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1

Ph203:

last time: what's Q# k?

Consider operator $\hat{K} = \gamma_0 (1 + \hat{\vec{\Sigma}} \cdot \hat{\vec{L}})$ 4x4!

$$\hat{\vec{\Sigma}} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

easy to show $[\hat{K}, H_{Dirac}] = 0$, $\hat{K}\psi_D = -k\,\psi_D$, $k = \pm(j+\frac{1}{2})$

→ see Thad. Ch. 2.4 & Exercise 2.6

but while $[\hat{L}^2, \hat{H}_{Schrod.}] = 0$ since $[\hat{L}^2, \hat{p}^2] = 0$

$[\hat{L}^2, \hat{H}_D] \neq 0$ since $[\hat{L}^2, p_i] \neq 0$

thus for H_D rotational Quan. #'s are $\hat{J}^2, \hat{S}^2, \hat{K}$

but not $\hat{L}^2$

however $\hat{L}^2 \phi = l(l+1)\phi$

$\hat{L}^2 \chi = l'(l'+1)\chi$

Note for $H_{Schröd}$

define $\hat{K}_{NR} = \hat{\vec{\sigma}} \cdot \hat{\vec{L}} + 1 = \hat{J}^2 - \hat{L}^2 - \hat{S}^2 + 1$

then for simul. eigenstate of $\hat{L}^2, \hat{S}^2, \hat{J}^2 \Rightarrow |l s j m_j\rangle$

$$\hat{K}|l s j m_j\rangle = [j(j+1) - l(l+1) - s(s+1) + 1]\,|l s j m_j\rangle$$
$$= k\,|l s j m_j\rangle$$

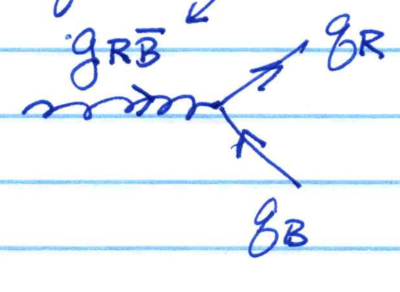
∴ k is redundant in NRQM

QCD Overview

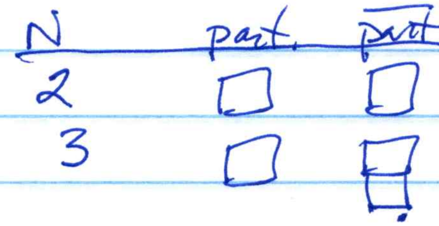
Gluons: Since $SU(3)_c = \begin{pmatrix} R \\ G \\ B \end{pmatrix}$ = 3 color charges,

need color states (W.F.) for gluons

assume gluon quark interaction needs:

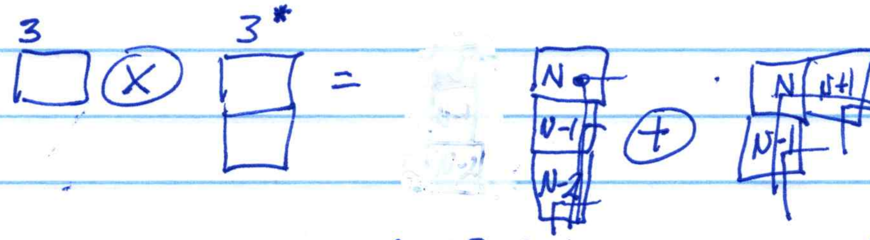


Build gluon states via Young Tableaux:
For particle/antipart. states use "conjugate" rep. = vertical column w N-1 boxes for $\bar{p}$ in SU(N)



⋮

Thus to build $c\bar{c}$ states of gluons (e.g. $R\bar{B}$, ...) in 3 colors [SU(3)]:



$n_D = 3\times2\times1$ (hooks) $\qquad n_D = 3\times1\times1$

$$\Leftrightarrow \frac{N(N-1)(N-2)}{3\times2\times1} \oplus \frac{3\times2\times4}{3}$$

states $\quad 6 \qquad\qquad 3$

$$\Leftrightarrow \underset{\sim}{1} \oplus \underset{\sim}{8}$$

no net color
no short range force
(only dipole-dipole)
(but OK for Mesons!)

Octet of gluons!

where

$$\underset{\sim}{1} = \frac{R\bar{R}+G\bar{G}+B\bar{B}}{\sqrt{3}}$$

Not! gluons

↑ colorless

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<u>Gluons</u>: $\bar{R}B, \bar{B}R, \bar{R}G, \bar{G}R, \bar{B}G, \bar{G}B, \frac{\bar{R}R + \bar{B}B - 2\bar{G}G}{\sqrt{6}}, \frac{\bar{R}R - \bar{B}B}{\sqrt{2}}$

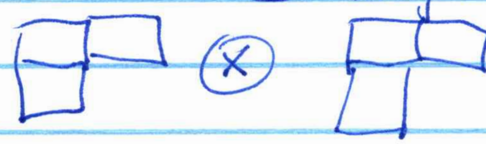
all w color & linearly indep.

Glueballs:

Consider state of $g+g$ formed from octet of colors:

see Lecture 10 Young Tab.

