Ph. 203 Lect 1

No Exams → 9 HW sets 50% to pass P/F

See Canvas for details

Parts from 2 texts see Bertulani
Bhaduri

Coverage: N-N Force — Nuclear Models
Nuclei Props. — + QCD
Nuclear Models — Nucl. Astro. & Elec. Weak

QM prelims: Ang. Mom. & Scattering

Generalized Ang. Mom. defined by

$$[\hat{J}_i, \hat{J}_j] = i\,\epsilon_{ijk}\,\hat{J}_k \quad ; \quad i, j, k = 1, 2, 3$$

$\hat{A}$ = operator

ϵ_{ijk}: anti-sym. unit tensor

$$\epsilon_{123} = \epsilon_{312} = \epsilon_{231} = 1$$

$$\epsilon_{321} = \epsilon_{132} = \epsilon_{213} = -1$$

all others = 0

Eigenstates of $\hat{J}^2$ & $\hat{J}_3$ are $|j\,m\rangle$

$\hat{J}_\pm |j\,m\rangle \propto |j\,m\pm1\rangle$ if $|j\,m\pm1\rangle$ exist

$= 0$ otherwise

Addition of Ang. Mom.

can combine $\hat{\vec{j}}_1$ & $\hat{\vec{j}}_2$ (e.g. spin + orbit, spin + spin, ...)

to form eigenstates of J_{total}: $\hat{\vec{J}} = \hat{\vec{j}}_1 + \hat{\vec{j}}_2$

$$\Leftrightarrow |J\,M\rangle = \sum_{m_1, m_2} \underbrace{(j_1 j_2 m_1 m_2 | J M)}_{\text{C.G. coefficient}} |j_1 m_1\rangle |j_2 m_2\rangle$$

