CHAPTER 3

The M.I.T. Bag Model

3.1 PHYSICAL BASIS

In the last chapter, the independent motion of quarks in a cavity was considered at length. It will turn out that the working version of the M.I.T. bag model is almost identical, except that an extra term is introduced to balance the outward pressure on the walls of the cavity. One is naturally faced with the question of justifying such a naive model. We proceed to do this briefly, starting with the nature of QED and QCD vacuum. A good discussion on this topic is given in the text by Lee[1], and in Nielsen[2]. More details will be found in Sections 5.4 and 5.5 .

We first discuss the term polarizability as used in nonrelativistic physics. The Coulomb potential between two electric charges gets "screened" in a polarizable medium:

$$V(r) = \frac{e^2}{4\pi\epsilon r} \quad ,$$

where ϵ is the dielectric constant for the medium, and r is the spatial distance between charges, $r = |\mathbf{r}|$. In vacuum, $\epsilon = 1$. Screening of the charges due to the polarizability of the molecules makes $\epsilon > 1$. Also, in a paramagnetic medium, an external magnetic field **B** itself induces magnetization, with the result that the energy density changes by

$$\Delta E = -\frac{1}{2} 4\pi \chi B^2$$

where χ (> 0) is the magnetic susceptibility of the medium. The magnetic permeability μ is defined as $\mu = 1+4\pi\chi$, and for a paramagnetic substance $\mu > 1$. In nonrelativistic physics, it is possible to have both ϵ and μ greater than unity, as, for example, in a free electron gas at zero temperature.

Exercise 3.1

Use the nonrelativitic quark-model to calculate the magnetic susceptibility of a proton. Use perturbation theory. With a uniform magnetic field B in the z-direction, the perturbing Hamiltonian is $\mathcal{H}_{\text{int}} = -\hat{\mu}_z B + (e^2/8m_c)(x^2+y^2)B^2$, and $\mu_z = \sum_q (e_q/2m_c)\sigma_z$, where m_c is the constituent quark mass. We have taken $\mathbf{A} = \frac{1}{2}(\mathbf{B} \times \mathbf{r})$. The first term, taken to second order, gives a paramagnetic contribution. The diagonal matrix-element of the second term is diamagnetic. Show that the proton magnetic susceptibility χ_p is given by

$$4\pi\chi_P = \frac{2\mu_{P\Delta}^2}{(E_\Delta - E_N)} - \frac{e^2}{6m_c}\langle r^2\rangle_P \,.$$

Here $\mu_{P\Delta} = \langle P|\mu_z|\Delta\rangle = 2\sqrt{2/3}\,\mu_P$, μ_P being the magnetic moment of the proton. Transitions to states other than $\Delta(1230)$ are ignored because of negligible spatial overlap with the ground state. Take $m_c = 336\,\text{MeV}$, $\langle r^2\rangle_P^{1/2} = 0.86\,\text{fm}$, and $E_\Delta - E_N = 300\,\text{MeV}$. Show that $\chi_P \approx 2\times10^{-4}\,\text{fm}^{+3}$. How do you think it can be measured? In the literature, χ_P is also called the magnetic polarizability and denoted by $\bar{\beta}$. The corresponding response function for the electric field is termed electric polarizability and denoted by $\bar{\alpha}$ (see Exercise 5.11).

In QED, the vacuum itself behaves like a polarizable medium. This is so because the interaction between two electrons takes place via the exchange of photons. The photon, although uncharged, can create virtual electron-positron pairs, causing partial screening of the test electron charge. This means that $\epsilon > 1$ as long as the distance r is large enough so that the virtual cloud around the test charge is not penetrated. This distance is ridiculously small, so in practice the larger than unity value of

