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Ph 203 L3

Last time:

pp: 1S_0 , $a_{pp}^N = -17.1 \pm 0.2 \text{ fm}$, $r_e = 2.79 \pm 0.02 \text{ fm}$

nn: 1S_0 , $a_{nn}^N = -18.7 \pm 0.6 \text{ fm}$, $r_e = 2.84 \pm 0.03 \text{ fm}$

Info from Low E n-p Scattering

→ EM corrections are modest

→ Scattering can occur in both 1S_0 or 3S_1 ,

since $t=0$ or 1 are possible

↳ thus could be different "a" for $s=0$ (singlet)

" a_s " & $s=1$ (triplet) " a_t ":

∴ for $l=0$ can define 4×4 matrix for f_0 :

$$f_0 = - \begin{pmatrix} |11\rangle & |10\rangle & |1-1\rangle & |00\rangle \\ \hline a_t & & \phi & \\ & a_t & & \\ \phi & & a_t & \\ & & & a_s \end{pmatrix} \begin{matrix} |11\rangle \\ |10\rangle \\ |1-1\rangle \\ |00\rangle \end{matrix}$$

⇒ matrix must be diagonal since J^2 & J_z are conserved

⇒ Can't depend on S_z ($\hat{H} \rightarrow$ Hamiltonian \Rightarrow is invariant under rotations)

⇒ but $a_s \neq a_t$ is possible

then for unpolarized np scattering expect an incoherent sum over spin states

$$\sigma_{np}(E \rightarrow 0) = 4\pi \left(\frac{3a_t^2 + a_s^2}{4} \right)$$

↳ probabilities add, not

amplitudes

Exp $\sigma_{np}(E \rightarrow 0) = 20.44 \pm 0.23 \text{ b}$

↳ barn = 10^{-28} m^2
= 100 fm^2

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To separately determine a_s & a_t can scatter "cold" neutrons $\lambda_n \approx 10 \text{ \AA}$ from H_2 molecules
 \hookrightarrow note "thermal" neutrons $\lambda \sim 1 \text{ \AA}$
 MB dist. @ 300K

But... use para- H_2 (recall para-H $\uparrow\downarrow$ pp gnd state
 ortho-H $\uparrow\uparrow$ l=1 state ortho)

then

Both protons contribute coherently (amplitudes add)

$\therefore \sigma_{H_2}^{para} = 4\pi \left(\frac{3a_t + a_s}{4} \right)^2 ; \sigma_{H_2}^{para} = 1.73 \pm 0.01 \text{ b}$

$\therefore 3a_t = -a_s$, but which is negative?

\hookrightarrow since deuteron is bound, need $a_t > 0$

$\therefore a_t^N = 5.423 \pm 0.005 \text{ fm} ; r_e^t = 1.73 \pm 0.02 \text{ fm}$

$a_s^N = -18.5 \pm 1.5 \text{ fm} ; r_e^s = 2.73 \pm 0.03 \text{ fm}$

\hookrightarrow consistent w a_{nn}, a_{pp}^N

\hookrightarrow suggest charge independence of NN force

Conclusions on N-N interaction

1. $[\hat{H}, \hat{T}_3] = 0 \Rightarrow$ isospin symmetry
2. Is spin and/or isospin dependent $a_s \neq a_t$
3. 2 nucleons are unbound in 1S_0 , but bound in 3S_1 , but something's missing \Rightarrow how to get (np)
 $Q_d > 0 ??$

\hookrightarrow must have $V(\vec{r})$ non-central: $V(\vec{r}) \neq V(r)$

\hookrightarrow Consider "Tensor" Force: $V_{\text{tensor}} = V_T(r) \hat{S}_{12}$

Note:
 $a_s^{N+EM} = -23.72 \pm 0.02 \text{ fm}$

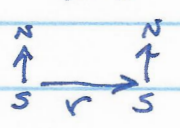
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with $\hat{S}_{12} = \frac{3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$

this is scalar product of 2 rank 2 tensors (see HW 3)

↳ Example of "Tensor" force: 2 magnetic dipoles

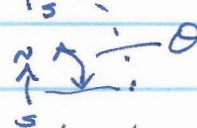
$\theta = 0$



↑^N ↑^N
↓^S ↓^S
r

repulsive

$\theta = 90^\circ$



↑^N ↓^N
↓^S ↓^S
r

attractive

What other terms in N-N Force are allowed/required?
Consider energy dependence of phase shifts