46. Clebsch-Gordan Coefficients, Spherical Harmonics, and d Functions

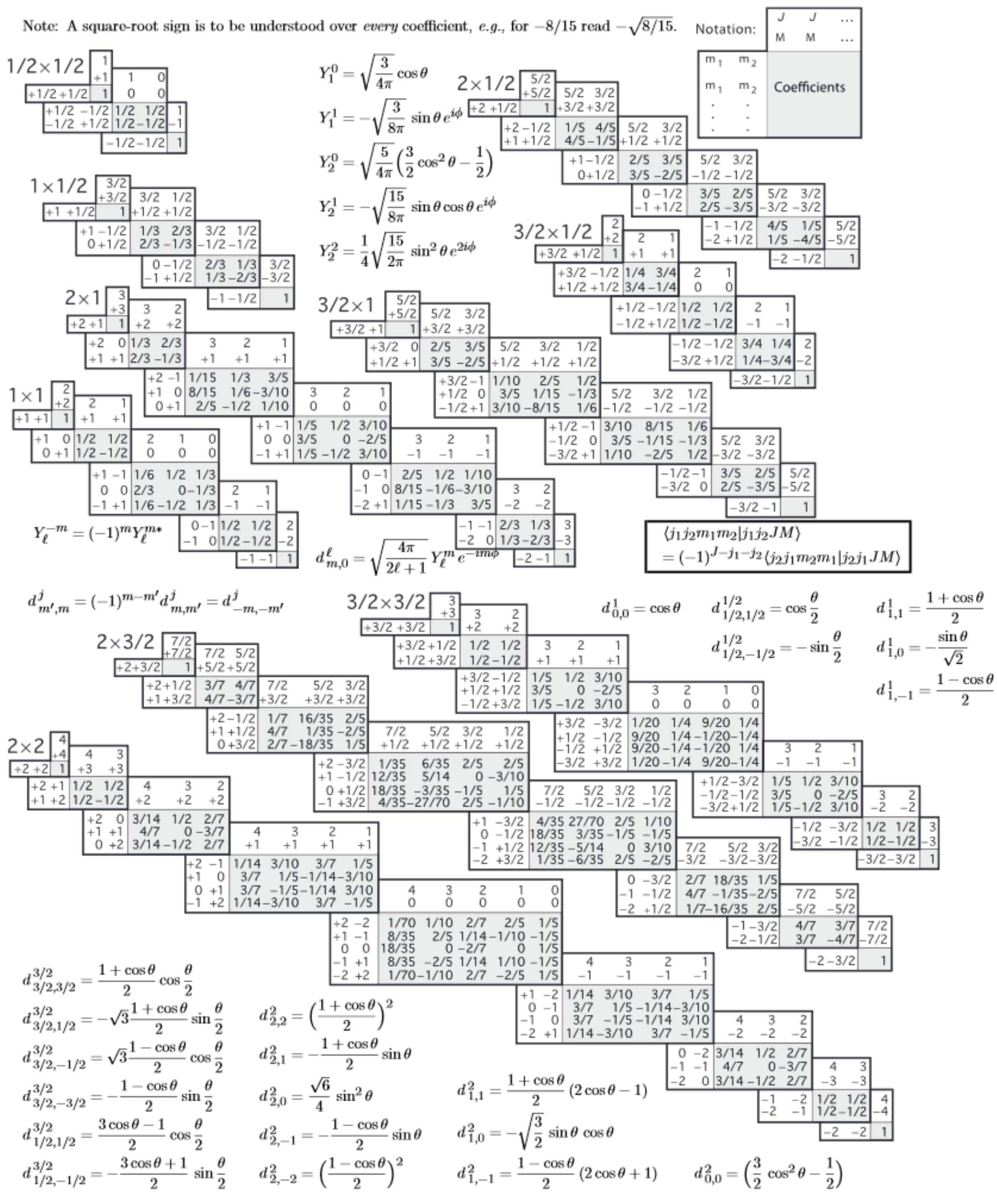


Figure 46.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

4/1/24 2

include C.G pic

Example 1 combine 2 spin $\frac{1}{2}$ nucleons (n or p)

$$\vec{J} = \underbrace{\underbrace{\vec{s}_1 + \vec{s}_2}_{S} + \vec{l}}_{J}$$ → relat. orb. ang. mom.

∴

$l=0$			$l=1$	
$\lvert s\, m_s\rangle$	$\lvert J M_J\rangle$		$\lvert s\, m_s\rangle$	$\lvert J m_J\rangle$
$\lvert 11\rangle$	$\lvert 11\rangle$		$\lvert 11\rangle$	$\lvert 2, \begin{smallmatrix}2\\1\\0\\-1\\-2\end{smallmatrix}\rangle \Rightarrow {}^3P_2$
$\lvert 10\rangle$ →	$\lvert 10\rangle$	${}^3S_1$	$\lvert 10\rangle$	$\lvert 1 \begin{smallmatrix}1\\0\\-1\end{smallmatrix}\rangle \Rightarrow {}^3P_1$
$\lvert 1\,{-1}\rangle$	$\lvert 1\,{-1}\rangle$		$\lvert 1\,{-1}\rangle$	$\lvert 00\rangle \Rightarrow {}^3P_0$

$\lvert 00\rangle \rightarrow \lvert 00\rangle \Rightarrow {}^1S_0$ $\qquad$ $\lvert 00\rangle \rightarrow \lvert 1 \begin{smallmatrix}1\\0\\-1\end{smallmatrix}\rangle \Rightarrow {}^1P_1$

where ↪ ${}^{2s+1}l_j$

& for $l=2 \Rightarrow \longrightarrow {}^3D_3, {}^3D_2, {}^3D_1, {}^1D_2$

then 2N W.F.:

e.g. $\lvert {}^3P_0\rangle_{M=0} = \sum (C.G.) \lvert s m_s\rangle \lvert l m_l\rangle = \lvert JM\rangle = \lvert 00\rangle$

$= \frac{1}{\sqrt{3}} \left(\lvert 11\rangle Y_{1-1} - \lvert 10\rangle Y_{10} + \lvert 1\,{-1}\rangle Y_{11} \right)$

see C.G.

or

$\lvert {}^3P_2\rangle_{M=2} = \underset{JM}{\lvert 22\rangle} = \underset{sm}{\lvert 11\rangle} Y_{11}$

Ex. 2: Isospin as Gen. Ang. Mom.