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ϵ is absorbed in the definition of the charge e. For a detailed discussion of this point, see Section 5.4 . It does mean that the electromagnetic coupling constant should become stronger as $r \to 0$. In a Lorentz invariant theory, moreover, $\mu\epsilon = 1$, so $\epsilon > 1$ implies $\mu < 1$ for the QED vacuum, This is why the QED vacuum is termed diamagnetic. In QCD, the quarks have color charge, and interact by exchanging gluons. Gluons, like photons, are spin-one objects, but unlike photons, they also carry (color) charge. In a QCD vacuum, a gluon can produce virtual $q\bar{q}$ pairs, which would screen the interaction, and should make it diamagnetic as in the QED case. However, since the gluons have color charge as well as spin, they can cause color magnetization of the medium and make the medium paramagnetic. This effect actually overcomes the diamagnetic property of the $q\bar{q}$ pairs, and the overall result is that $\mu_c > 1$ for the QCD vacuum where the subscript refers to color. The situation is somewhat like the paramagnetism of the electron gas, where the intrinsic spin alignment of the electrons overrides the diamagetism of orbital motion.

Since $\mu_c > 1$ for the QCD vacuum (for large enough r), it follows that $\epsilon_c < 1$, so that the color electric interaction between the charged objects becomes stronger for larger distances. In this sense it is an "antiscreening" medium. As the distance $r \to 0$, on the other hand, μ_c and $\epsilon_c \to 1$, and the interaction becomes weaker. This is called asymptotic freedom, in the sense that $r \to 0$ corresponds to asymptotically large values of momentum transfer $\mathbf{q}^2$. For large r or small $\mathbf{q}^2$, if $\mu_c \gg 1$, this should also explain confinement of color charge. An idealization of this picture, leading to the bag model, is shown in Fig. 3.1 (a). Inside the hadron, $\mu_c = \epsilon_c = 1$, the

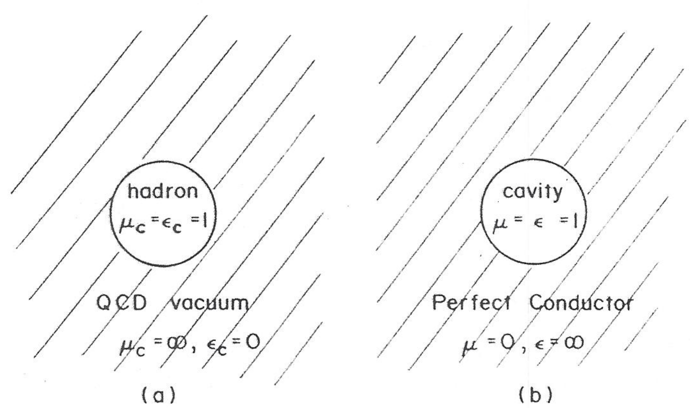

Figure 3.1 The idealized picture of a hadron surrounded by the QCD vacuum is analogous to a cavity in a perfect conductor.

interaction between color charges is weak. Outside the hadron radius, the medium is a perfect paramagnet, with $\mu_c = \infty$ and $\epsilon_c = 0$. Consequently the color fields are totally confined within the hadron. The situation is analogous to the electrodynamic case, Fig. 3.1 (b), where a cavity is dug into a perfect conductor (*i.e.*, a superconductor). Inside this cavity, on the boundary of the surface, $\hat{\mathbf{n}} \times \mathbf{E} = 0$ and $\hat{\mathbf{n}} \cdot \mathbf{B} = 0$, where $\hat{\mathbf{n}}$ is an outward normal unit vector to the surface. The QCD vacuum is analogous to the superconductor in electrodynamics, but with the roles of μ and ϵ interchanged. Accordingly, on the surface of the hadron, the boundary conditions (for the gluon color fields) are (see Fig. 3.2)

$$\hat{\mathbf{n}} \times \mathbf{B} = 0 \quad , \quad \hat{\mathbf{n}} \cdot \mathbf{E} = 0 \, . \tag{3.1.1}$$