$\underset{\sim}{8} \otimes \underset{\sim}{8} =$



$= \underset{\sim}{27} \oplus \underset{\sim}{10} \oplus \underset{\sim}{\overline{10}} \oplus \underset{\sim}{8} \oplus \underset{\sim}{8} \oplus \underset{\sim}{1} = 64$ ✓

singlet colorless

recent evidence for 0^- glueball:

$M_{gg} = 2395 \pm 70$ MeV/c^2

$\Gamma_{gg} \simeq 190$ MeV

@ BEPC II using BES III detector

Lattice QCD (see next time) predicts (PRL 2019)

$J/\psi \to \gamma G_{0^-}$ w $M_{G_{0^-}} = 2395(14)$ MeV/c^2

Consider Exp. PICS: $= 2395(14)$ MeV/c^2

Glueballs

Determination of Spin-Parity Quantum Numbers of $X(2370)$ as 0^{-+} from $J/\psi \to \gamma K_S^0 K_S^0 \eta'$

M. Ablikim *et al.*[*]
(BESIII Collaboration)

(Received 8 December 2023; revised 5 March 2024; accepted 28 March 2024; published 2 May 2024)

Based on $(10087 \pm 44) \times 10^6$ J/ψ events collected with the BESIII detector, a partial wave analysis of the decay $J/\psi \to \gamma K_S^0 K_S^0 \eta'$ is performed. The mass and width of the $X(2370)$ are measured to be $2395 \pm 11(\text{stat})^{+26}_{-94}(\text{syst})$ MeV$/c^2$ and $188^{+18}_{-17}(\text{stat})^{+124}_{-33}(\text{syst})$ MeV, respectively. The corresponding product branching fraction is $\mathcal{B}[J/\psi \to \gamma X(2370)] \times \mathcal{B}[X(2370) \to f_0(980)\eta'] \times \mathcal{B}[f_0(980) \to K_S^0 K_S^0] = (1.31 \pm 0.22(\text{stat})^{+2.85}_{-0.84}(\text{syst})) \times 10^{-5}$. The statistical significance of the $X(2370)$ is greater than 11.7σ and the spin parity is determined to be 0^{-+} for the first time. The measured mass and spin parity of the $X(2370)$ are consistent with the predictions of the lightest pseudoscalar glueball.

DOI: 10.1103/PhysRevLett.132.181901

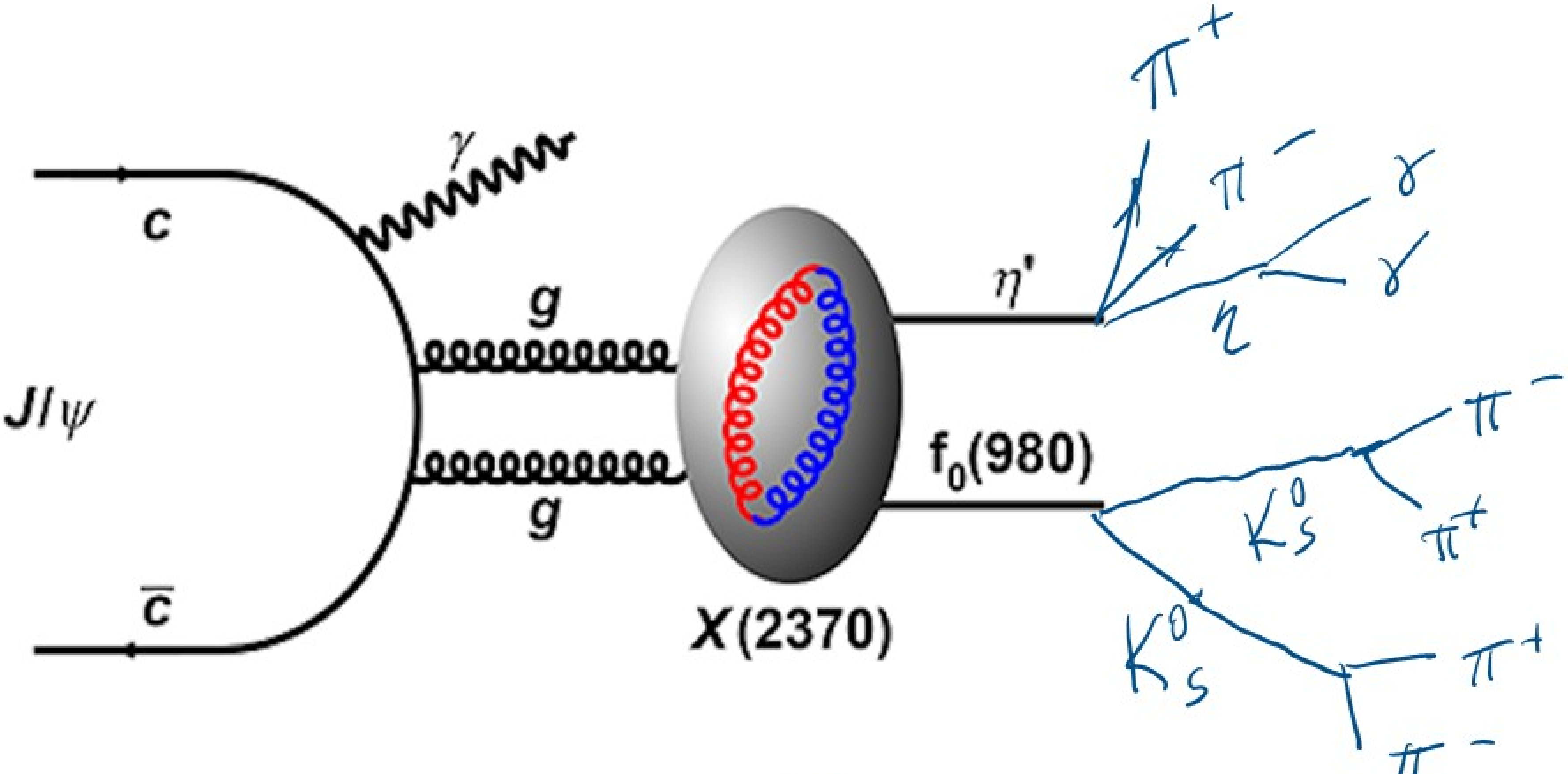
Note: Lattice QCD predicts

0^+ "Scalar" Glueball @ $M \simeq 1.9$ GeV

(some evidence of mixed state e.g. $gg\,q\bar{q}$

0^- "Pseudoscalar Glueball @ $M_{0^-} = 2.4$ GeV

The J/ψ system can decay to a photon and two gluons, where the two gluons can then combine to temporarily create an X(2370) exotic particle. Although its nature is still not 100% certain, the interpretation of the X(2370) as a glueball remains compelling, and if so, it would be the first glueball particle ever revealed by experiment.