Assume p & n are 2 states of same particle → nucleon

↪ see later

then isospin W.F. $\Rightarrow \lvert t\, t_3\rangle$ eigenstates of $\hat{T}^2, \hat{T}_3$

w $\lvert \frac{1}{2} \frac{1}{2}\rangle = p$

$\lvert \frac{1}{2} -\frac{1}{2}\rangle = n$

also works for u, d quarks

w $\hat{\vec{T}} = \frac{1}{2}\hat{\vec{\tau}}$, $\hat{\vec{\tau}} \equiv \hat{\vec{\sigma}}$ & W.F. is spinor

&

$\hat{T}^2|t\,t_3\rangle = t(t+1)|t\,t_3\rangle$; $\hat{T}_3|t\,t_3\rangle = t_3|t\,t_3\rangle$

$\therefore\ \hat{T}^2|p\rangle = \frac{3}{4}|p\rangle$

also have $\hat{T}_\pm$ w $\underbrace{\hat{T}_-|p\rangle = |n\rangle}_{\beta\text{-decay}}$, $\hat{T}_- = \hat{T}_1 - i\hat{T}_2$

also works for $t > \frac{1}{2}$

$\Leftrightarrow\ \pi^{+,0,-}$ $\quad |\pi^+\rangle = |11\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$|\pi^0\rangle = |10\rangle$

$|\pi^-\rangle = |1\,{-1}\rangle$

Ques: what's $|00\rangle$? $\Rightarrow \eta^0$

here 3x3 rep. of SU(2):

$$\hat{T}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix};\quad \hat{T}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix};\quad \hat{T}_3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$$

Rotations & Tensor Operators

Can change basis (e.g. rotate by 90°) of Ang. Mom. eigenstates via rotation operator:

$$\hat{R}(\vec{\theta}) = e^{-i\hat{\vec{J}}\cdot\hat{\vec{\theta}}} \rightarrow (\hbar = 1 \text{ here})$$

clearly $[\hat{R}, \hat{J}^2] = 0 \ \therefore$ j in $|jm\rangle$ unchanged

but, generally, m will change via

$$|jm\rangle' = \hat{R}(\vec{\theta})|jm\rangle = \sum_{m'=-j}^{+j} |j\,m'\rangle \underbrace{\mathcal{D}^{j}_{mm'}(\vec{\theta})}_{\text{Rotation Matrix}}$$

for arbitrary $\vec{\theta}$ can use Euler angles α, θ, γ w

$$\hat{\mathbb{R}}(\vec{\theta}) = \mathbb{R}(\alpha,\theta,\gamma) \equiv e^{-i\gamma\hat{J}_3''} e^{-i\theta\hat{J}_2'} e^{-i\alpha\hat{J}_3}$$

$\hookrightarrow \hat{J}_2'$ is in sys. rotated by α

OR!!

$$= e^{-i\alpha\hat{J}_3} e^{-i\theta\hat{J}_2} e^{-i\gamma\hat{J}_3}$$

$\hookrightarrow$ all in fixed coord. sys.! ($\hookrightarrow$ lab)

$$\therefore \mathcal{D}^j_{m'm}(\alpha,\theta,\gamma) = \langle jm'|\mathbb{R}(\alpha,\theta,\gamma)|jm\rangle$$

$$= e^{-im\gamma} e^{+im'\alpha} \underbrace{\langle jm'|e^{-i\theta\hat{J}_2}|jm\rangle}$$

reduced rotation matrix element $\equiv d^j_{m'm}(\theta)$

e.g. $d^{1/2}_{\frac{1}{2}\,-\frac{1}{2}} = -\sin\left(\frac{\theta}{2}\right)$

$d^2_{21} = -\frac{1}{2}\sin\theta(1+\cos\theta)$

see P.D.G.