This would suffice for the color electric and magnetic fields, but what boundary condition should be used for the quark fields if these are present? In the case of the static cavity, we found, in Eq. (2.5.7), $\bar{\psi}\psi\big|_{r=R} = 0$ as a boundary condition. To implement this in a Lorentz invariant way, we write

$$i\gamma^\mu n_\mu \psi = \psi \qquad \text{on the surface of the bag.} \tag{3.1.2 a}$$

Here n_μ is a space-like unit vector normal to the surface. Note that this implies

$$-i\bar{\psi}\gamma^\mu n_\mu = \bar{\psi} \qquad \text{on the surface.} \tag{3.1.2 b}$$

Then $\bar{\psi}\psi = i\bar{\psi}(\gamma^\mu n_\mu \psi) = -i(\bar{\psi}\gamma^\mu n_\mu)\psi = 0$ on the surface of the bag. In this form, it also means that there is no outward quark current accross

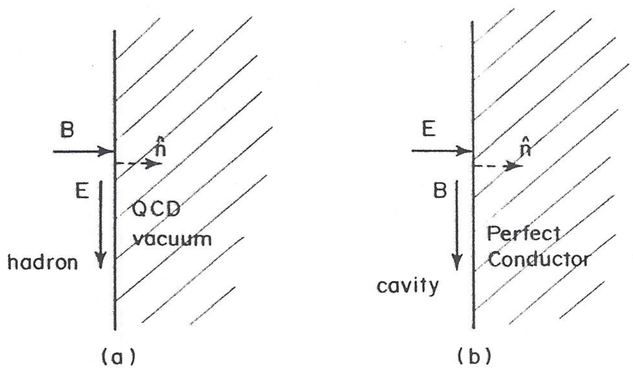

Figure 3.2 The boundary conditions (a) For the color electric and magnetic fields, $\hat{\mathbf{n}} \times \mathbf{B} = 0$, $\hat{\mathbf{n}} \cdot \mathbf{E} = 0$ on the interface. (b) For the electric and magnetic fields, $\hat{\mathbf{n}} \times \mathbf{E} = 0$, $\hat{\mathbf{n}} \cdot \mathbf{B} = 0$ on the interface.

3.5 THE STATIC SPHERICAL M.I.T. BAG

The M.I.T. bag model is particularly simple in the spherical, static approximation and is often used in other applications. The Hamiltonian of the three-quark system in the bag may be deduced from the Lagrangian (3.2.7). For massless quarks in the ground state, the mass of the baryon (N or Δ) is

$$M(R) = \frac{3 \times 2.04}{R} + \frac{4\pi}{3} B R^3 \mp \frac{3 \times 0.117\, \alpha_s}{R} + \frac{Z_0}{R} \,. \tag{3.5.1}$$

Here the first term is just the kinetic energy of the three quarks in a cavity of radius R, see Eq. (2.5.5) and Table 2.1 . The second term may be interpreted as the extra energy needed to create the perturbative vacuum of volume V, and the third term is the hyperfine interaction discussed in Section 3.3, which lowers the nucleon mass relative to the delta. The last term Z_0/R is supposed to take care of all effects which are difficult to calculate — like zero-point energy, centre-of-mass correction, color magnetic self-energy, *etc.* In the static spherical model, the pressure balance equation is equivalent to minimization of $M(R)$ as a function of R. To see this, write Eq. (3.5.1) as

$$M(V) = \mathcal{E}(V) + BV \quad ,$$

where V is the volume of the bag, and $\mathcal{E}(V)$ represents the total kinetic and interaction energy of the quarks. Then, at equilibrium volume $V_N = \frac{4\pi}{3} R_N^3$, the condition $\frac{\partial M}{\partial V}\big|_{V=V_N} = 0$ gives

$$-\frac{\partial \mathcal{E}}{\partial V}\bigg|_{V_N} = B \quad ,$$

which is nothing but the pressure balance equation. We have, for the equilibrium radius $R = R_N$,

$$\frac{\partial M}{\partial R}\bigg|_{R_N} = -\frac{a_N}{R_N^2} + 4\pi B R_N^2 = 0 \quad , \tag{3.5.2}$$