↳ see Next page

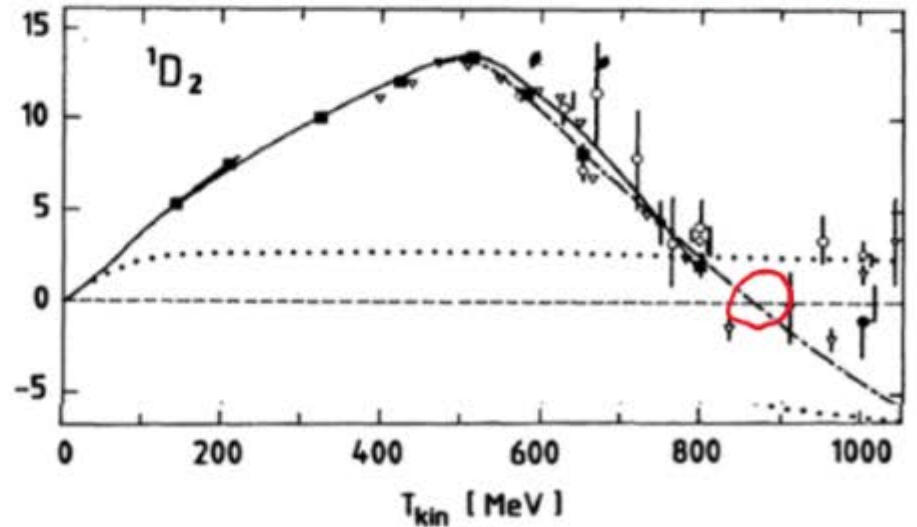
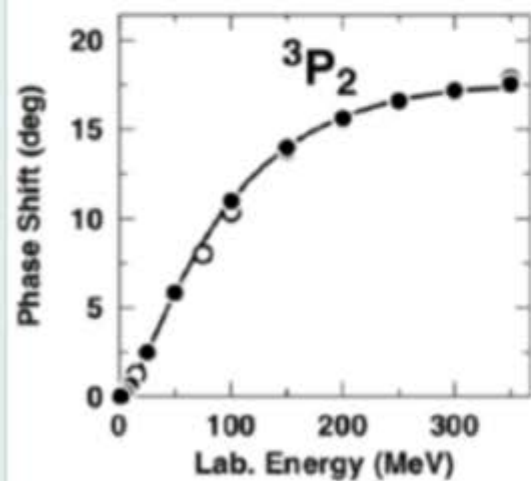
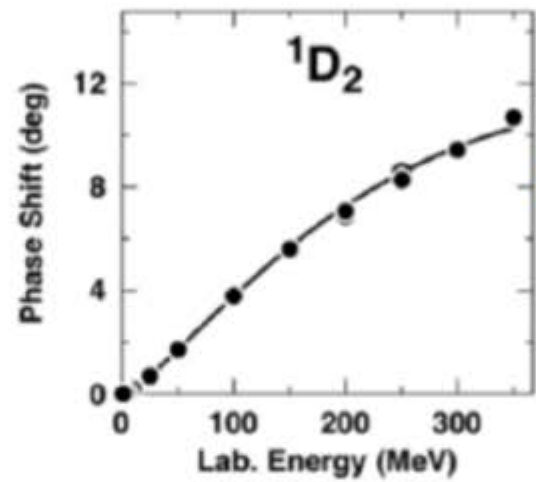
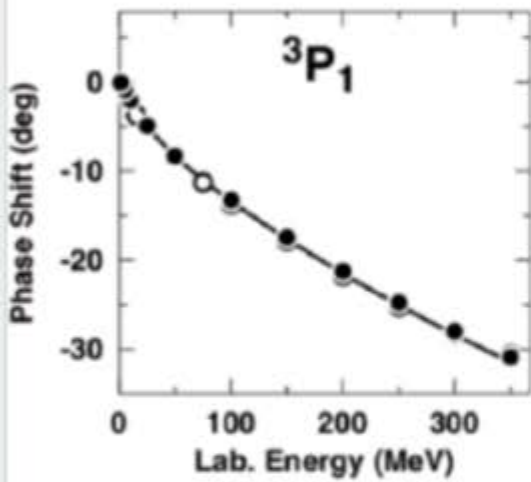
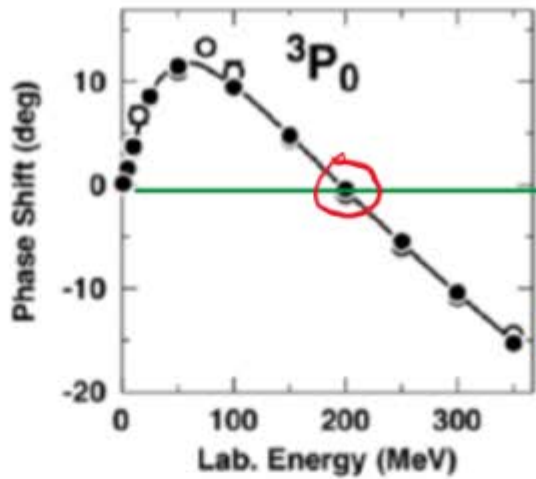
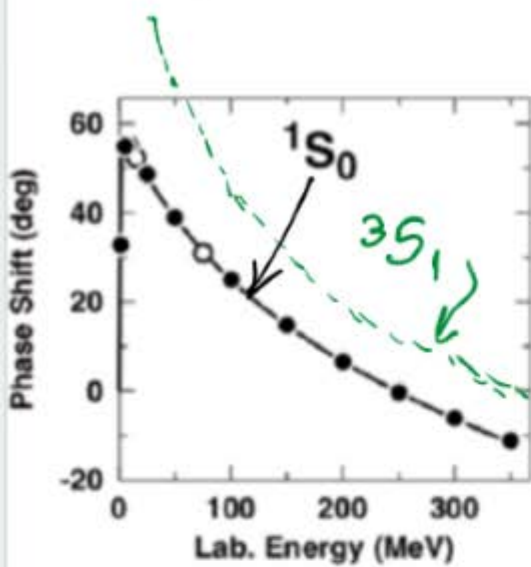
n-p Phase Shifts