Credit: Physical Review Letters/Twitter

e.g. Reconstruct:

$$M^2_{\pi^+\pi^-} = (E_{\pi_1} + E_{\pi_2})^2 - (\vec{P}_{\pi_1} + \vec{P}_{\pi})^2$$

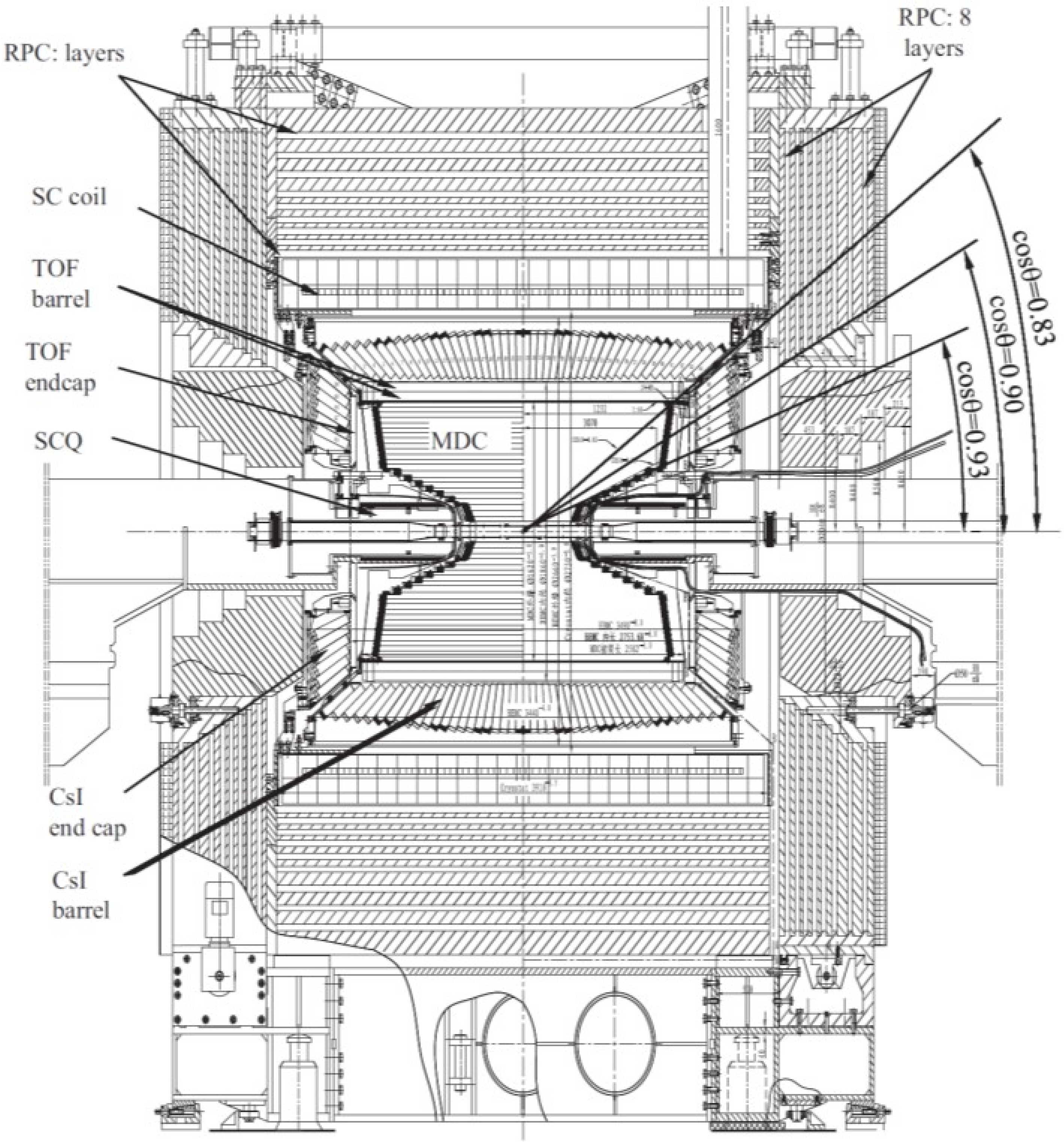


Fig. 1. Schematic drawing of the BESIII detector.

BESIII ~ 4π detector

- Tracks charged particles with Drift Chamber (MDC)
- Momentum via B-Field = 1 T Solenoid
- PID (Particle Identification) via CSI EM calorimeter & TOF (Time-Of-Flight) detectors
- CsI calorimeter also give total energy deposition
- RPC (Resistive Plate Chambers) measure muons that penetrate B-field coil

