SP2

$\vec{S} = \dfrac{\vec{\sigma}_1 + \sigma_2}{2}$ $\qquad$ $\vec{T} = \dfrac{\vec{\tau}_1 + \vec{\tau}_2}{2}$

~~8~~. For a velocity-independent two-body potential, the only two-body scalars that can be formed using operators $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $\boldsymbol{S} = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2$ and $\boldsymbol{T} = (\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)/2$ are r, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ and S_{12}, where $S_{12} = 3(\boldsymbol{r} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{r} \cdot \boldsymbol{\sigma}_2)/r^2 - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$. Show that the operators

(a) $\boldsymbol{S} \cdot \boldsymbol{S}$ $\qquad$ (b) $(\boldsymbol{r} \cdot \boldsymbol{S})^2$

(c) $(\boldsymbol{r} \times \boldsymbol{S}) \cdot (\boldsymbol{r} \times \boldsymbol{S})$ $\qquad$ (d) $(\boldsymbol{r} \times (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)) \cdot (\boldsymbol{r} \times (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$

can be reduced to functions of these scalars. Give the symmetry argument of why scalar products $\boldsymbol{r} \cdot \boldsymbol{S}$ and $\boldsymbol{r} \cdot \boldsymbol{T}$ are not allowed for a nuclear potential.

With velocity or momentum dependence, the only additional operator required is $\boldsymbol{L} \cdot \boldsymbol{S}$, where $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ and $\boldsymbol{p} = \frac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2)$. Show that the following terms do not form independent scalars either:

(e) $\boldsymbol{r} \times \boldsymbol{L} \cdot \boldsymbol{p}$ $\qquad$ (f) $(\boldsymbol{L} \cdot \boldsymbol{S})(\boldsymbol{L} \cdot \boldsymbol{L})$

(g) $(\boldsymbol{r} \cdot \boldsymbol{p})(\boldsymbol{r} \cdot \boldsymbol{S})$ $\qquad$ (h) $(\boldsymbol{r} \cdot \boldsymbol{p})(\boldsymbol{L} \cdot \boldsymbol{S})$