Notes:

1. Large energy scale obscures $k \rightarrow 0$ limit for $L = 0$, since need $\delta \rightarrow 0$ as $k \rightarrow 0$.

2. Both 3P_0 and 1D_2 δ go negative at high energy \rightarrow repulsive core.

3. ${}^3S_1 > {}^1S_0$ consistent with bound deuteron, but unbound di-neutron.



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Summary of Required terms in V_{NN} (2-nucleon interaction)

1. Nuclear Force (N.F.) is spin +/or isospin dependent

$$\hookrightarrow a_s \neq a_t$$
$$\begin{matrix} {}^1S_0 & {}^3S_1 \\ T=1 & T=0 \end{matrix}$$

2. N.F. has "Tensor" component ($Q_d \neq 0$)

3. N.F. is short range ($r \lesssim 2 \text{ fm}$)

4. N.F. is attractive @ $r \approx 1-2 \text{ fm}$

why: a) 1S_0 & 3S_1 have $S > 0$
b) Deuteron is bound

5. N.F. has repulsive core

$$p \approx 650 \text{ MeV}/c$$

why? a) 3P_0 d_x becomes < 0 for $E_k > 200 \text{ MeV}$

$$|\vec{L}| = \sqrt{L(L+1)} \hbar = \sqrt{2} \hbar = R_c p \Rightarrow R_c \approx 0.4 \text{ fm}$$

b) 1D_2 $S_x < 0$ for $E > 850 \text{ MeV}$

$$|\vec{L}| = \sqrt{6} \hbar = R_c p \quad \hookrightarrow p = 1500 \text{ MeV}/c$$