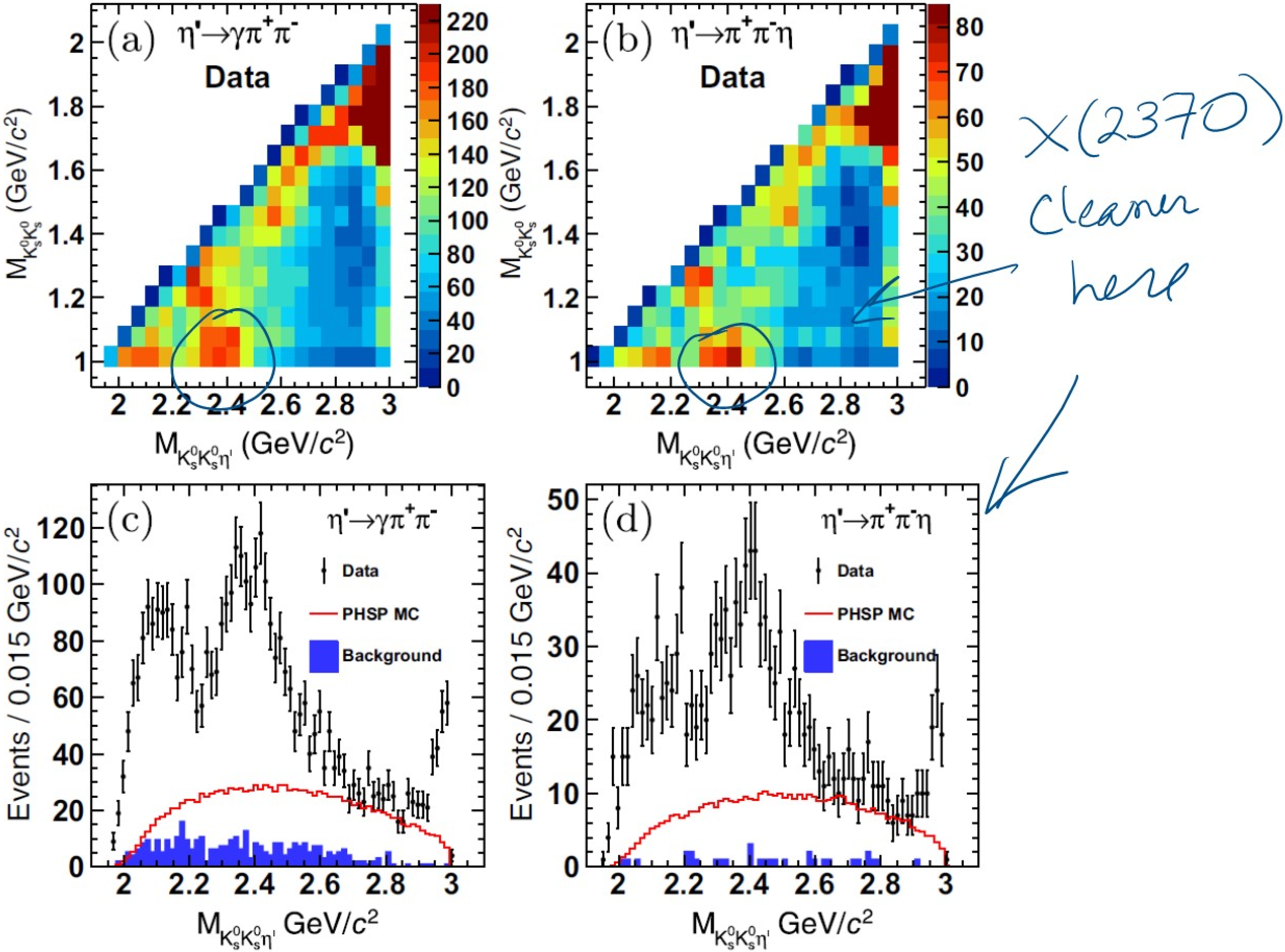
BEPCII Collider

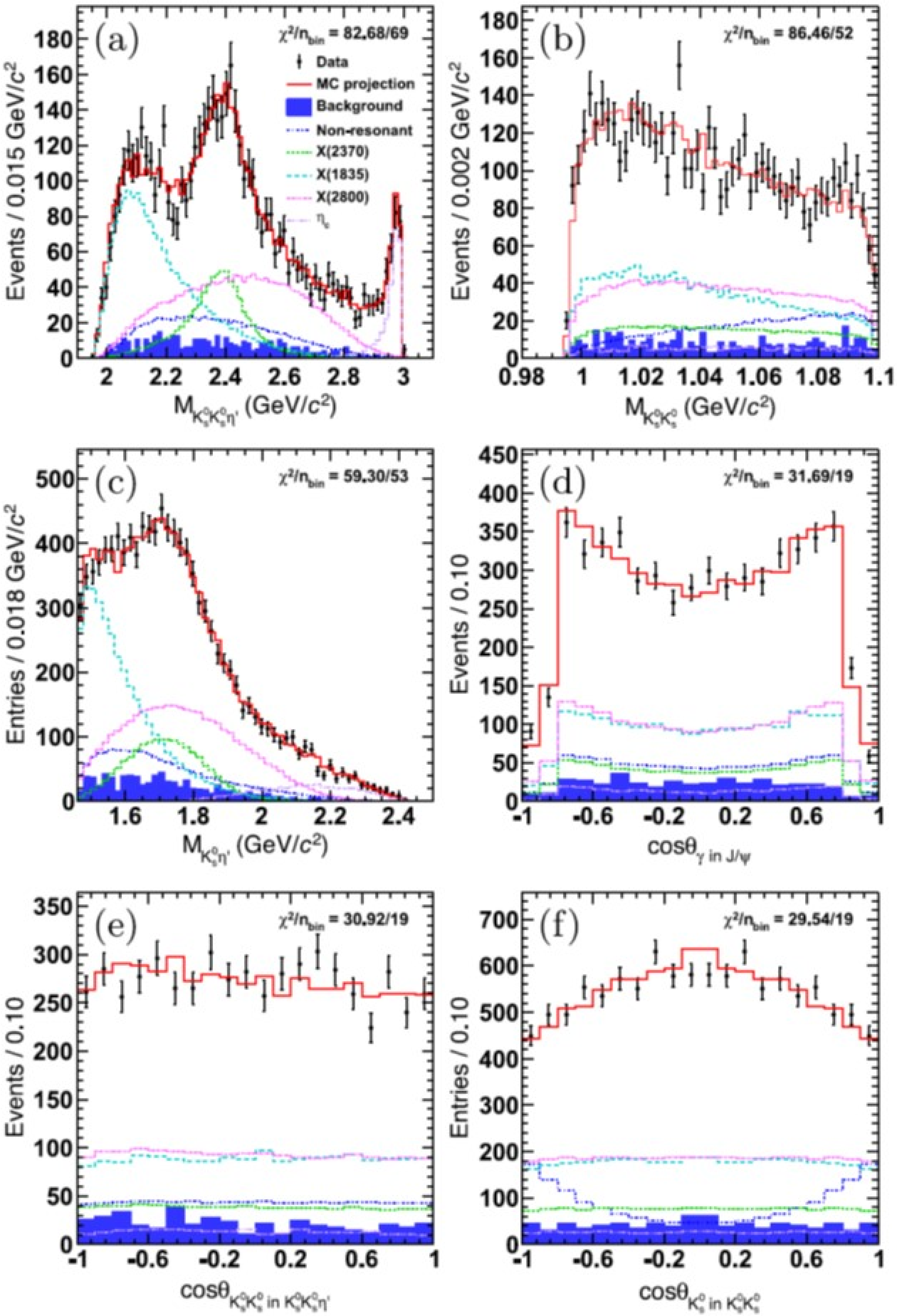
- e-/e+ collider at 2.4 GeV C of M
- "Charm Factory" since charm meson = charm-anticharm meson with q_charm mass = 1.3 GeV

