Homework #7

* **Problem 4.2** Use separation of variables in cartesian coordinates to solve the infinite *cubical* well (or "particle in a box"):

$$V(x, y, z) = \begin{cases} 0, & x, y, z \text{ all between } 0 \text{ and } a; \\ \infty, & \text{otherwise.} \end{cases}$$

(a) Find the stationary states, and the corresponding energies.

(b) Call the distinct energies $E_1, E_2, E_3, \ldots$, in order of increasing energy. Find E_1, E_2, E_3, E_4, E_5, and E_6. Determine their degeneracies (that is, the number of different states that share the same energy). *Comment*: In *one* dimension degenerate bound states do not occur (see Problem 2.44), but in three dimensions they are very common.

(c) What is the degeneracy of E_{14}, and why is this case interesting?

Hint for Problem 4.2: You can use results from Week 6 notes.

* **Problem 4.4** Use Equations 4.27, 4.28, and 4.32, to construct Y_0^0 and Y_2^1. Check that they are normalized and orthogonal.

Problem 4.16 What is the *most probable* value of r, in the ground state of hydrogen? (The answer is *not* zero!) *Hint:* First you must figure out the probability that the electron would be found between r and $r + dr$.

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* **Problem 4.22**

(a) Starting with the canonical commutation relations for position and momentum (Equation 4.10), work out the following commutators:

$$\left[L_z, x\right] = i\hbar y, \qquad \left[L_z, y\right] = -i\hbar x, \qquad \left[L_z, z\right] = 0$$
$$\left[L_z, p_x\right] = i\hbar p_y, \quad \left[L_z, p_y\right] = -i\hbar p_x, \quad \left[L_z, p_z\right] = 0. \tag{4.122}$$

(b) Use these results to obtain $\left[L_z, L_x\right] = i\hbar L_y$ directly from Equation 4.96.

(c) Find the commutators $\left[L_z, r^2\right]$ and $\left[L_z, p^2\right]$ (where, of course, $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$).

(d) Show that the Hamiltonian $H = \left(p^2/2m\right) + V$ commutes with all three components of **L**, provided that V depends only on r. (Thus H, L^2, and L_z are mutually compatible observables.)