$$\hookrightarrow R_c \approx 0.3 \text{ fm}$$

6. N.F. has spin-orbit component

why? a) 3P phase-shifts depend on $|\vec{J}| = |\vec{L} + \vec{S}|$

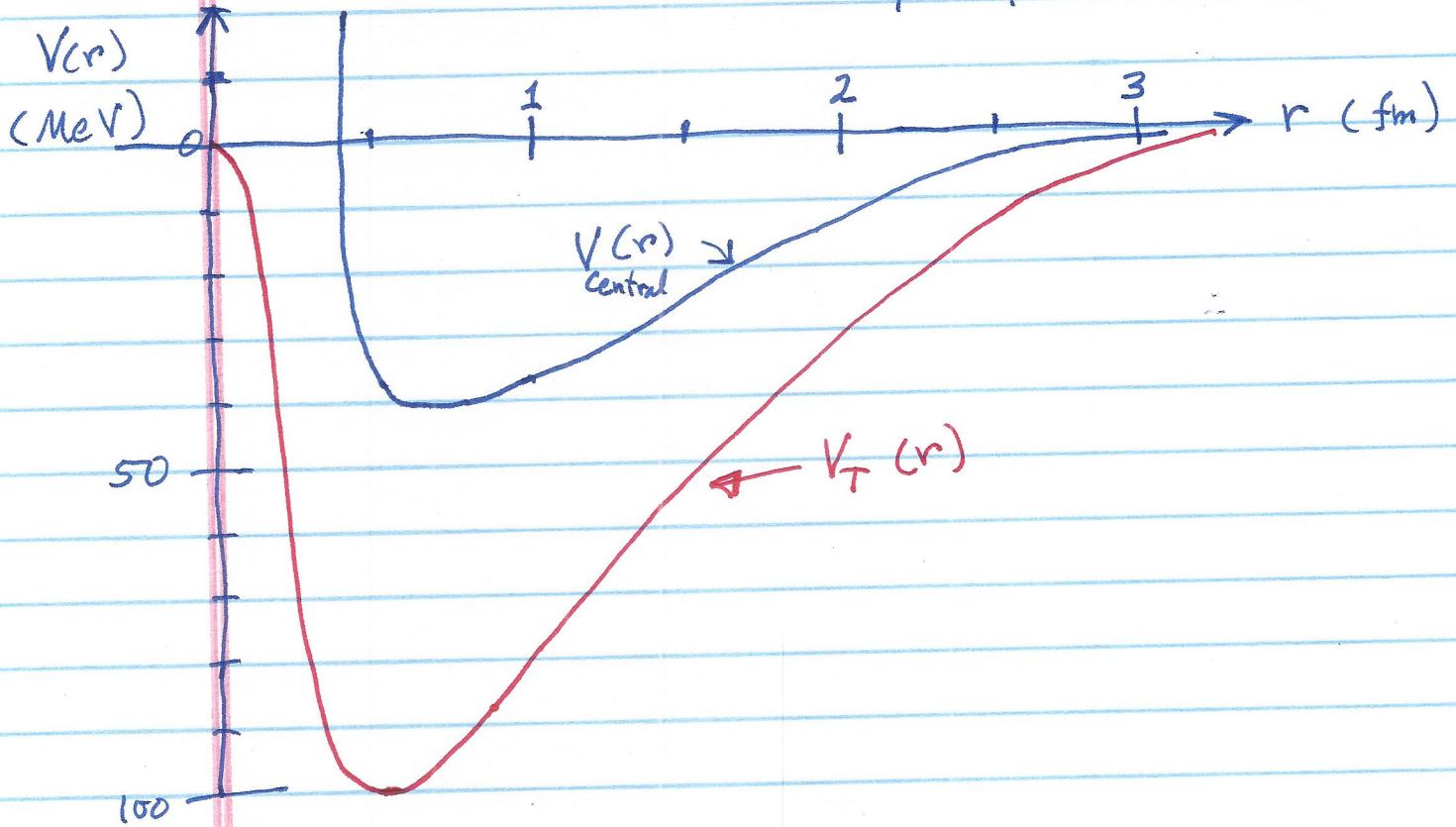
$$\hookrightarrow \vec{L} \cdot \vec{S} = \frac{1}{2} (J^2 - L^2 - S^2)$$

b) "Discovered" by Marie-Goeppert Mayer to explain "Magic" nuclei

(see next week)

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Realistic n-p Potential to explain Deuteron properties
& n-p Phase Shifts.



Given what the data requires for N-N Force, we can ask, given the symmetries & Invariance of Strong Int. what operators are possible?

Assumptions: (Basic phys. + exp)

(I) \hat{H} = Hamiltonian is invariant w.r.t. rotations
e.g. $\rightarrow J$ is conserved
∴ can't have S_z .

(II) \hat{H} invariant w.r.t. exchange of identical particles

III \hat{H} invariant w.r.t. Parity: $\hat{P} \vec{r} \rightarrow -\vec{r}$
 then since \hat{H} has \vec{p}^2 terms (even under parity)
 all terms must be even under parity

Recall: $\hat{P} \vec{p} = -\vec{p}$
 $\hat{P} \vec{\sigma} = +\vec{\sigma}$
 $\hat{P} \vec{L} = +\vec{L}$ } Axial Vectors

$\vec{\sigma}_1 \cdot \vec{r}$ not permitted
 (P-odd)

IV \hat{H} appears to be charge independent

but this allows $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, $(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{L}_1 \cdot \vec{L}_2)$
 but $\vec{\sigma}_1 \cdot \vec{\sigma}_1$, $\vec{\sigma}_1 \cdot \vec{r}$ are meaningless

V Velocity-dependent terms needed
 e.g. $\vec{L} \cdot \vec{S}$; $\vec{S} = \frac{\hbar}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$

other terms are possible:
 $(\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})$; $\vec{p} = \vec{p}_1 - \vec{p}_2$
 $(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L})$ usually ignored

Next time: most general V_{NN}
 consistent w symmetries, invariance & data.