Mesons as $q\bar{q}$ states

State	J^π	M(MeV)	τ/Γ	$c\tau$	$q\bar{q}$ content
$\pi^\pm$	0^-	139	$3\cdot 10^{-8}$ s	8m	$u\bar{d}$, $d\bar{u}$
K_s^0	0^-	498	$1\cdot 10^{-10}$ s	3cm	$d\bar{s}+s\bar{d}$
η	0^-	548	1.3 keV	—	$u\bar{u}\pm d\bar{d}$
η'	0^-	958	188 keV	—	"
f_0	0^+	980	50 MeV	—	$u\bar{u}$, $d\bar{d}$, $s\bar{s}$
J/ψ	1^-	3100	92 keV	—	$c\bar{c}$

↑ Vector Meson


(a) η'→γπ⁺π⁻
Data
(b) η'→π⁺π⁻η
Data
$M_{K_S^0K_S^0}$ (GeV/c^2)
$M_{K_S^0K_S^0\eta'}$ (GeV/c^2)
X(2370)
Cleaner here
(c) η'→γπ⁺π⁻
(d) η'→π⁺π⁻η
Data
PHSP MC
Background
Events / 0.015 GeV/c^2
$M_{K_S^0K_S^0\eta'}$ GeV/c^2



Partial Wave Analysis (PWA) for J^π

include "all" states & decay distrib.

Note:

$e^+ \rightarrow \quad \leftarrow e^-$ $\quad E_{e^+} + E_{e^-} = M_{J/\psi}$

$\Downarrow$

J/ψ

$\Downarrow$ $\quad \gamma \quad E_\gamma \approx 0.7\,\text{GeV}$

θ

$p \quad X$

$KE < 1\,\text{MeV}$

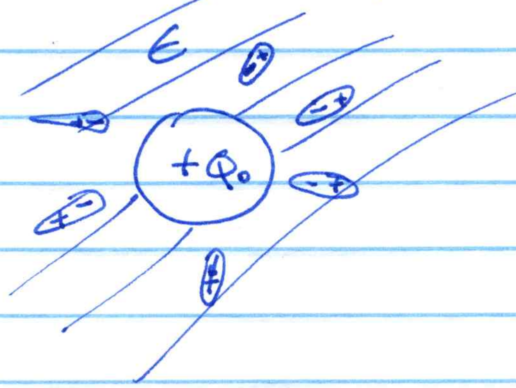
J^π influences decay θ

$J^\pi = 0^-$ to 10σ

Confinement: Key feature of QCD

Consider simple E & M analogy:

Classical point charge in hole of polarizable medium $\hookrightarrow \epsilon > 1$



Outside of hole $\vec{E}$-field is $\dfrac{Q_0}{\epsilon r}$ $\therefore$ charge is screened by material

$Q_{out} = \dfrac{Q}{\epsilon} < Q_0$

Note:

A similar effect occurs in QED due to polarizability of Vacuum $\Rightarrow$ vacuum contains $\gamma \to e^+e^-$ pairs that "screen" bare charge:

e.g. QED: 

$\Rightarrow$ screening

could be any charged fermion

observed charge $q < q_{bare}$ if probed @ large distance $\Rightarrow$ QED vacuum has $\epsilon_{vac}^{QED} > 1$

But for QCD, exp. says color charge is not directly observed $\Leftrightarrow$ confined

but gluon's have color charge (Non-Abelian) & can interact $\Rightarrow$ this gives $\epsilon_{vac}^{QCD} \ll 1$

$\therefore$ consider hadron (meson/baryon...) as hole in QCD Vacuum

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then if color change exist in hole effective charge outside is $Q^{QCD}_{out} = \frac{Q^{QCD}_0}{\epsilon} >> 1$

$\Rightarrow$ Gives <u>huge</u> long-range Strong Force (not observed!)

$\therefore$ Hole <u>must</u> have no net color charge & gluons/quarks confined inside hadrons

$\hookrightarrow$ suggests QCD is "anti-screening"

$\hookrightarrow$ see paper on web page

$\hookrightarrow$ Due to "β-function"

where $\beta = \left(\frac{\partial g}{\partial Q^2}\right)\Big|_{Q^2=M^2}$ renormalization scale needed to cancel ∞'s

g is charge, e.g.

QED $\alpha_{EM} = \frac{e^2}{\epsilon_0 4\pi \hbar c} = \frac{e^2}{4\pi}\left(\frac{1}{\epsilon_0}\right)$ if $\hbar = c = 1$

$\hookrightarrow g = e$

& ϵ_0 is polariz. of Vac.

~~QCD $\alpha_{QCD} = \frac{g^2_{strong}}{4\pi}$~~

$\hookrightarrow$ & for QED $\frac{\partial g}{\partial Q^2} > 0$ via

$$\alpha_{EM} = \frac{\alpha_0(Q_0^2)}{1 - \frac{\alpha_0(Q_0^2)}{3\pi}\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

gives screening $\alpha \uparrow$ as $Q \uparrow$

due to γ e^- e^+ γ

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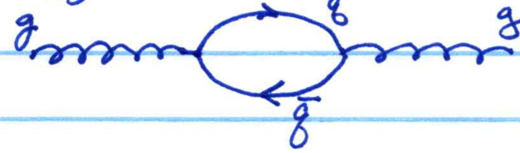
For QCD: $\alpha_{QCD} = \frac{g^2_{strong}}{4\pi}$

for α_{QCD} there are more terms for QFT Vac. Polariz.:



$\Rightarrow$ Screening

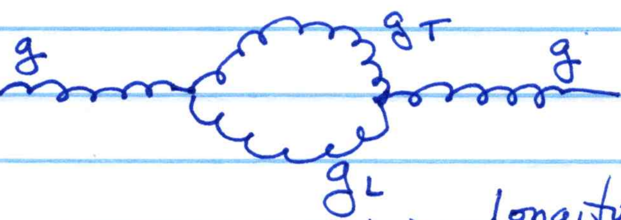
+ 

$\Rightarrow$ Screening

$g_T \rightarrow$ Transversely Polarized g

but...