where $a_N = (6.12 - 0.351\alpha_s + Z_0)$ for the nucleon. This shows that the equilibrium radius R_N is determined by the parameters α_s, Z_0 and B. The choice of R_N determines the $r.m.s$ radius $\langle r^2 \rangle^{1/2}$ and the magnetic moment of the nucleon. We have noted before that the two cannot be fitted simultaneously for the proton. Nor can one get the negative mean square charge radius of the neutron. Substituting the equilibrium condition (3.5.2) in Eq. (3.5.1) yields

$$M_N = \frac{16\pi}{3} B R_N^3 = \frac{4}{3} \frac{a_N}{R_N} \quad , \tag{3.5.3}$$

and a similar equation for M_Δ. DeGrand[5] *et al.* introduced another parameter, the mass of the strange quark, m_s. Taking

$$B^{1/4} = 145\,\text{MeV}\ ,\ Z_0 = -1.84\ ,\ \alpha_s = 2.2 \quad \text{and}\ m_s = 279\,\text{MeV} \quad , \tag{3.5.4}$$

they could fit the masses of the ground-state baryons and mesons reasonably well, except the pion mass. The set of parameters (3.5.4) is by no means unique, and many other sets exist in the literature. Generally speaking, the bag model in this simple form cannot give the correct pion mass. It also does poorly in the description of the excited states[9]. But apart from these details, there are a couple of serious drawbacks in this model from the nuclear physics point of view. In nucleon-nucleon scattering, the most well-established part of the force is the tail due to the one-pion exchange process. There is no provision for this mechanism in the M.I.T. version of the bag model. The other problem is the size of the bag, which is too big. At normal nuclear matter density, a nucleon on the average occupies a volume of about $6\,\text{fm}^3$. But this means that for $R \sim 1.1\,\text{fm}$ the bags would already touch each other at normal nuclear density. Even moderate compression of nuclear matter in nuclear reactions should then bring out quark degrees of freedom due to overlapping 6-quark, 9-quark bags *etc.* But such explicit quark degrees of freedom are not manifest in nuclear physics. These considerations prompted the development of the chiral bag[10], where the quark-bag is smaller in size and is surrounded by a pion cloud. In the next chapter we proceed to develop the idea of chiral symmetry and the chiral and cloudy bags.

Exercise 3.4: Six-quark Bags

It is interesting to calculate the energy of 6-quark, 9-quark *etc.* bags and compare them with the corresponding nuclear masses like deuteron *etc.* For example, the 6-quark bag with the quantum numbers of the deuteron is about 300 MeV heavier than the deutron in such calculations. It turns out that ΔE_M of Eq. (3.3.19) is attractive only for the nucleon ($n = 3, I = S = \frac{1}{2}$), but repulsive for the other cases. Color singlet wave functions for 6-quark, 9-quark *etc.* are of mixed permutation symmetry. The total wave function cannot be written as a product of the color part

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Using this property,
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(d) Is it possible to hav
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For $B^{1/4} = 145\,\text{MeV}$ (Section 3.5), $\tau_0 \sim 100\,\text{MeV}$, which is too low a value. High energy heavy-ion collisions at $E_{\text{lab}} = 2.1\,\text{GeV}/A$ already indicates proton temperatures higher than this[21]. Although the counting of hadronic states at high excitation is very uncertain, a fit to the data may be obtained[22] if $\tau_0 \approx 150$–$200\,\text{MeV}$. It is believed that near the limiting hadronic temperature the quarks and gluons in individual hadrons may get deconfined and form a quark-gluon plasma. This topic is further discussed in Section 5.5 in the context of the QCD vacuum. Ultrarelativistic heavy-ion collisions of nuclei may form pockets of quark-gluon plasma which hadronize in the time scale of 10^{-22} sec. For a review of the extensive literature on this topic, see the article by McLerren[23]. A comprehensive and general review on "quarks in nuclei" is given by C. W. Wong[24].

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