Spherical Tensors:

An irreducible (spherical) tensor of rank n $[T_{n\mu}(c);\ \mu=-n,\ldots+n]$ (c: coordin.) transforms like Ang. Mom. under coord. rotation:

$$T_{n\mu}(c) = \sum_{\mu'} T_{n\mu'}(c')\,\mathcal{D}^n_{\mu'\mu}(\vec{\theta})$$

Note:

① Scalar is sph. Ten. of rank $\phi \Rightarrow T_{00} = a$

② Vector w components A_1, A_2, A_3 can be a spherical tens. of rank 1 via

$$T_{10} = A_3, \quad T_{1\pm1} = \mp\frac{1}{\sqrt{2}}(A_x - iA_y)$$

5 4/1/24

(3) $T_{n\mu}$ can be quantum States or operators

leads to (no proof) <u>Wigner-Eckart Thm</u>

Matrix element of $T_{n\mu}$:

$$\langle \sigma j' m' | \hat{T}_{n\mu} | \sigma j m \rangle = \langle \sigma j' \| \hat{T}_n \| \sigma j \rangle \times \left[\frac{\langle j n m \mu | j' m' \rangle}{\sqrt{2j'+1}} (-1)^{2n} \right]$$

other quant. #'s

C.G.

Can lead to selection rules in γ or β decay if C.G $= \phi$

<u>Ex</u>: spin $\frac{1}{2}$ particle cannot have Quad. Moment

$$\hat{Q} = e(3z^2 - r^2) = \left(\frac{16\pi}{5}\right)^{1/2} e\, Y_{20}$$

(see HW)

<u>N.R. Scattering</u>

Consider upolarized, elastic scattering

$$\psi_{tot}(\vec{r}) = \underbrace{e^{ikz}}_{\text{incident plane wave}} + \underbrace{f(\theta)}_{\text{scattering amplitude}} \frac{e^{ikr}}{r}$$

scattered wave

Due to finite range of NN force [($r_0 \sim$ few fm) $\Rightarrow V = 0$ if $r > r_0$]

partial wave expansion of $f(\theta)$ is <u>very</u> <u>useful</u>

Semi-classically w $|\vec{L}_{max}| \simeq r_0 P$
$$\simeq \sqrt{\ell(\ell+1)}\,\hbar$$
$$\ell_{max} \simeq \frac{r_0 P}{\hbar} = k r_0$$
$\therefore$ if $k r_0$ is small, only few ℓ values needed for $f(\theta)$

see link to 3D scattering

phase shift

$$\Leftrightarrow \quad f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty} Y_{\ell 0}(\theta)\sqrt{4\pi(2\ell+1)}\, e^{i\delta_\ell}\sin\delta_\ell$$

Note:

1. Since $\frac{d\sigma}{d\Omega} \equiv |f(\theta)|^2$
$$\sigma_{tot} = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$$
since cross terms vanish b/c $Y_{\ell m}$ are orthog.

2. δ_ℓ completely specifies elastic scattering w $\delta_\ell > 0$ for attractive V
$\delta_\ell < 0$ " repulsive "

3. Since $f(0) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)(\cos\delta_\ell + i\sin\delta_\ell)\sin\delta_\ell$

then $\underbrace{\sigma_{tot} = \frac{4\pi}{k}\,\mathrm{Im}\, f(0)}_{}$

special case of Optical Thm

Ph 203: Lect 2

⇒ Finished w/ Review now "Real" NP... Consider:

Deuteron : simplest nucleus

Properties:

I. Binding Energy: 2.22 MeV = E_b ← slow n capture on p
⇒ only 1 bound state ($\frac{E_b}{V_0} \sim 5\%$!)
↳ see HW 2

II. Intrinsic Spin/Parity: $J^{\pi} = 1^{+}$

III. Isospin: $t = 0$, $t_3 = 0$

IV. Mag. Moment $\mu_d = 0.857\,\mu_N$
↳ μ_N = nuclear magneton
$\equiv \frac{e\hbar}{2m_p c}$ (Dirac moment)
$\ll \mu_B$