RCP/LCP

+ 

$\Rightarrow$ Anti-Screening

$g_L \rightarrow$ longitudinally polarized gluons $\Rightarrow$ OK since virtual

last term larger than previous 2 gives

$$\alpha_s(Q^2) = \frac{\alpha_0(Q_0^2)}{1 + \frac{\alpha_0}{12\pi}\underbrace{(11N_c - 2n_f)}_{=+21}\ln\left(\frac{Q^2}{Q_0^2}\right)} \quad \therefore \beta < 0!$$

$\therefore \alpha_s < 1$ @ $Q^2 \gg Q_0^2$

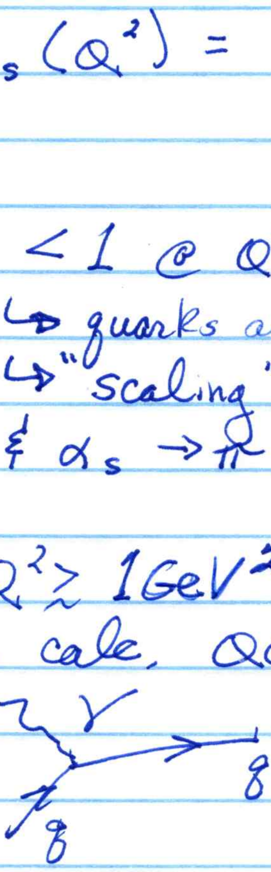
↳ quarks are "asymptotically free"

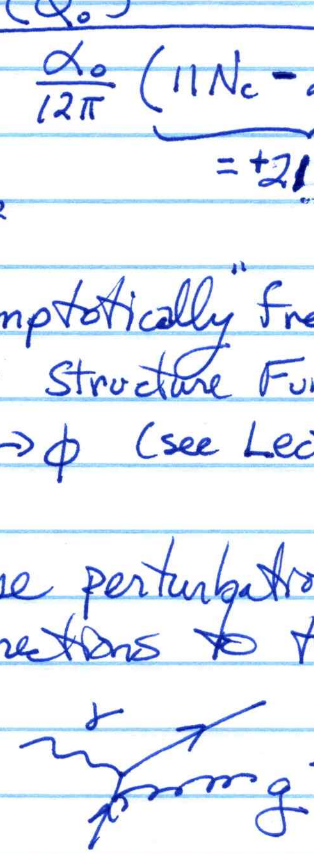
↳ "scaling" of **DIS** Structure Functs. (Lect.)

& $\alpha_s \to \pi$ @ $Q^2 \to \phi$ (see Lect. 5)

&

for $Q^2 \gtrsim 1\,GeV^2$ can use perturbation Theory to calc. QCD corrections to tree level:



+ 

= gluon bremsstrahlung

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1

Ph 203 L14

"what's the

Ques: Meson Nonet?"

How to make a meson?

$SU(2)_{Isospin}$, $SU(3)_c$, $SU(2)_{spin}$, $SU(3)_{\text{"Flavor"}} = u, d, s$

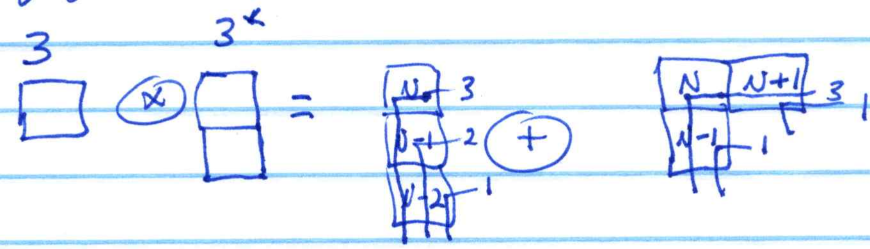
$\Rightarrow u, d$

Last time made color-$\overline{\text{color}}$ states for gluons in $SU(3)_c$ $\quad 3 \otimes 3^*$

For mesons "flavor" symmetry was tried

$\hookrightarrow$ u, d, s are diff. flavors of same particle (but $m_s \gg m_{u,d}$)

$\therefore$ build $q\bar{q}$ states from $SU(3)_{flavor}$



$$= \frac{3\times2\times1}{3\times2\times1} \oplus \frac{3\times2\times4}{3\times1\times1}$$

$$= \underbrace{\underset{\sim}{8} \oplus \underset{\sim}{1}}_{\text{Nonet}}$$

Including strange mesons, can find several nonets using "strangeness"

s quark has $S = -1$

$\bar{s}$ $\quad S = +1$

u, d $\quad S = 0$

u has $t_3 = \frac{1}{2}$

d " " $= -\frac{1}{2}$ $\quad \therefore$ e.g.

