \frac{+a}{2}$$
$$\psi(x,0) = 0 \qquad , \qquad \text{elsewhere}$$

(a) **(2 points)** Determine the normalization constant A for this state.

(b) **(2 points)** Technically this is **NOT** an allowable wave function according to the postulates of QM. Briefly explain why this is the case.

(c) **(2 points)** What is the "k-space" wave function - $b(k)$ - for this state?

(d) **(2 points)** Use your results from part (c) to calculate the probability distribution, as a function of k, for measuring a particlular value of k and sketch your result. Be sure to include the scale on the k-axis.

Now consider a different free particle wave packet - the gaussian wave packet. Recall from HW and lecture that such a wave function broadens as a function of time. Perhaps we can observe this broadening to *Prove* Quantum Mechanics.

(e) **(3 points)** Use the results from the second HW set (4^{th} problem in BP2 - labeled 2.22) to estimate the time for a regular Ping-Pong ball, $m = 2.7$ g, to double in size due to wave packet broadening by assuming that the ball's radius, $r = 2$ cm is equal to Δx at $t = 0$. Is this an experiment worth doing?

Problem 3 (Relative Phase)

In lecture we commented that the overall phase of a wave function has no physical significance, but that a relative phase between components of a superposition of states can have real physical significance.

Consider the following one-dimensional wave function for a particle:

$$\psi(x) = A\left[\phi_a(x) + e^{i\theta}\phi_b(x)\right]$$

where ϕ_a and ϕ_b are purely real energy eigenstates with energy eigenvalues E_a and E_b respectively and θ is a relative phase angle.

(a) (2 points) Write down an expression for the fully time-dependent probability density $\psi^*(x,t)\psi(x,t)$ for the cases $\theta = 0$ and $\theta = \pi/2$ in terms of $\phi_a(x), \phi_b(x), E_a, E_b$ and $\hbar$.

(b) (2 points) Use your result from part (b) to predict how long you have to wait until the initial probability density is reproduced for the first time for the two cases $\theta = 0$ and $\theta = \pi/2$. Express this time in terms of E_a, E_b and $\hbar$. If the Prof was correct the answers should be clearly different for the two cases.

Problem 4 (Infinite Square Well)

Consider an infinite square well where the ends of the well are at $-a/2$ and $a/2$ instead of at 0 and a (as in Lecture or Griffiths), i.e. consider the potential

$$V(x) = 0 \text{ if } \ -\frac{a}{2} < x < \frac{a}{2}$$

$$V(x) = \infty \text{ otherwise}$$

This change modifies the form of the solution compared to Griffiths and what we did in Lecture.

(a) (2 points) Determine the normalized energy eigenstates for this well.

(b) (2 points) Determine energy eigenvalues for this well.

We now introduce the "parity" operator $\hat{P}$ which has the property $\hat{P}\psi(x) = \psi(-x)$ for an abitrary wave function $\psi(x)$. This implies that $\hat{P}\hat{P}\psi(x) = \hat{P}\psi(-x) = \psi(x)$. If now $\phi_P(x)$ is an eigenstate of $\hat{P}$ then $\hat{P}\phi_P(x) = \alpha\phi_P(x)$.

(c) (2 points) Use repeated operations with the $\hat{P}$ operator to determine the eigenvalues of the $\hat{P}$ operator.

(d) (2 points) Show that the eigenstates from part (a) are also eignenstates of $\hat{P}$.

(e) (2 points) What are the corresponding eigenvalues of $\hat{P}$ for these eigenstates?