⇒ note $\mu_p = 2.79\,\mu_N$; $\mu_n = -1.91\,\mu_N$

V. Quadrupole Moment: $Q_d = \underline{0.286 \pm .002}\ e\,fm^2$
recall $\hat{Q} = e(3z^2 - r^2)$

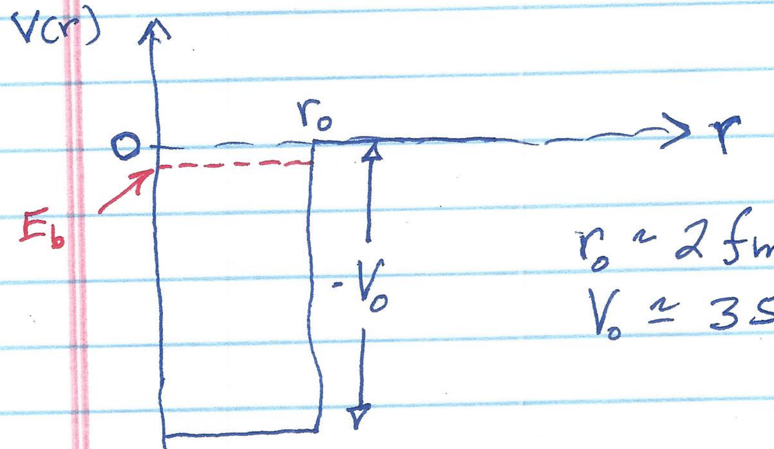
$\boxed{fm = 10^{-15} m}$

VI. Charge Radius $r_d^{\delta} = 2.128 \pm 0.001$ fm
↳ note $r_p^{\delta} = 0.842 \pm 0.001$ fm (Maybe!) see later
$r_n^{\delta} = -0.106 \pm 0.006$ fm

Discuss each of above...

I. $E_b = 2.22$ MeV ≃ γ-ray energy from $n + p \rightarrow d + \gamma$ (slow n)
NP lingo:
↳ $p(n,\gamma)d$
↑ target ↑ beam ↑ detect

Can reproduce this via finite Square Well (HW 2).



$r_0 \sim 2\,fm$
$V_0 \simeq 35\,MeV$

II $J^\pi = 1^+ \Rightarrow$ consistent w 2 spin $\frac{1}{2}$ particles coupled to $J=1$ in $l=0, 2, \ldots$ state (see later $\Rightarrow$ no $l=1$)

$J^\pi_{n,p} = \frac{1}{2}^+$ & $l = 0, 2 \Rightarrow \pi = +$

III $|t\,t_3\rangle = |00\rangle$ why not $|10\rangle$?

$t_3 = t_3^p + t_3^n = 0$

Assuming d is composed of 2 "identical" fermions

$$\psi_d = \psi_{space} \times \psi_{spin} \times \psi_{isospin}$$

need ψ_d anti sym. under $n \leftrightarrow p$ exchange

$\therefore$ if ψ_{space} is $l = 0, 2 \Rightarrow$ sym!

ψ_{spin} is triplet $(S=1) \Rightarrow$ sym!

$\therefore$ $\psi_{isospin}$ is singlet $t=0 \Rightarrow$ antisym.!

IV Mag. Moment $\Rightarrow$ due to $\mu_n + \mu_p$ + orbital motion of p

thus if d is pure $l=0$ $\mu_d = \mu_n + \mu_p$

$= 0.880\,\mu_N$

③ 4/3/24

& $\mu_d^{exp} = 0.857\,\mu_N$ ⇒ ~3% difference → Why?

Ⅵ) Quad Moment (deduced from hyperfine splitting in HD & D_2)

$$Q_d \equiv \langle JM=J|\hat{Q}|JM=J\rangle$$

↳ likewise for μ_d

$Q_d > 0$ requires non-spherical → spin can't do this

∴ need $l > 0$ ⇒ $l=1$ has wrong parity ∴

try $\psi_d = a\varphi_s + b\varphi_D$

w $\varphi_s = R_s(r)\,\mathcal{Y}^1_{110}$; $\varphi_D = R_D(r)\,\mathcal{Y}^1_{112}$

↑ space (θ,φ) & spin via

$$\mathcal{Y}^{M_J}_{JSL}(\theta,\varphi) = |JM\rangle_{LS} = \sum_{m_L m_S} Y_{Lm_L}(\theta,\varphi)(C.G.)|S m_S\rangle$$

then $\mathcal{Y}^1_{110} = Y_{00}|11\rangle$; $\mathcal{Y}^1_{112}$ = HW

then if $|b|^2 \simeq 4\%$

can get $Q_d^{thy} = Q_d^{exp} = 0.286\ e\cdot fm^2$

Note!

① Pure D-state $|b|^2=1$ gives $Q_d < 0$ oblate $(3z^2 - r^2)$

② Why D-state in gnd state? ⇒ "Tensor" Force = next week

Info on N-N Force: from low E N-N scattering

@ low Energy only $l=0$ important

How low?