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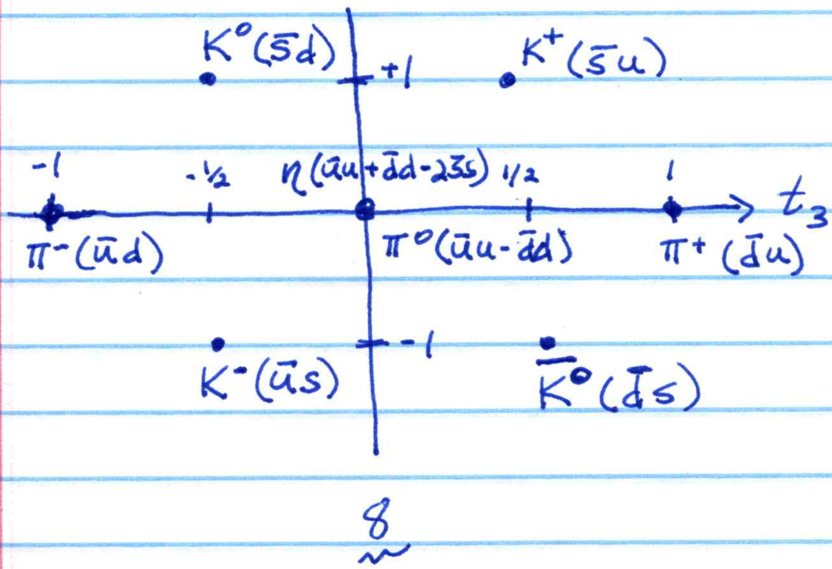
J^π: 0^+ = Scalar
Note 1^- = vector
1^+ = pseudovector

For Light Pseudoscalar (0^-) Mesons



$\Rightarrow$ and $\underset{\sim}{1}$ $\eta'(u\bar{u}+d\bar{d}+s\bar{s})$

$s=0, t=0$

Note: π's $\simeq$ 140 MeV
K's $\simeq$ 495 MeV
η $\simeq$ 550 MeV
η' $\simeq$ 960 MeV

Also a nonet of Vector Mesons: 1^-

Q2: Longitudinal polariz for massless vector bosons γ, g

"virtual/off-mass-shell" massless particles

e.g. $\gamma^* \sim P_\mu = (\nu, \vec{q})$

$M_*^2 = \nu^2 - |\vec{q}|^2 \neq 0$

$\gamma \sim P_\mu = (q, \vec{q})$; $M_\gamma^2 = q^2 - q^2 = 0$

can have ϕ helicity $\Rightarrow$ longitudinal γ's, g's

$\Leftrightarrow$ depends on choice of gauge

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3

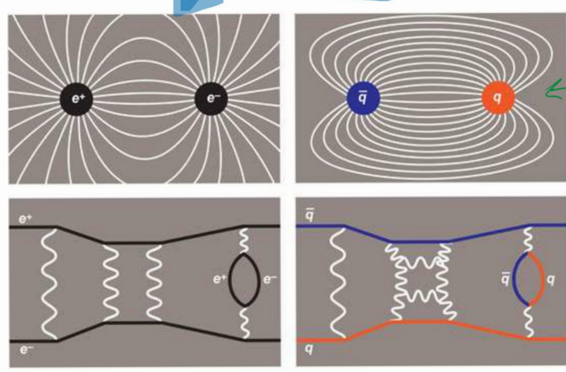
QCD: from Perturb. to Non-Pert.

Last time: discussed Confinement & Asymptotic Freedom

Compare QED & QCD $\vec{E}/\vec{B}$ fields

QCD vacuum prevents $\vec{E}_c/\vec{B}_c$ leakage

QED vs QCD



$\leftarrow r \simeq 2\,\text{fm} \rightarrow$

E-field to $r = \infty$

E_c- field confined

Approach to Non-Perturb.:

Start w QCD Lagrangian

see PDG → next page

9. Quantum Chromodynamics

Revised August 2023 by J. Huston (Michigan State U.), K. Rabbertz (KIT) and G. Zanderighi (MPI Munich).

quark-gluon interaction

gluon self-interaction

9.1 Basics

Quantum Chromodynamics (QCD), the gauge field theory that describes the strong interactions of colored quarks and gluons, is the SU(3) component of the SU(3)×SU(2)×U(1) Standard Model of Particle Physics. The Lagrangian of QCD is given by

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu}\,, \tag{9.1}$$

quarks

where repeated indices are summed over. The γ^μ are the Dirac γ-matrices. The $\psi_{q,a}$ are quark-field spinors for a quark of flavor q and mass m_q, with a color-index a that runs from $a = 1$ to $N_c = 3$, *i.e.* quarks come in three "colors." Quarks are said to be in the fundamental representation of the SU(3) color group.

The $\mathcal{A}^C_\mu$ correspond to the gluon fields, with C running from 1 to $N_c^2 - 1 = 8$, *i.e.* there are eight kinds of gluon. Gluons transform under the adjoint representation of the SU(3) color group. The t^C_{ab} correspond to eight 3×3 matrices and are the generators of the SU(3) group (*cf.* the section on "SU(3) isoscalar factors and representation matrices" in this *Review*, with $t^C_{ab} \equiv \lambda^C_{ab}/2$). They encode the fact that a gluon's interaction with a quark rotates the quark's color in SU(3) space. The quantity g_s (or $\alpha_s = \frac{g_s^2}{4\pi}$) is the QCD coupling constant. Besides quark masses, which have electroweak origin, it is the only fundamental parameter of QCD. Finally, the field tensor $F^A_{\mu\nu}$ is given by

$$\begin{aligned} F^A_{\mu\nu} &= \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu\,, \\ [t^A, t^B] &= i f_{ABC} t^C\,, \end{aligned} \tag{9.2}$$

where the f_{ABC} are the structure constants of the SU(3) group.

Neither quarks nor gluons are observed as free particles. Hadrons are color-singlet (*i.e.* color-neutral) combinations of quarks, anti-quarks, and gluons.

5

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Lattice Gauge Theory (LGT)

Refs:

QCD: Greiner, Schramm & Stein (GSS)

Intro LGT: U. Weise CFT + Path Integral + Stat Mech → LGT

Overview : Kronfeld 2012

Why Lattice Gauge Theory?

Kronfeld - 2012

the total "vacuum angle" $\theta = 0$; chiral symmetries emerge when two or more quark masses vanish (1, 2); and heavy-quark symmetries are revealed as one or more quark masses go to infinity (3, 4). More remarkable still are the phenomena that emerge at a dynamically generated energy scale Λ_{QCD}, the "typical scale of QCD." Much of what is known about QCD in this nonperturbative regime has long been based on belief. Evidence from high-energy scattering fostered the opinion that QCD explains the strong interactions and, therefore, the belief that QCD exhibits certain properties; otherwise, it would not be consistent with lower-energy observations. These emergent phenomena—such as chiral symmetry breaking, the generation of hadron masses that are much larger than the quark masses, and the thermodynamic phase structure—are the most profound phenomena of gauge theories. The primary aim of this review is to survey how lattice QCD has enabled us to replace beliefs with knowledge. To do so, we cover results that are interesting in their own right, influential in a wider arena, qualitatively noteworthy, and/or quantitatively impressive.

