Homework Set #3

Problem 2.19 Find the probability current, J (Problem 1.14) for the free particle wave function Equation 2.94. Which direction does the probability current flow?
2nd Ed : Eq. 2.95 3rd Ed.

** **Problem 2.27** Consider the *double* delta-function potential

$$V(x) = -\alpha\,[\delta(x+a) + \delta(x-a)],$$

where α and a are positive constants.

(a) Sketch this potential.

(b) How many bound states does it possess? Find the allowed energies, for $\alpha = \hbar^2/ma$ and for $\alpha = \hbar^2/4ma$, and sketch the wave functions.

(c) What are the bound state energies in the limiting cases (i) $a \to 0$ and (ii) $a \to \infty$ (holding α fixed)? Explain why your answers are reasonable, by comparison with the single delta-function well.

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* **Problem 2.21 The gaussian wave packet.** A free particle has the initial wave function

$$\Psi(x,0) = Ae^{-ax^2},$$

where A and a are (real and positive) constants.

(a) Normalize $\Psi(x,0)$.

(b) Find $\Psi(x,t)$. *Hint:* Integrals of the form

$$\int_{-\infty}^{+\infty} e^{-(ax^2+bx)}dx$$

can be handled by "completing the square": Let $y \equiv \sqrt{a}\left[x + (b/2a)\right]$, and note that $\left(ax^2+bx\right) = y^2 - \left(b^2/4a\right)$. *Answer:*

$$\Psi(x,t) = \left(\frac{2a}{\pi}\right)^{1/4} \frac{1}{\gamma} e^{-ax^2/\gamma^2}, \quad \text{where} \quad \gamma \equiv \sqrt{1 + (2i\hbar at/m)}. \tag{2.111}$$

(c) Find $|\Psi(x,t)|^2$. Express your answer in terms of the quantity

$$w \equiv \sqrt{a/\left[1 + (2\hbar at/m)^2\right]}.$$

Sketch $|\Psi|^2$ (as a function of x) at $t = 0$, and again for some very large t. Qualitatively, what happens to $|\Psi|^2$, as time goes on?

(d) Find $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$, σ_x, and σ_p. *Partial answer:* $\langle p^2\rangle = a\hbar^2$, but it may take some algebra to reduce it to this simple form.

(e) Does the uncertainty principle hold? At what time t does the system come closest to the uncertainty limit?

(This is problem #2.22 in the 2nd edition of Introduction to Quantum Mechanics.)