④ 4/3/24

$\Rightarrow$ need $kr_0 << 1$, $r_0 \sim 2\,fm$, $E_{cm} = \frac{\hbar^2 k^2}{2\left(\frac{M_N}{2}\right)}$ $\quad M_N = \frac{m_n + m_p}{2}$

$\therefore E_{cm} << 10\ MeV$ $\quad$ note $\frac{m_n}{m_p} \doteq 1.001$

$l=0$ Scattering

$$f_{l=0} = \frac{e^{i\delta_0}\sin\delta_0}{k} \; ; \quad \sigma_0^{tot} = \frac{4\pi}{k^2}\sin^2\delta_0$$

Expect $\sigma_{tot}(k\to 0) \simeq$ const. (not $=\infty$)

$\hookrightarrow \therefore \delta_0 \propto k$ as $k \to 0$

$$\& \lim_{k\to 0} \sigma^{TOT} = 4\pi a^2$$

$\hookrightarrow$ scattering length

w/ $f_0 = \dfrac{\sin\delta_0}{k(\cos\delta_0 - i\sin\delta_0)}$

define $a \equiv \lim_{k\to 0}\left(\frac{-1}{k\cot\delta_0}\right) = -\lim_{k\to 0}(f_0)$

$\hookrightarrow$ since $\sin\delta_0 << \cos\delta_0$ for $k\to 0$

Pics of a $\leftarrow$

see next page

Note

① For attractive potential

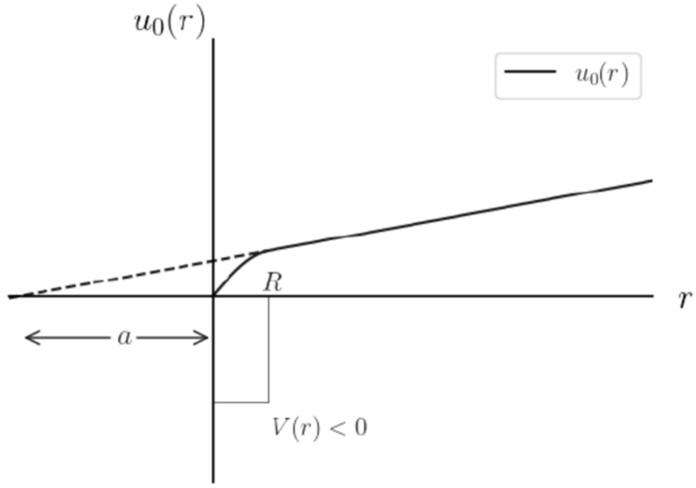
if $a<0 \Rightarrow$ no bound states

if $a>0 \Rightarrow$ bound states possible

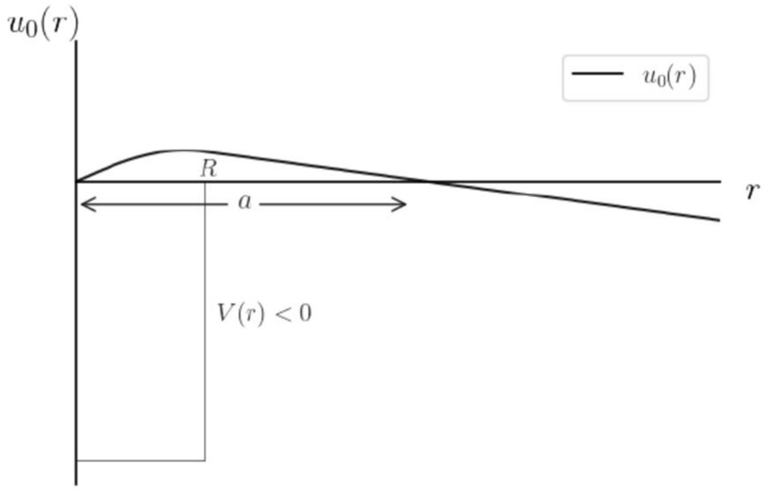
② For $k \gtrsim 0$, but still dominated by $l=0$, can expand $k\cot\delta_0$ in powers of k:

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}r_e k^2 + \ldots$$

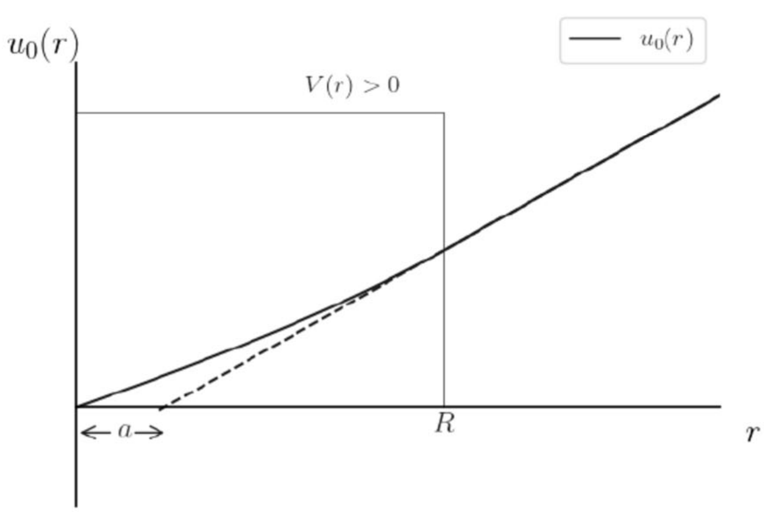
$\hookrightarrow$ called effective range for localized potential



(a) An attractive potential that is not strong enough to produce a bound state. In this case, $a < 0$ because we need to extrapolate the radial function to negative values to intercept the r-axis.



(b) A stronger attractive potential produces a bound state, and $a > 0$.



(c) For a repulsive potential, we always have $a > 0$.

4/3/24

Examples:

Gaussian: $V = -V_G\, e^{-r^2/r_G}$

Exponential: $V = -V_E\, e^{-r/r_E}$

Yukawa: $V = -V_Y \dfrac{e^{-r/r_Y}}{r/r_Y}$ → motivated by meson exchange (e.g. $\pi^{\pm}_{0}$)

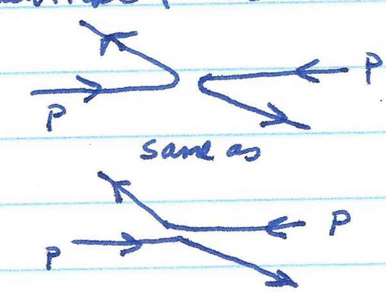
"a" from low E N-N Scattering

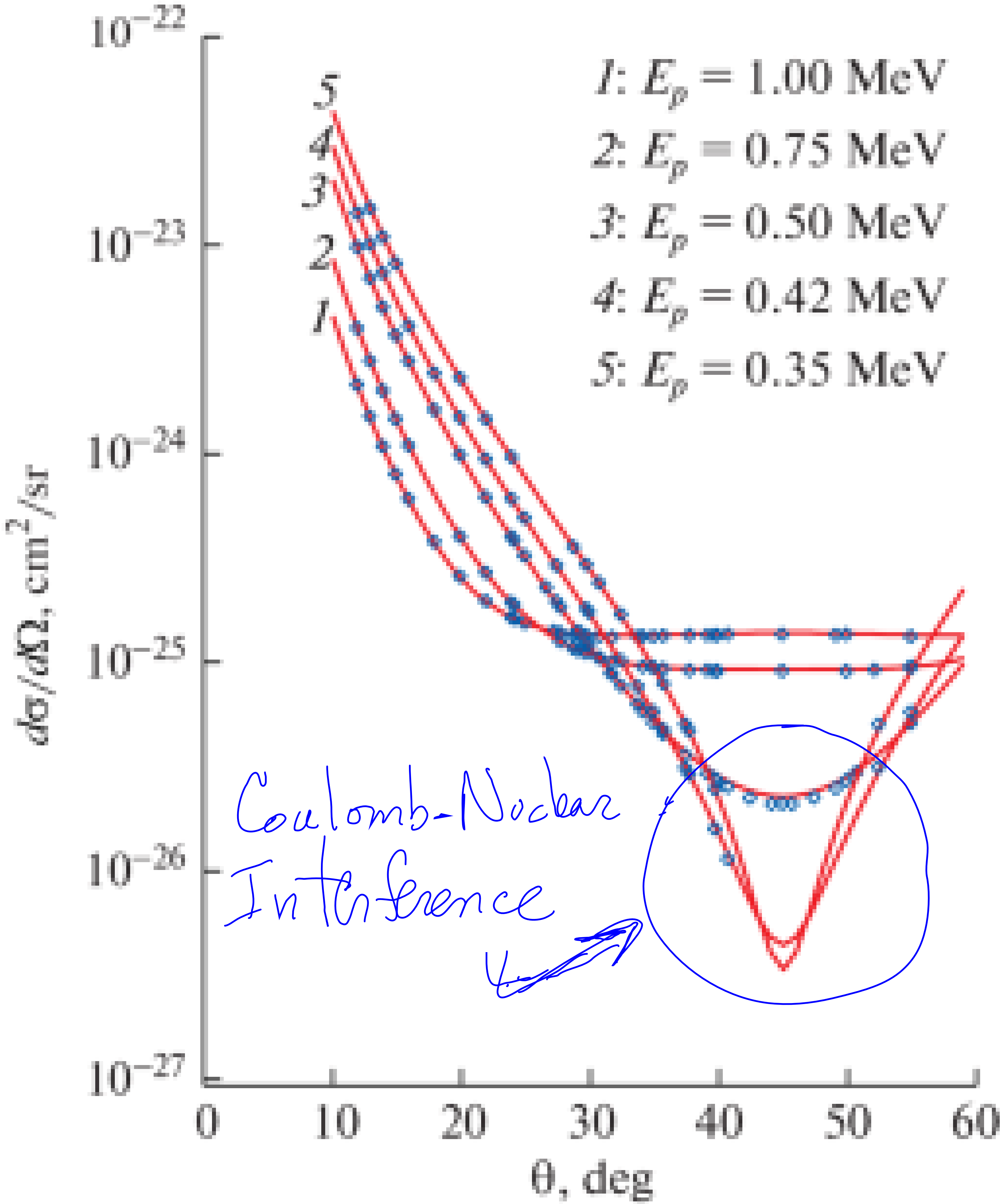
① p p scattering
for $l=0$, proton spins must be in singlet state ($^1S_0$), since $t = t_3 = 1$
(antisym.)
sym.

Coulomb interaction dominates for $E \lesssim 0.2$ MeV
Data for 0.3 MeV → 1 MeV

see next page

Note: $\frac{d\sigma}{d\Omega}$ must be sym. wrt 90° since identical particles





10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
$d\sigma/d\Omega$, cm^2/sr
0
10
20
30
40
50
60
θ, deg
1: E_p = 1.00 MeV
2: E_p = 0.75 MeV
3: E_p = 0.50 MeV
4: E_p = 0.42 MeV
5: E_p = 0.35 MeV
5
4
3
2
1
Coulomb-Nuclear Interference

Extract a_{pp} from Coulomb-Nuclear interference
also gives sign

w EM corrections:

$\underline{a^N_{pp} = -17.1 \pm 0.2 \text{ fm}}$, also $\underline{r_e^{pp} = 2.794 \pm 0.015 \text{ fm}}$

implies no bound state for attract. V

(2) nn Scattering

→ Again only $^1S_0$ possible

→ EM corrections are small @ low E
(only $\vec{\mu}_n$ dipole-dipole)

→ Use indirect exps. via nn final states

e.g. $n + d \to n n p$
or
$\pi^- + d \to n n \gamma$

} p/γ gives info on nn distrib.
↳ $\frac{d\sigma_{nn}}{d\Omega}$

Results: $\underline{a_{nn} = -18.7 \pm 0.6 \text{ fm}}$

& $\underline{r_e^{nn} = 2.84 \pm 0.03 \text{ fm}}$

∴ $a^N_{pp} \simeq a_{nn}$ ∴ N-N interaction is "charge" symmetric

↳ small charge-symmetry breaking also possible

This is consistent w isospin symmetry but isospin symmetry also requires charge independence

⟺ $a_{nn} = a^N_{pp} = a_{pn}^{^1S_0}$

↳ see Next Time