The rest of this article is organized as follows. Section 2 introduces the QCD

$\Lambda_{QCD} \sim 150\,\text{MeV}$

$E \gg \Lambda_{QCD}$ Perturbative

$E \lesssim \Lambda_{QCD}$ Non-Perturb.

I) Basics:

⇒ Path Integral:

Calc. Observables (e.g. $\langle \hat{\theta} \rangle$) via

$$\langle \hat{\theta} \rangle = \frac{1}{Z} \int \underbrace{DA_\mu\, D\psi\, D\bar{\psi}}_{\text{differentials for } g \,\&\, q}\; \hat{\theta}\, e^{-\overset{\text{action}}{S}}$$

$$S \equiv \int d^4x\, \mathcal{L}_{QCD}(x)$$

$$Z = \int DA_\mu \ldots e^{-S} \quad \text{(without } \hat{\theta})$$

to normalize $\langle \hat{\theta} \rangle$

$\Rightarrow$ Euclidean Space-Time to start
$x_\mu ; \mu = 1-4$: $x_4 \equiv \tau = it$
↳ convenient for M.C. integration
sothat QM time propagation vial Hamiltonian : $\psi(t) = e^{-iHt}\psi(0)$
becomes: $\psi(\tau) = e^{-H\tau}\psi(0)$
w τ = Boltzman factor
$\tau = \frac{1}{kT}$

II How to code it?

1. Discretize space-time $\Delta x_i \simeq 0.05$ fm, Δt variable
$i = 1-3$

try $\underset{x_i}{64^3}, \underset{\tau}{\times 128}$ w $a = \Delta x_i$

⇔ τ dependence imp. to stabilize non-t dep. observables m_q, m_H, α_s, ...

2. Discretize Action $\Rightarrow \int \mathcal{L}\, d^4x \equiv \sum_{x_i, \tau} \mathcal{L}_{\text{Discrete}}$

3. Do Monte Carlo Integration
$Z \quad \xi . \int DA_\mu \cdots \hat{O}\, e^{-S}$

4. Fit one (or a few) masses to set scale
⇔ $M_{exp.}$

5. Redo Calc at diff a & extrapolate to
$a \to 0 \Rightarrow$ continuum limit

e.g.

III Challenges:

MC integration (Metropolis) requires:

$\Rightarrow$ For "Quenched" (only valence quarks) need T flops
10^{12} floating points ops/s

Unquenched calcs (includes sea quarks) need Exaflops
10^{15} Flops

$\Rightarrow$ Challenging to maintain Gauge Symmetry (gluons) & Chiral Symmetry (quarks) while Discretizing Action

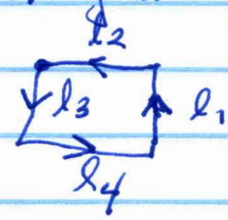
① Gluon Action S^G

Discretize Space-time 4D hypercube

e.g. in 2D

quarks @ vertices
gluons "link" quarks

For gauge-inv. gluon "propagation" use plaquette



w $S^G_{QCD} = \frac{1}{4}\int d^4x\, F^c_{\mu c} F^{\mu\nu}_c$

@ each link get link variable:

$$U_\mu(x) = 1 + i a A_\mu(x)$$

& Trace around Plaquette gives Action via

$$W_\square \equiv Tr\left[U(l_1)U(l_2)U(l_3)U(l_4)\right]$$

$$\Leftrightarrow S^G_{LGT} = \sum_\square \frac{2}{g^2}(3 - W_\square) + \mathcal{O}(a^5)$$

↳ see GSS

All is OK as long as

$$\lim_{a\to 0} S^G_{LGT} = S^G_{QCD}$$

② Quark Action

$$S^q_{QCD} = \int d^4x \left[i\bar\psi \gamma^\mu \partial_\mu \psi + m\bar\psi\psi\right]$$

For LGT try

$$S^q_{LGT} = \sum_{n,\mu}\left[i\bar\psi_n \gamma^\mu \frac{(\psi_{n,+\mu} - \psi_{n,-\mu})}{2a} + m\bar\psi_n\psi_n\right]$$

However, due to periodic structure of lattice quark propagator:

$$\frac{1}{\frac{i}{a}\gamma^\mu \sin(ap^\mu) - m} = \frac{-i\frac{\gamma^\mu}{a}\sin(ap^\mu) - m}{\frac{1}{a^2}\sum_\mu \sin^2(p^\mu a) + m^2}$$

w pole @ Physical quark mass $p_0 = -iE$:

@ $(p_x, p_y, p_z) = (0,0,0) \Rightarrow E = m$

but get extra fermions when

$$(ap_x, ap_y, ap_z) = (n_x\pi, n_y\pi, n_z\pi)$$

$\Rightarrow$ Called Fermion Doubling Problem
& they don't vanish as $a \to 0$
(ouch!)

K. Wilson fixed this "Wilson fermion"
by adding term so that $m_{doub.} = m + \frac{\delta}{a}$
& $m_{doublers} \to \infty$ as $a \to 0$

but this broke chiral Symmetry

$\Rightarrow$ Domain Wall (add 5th dimension w/ size L_5)
& trap chiral fermions on Wall
then as $L_5 \to \infty$, $m_{double} \sim \frac{1}{a}$
& Chiral Sym. OK

Now including $\sum_n' \bar{\psi}_n \gamma_\mu U_{n\mu} \psi_{x+\mu} + \dots$
gives q-g interactions...

Leading to

IV Results

$\to$ see PICS $\to$

Results from LGT:

"Real" Confinement!

"♫ More than a feeling..."

M. CARDOSO, N. CARDOSO, AND P. BICUDO

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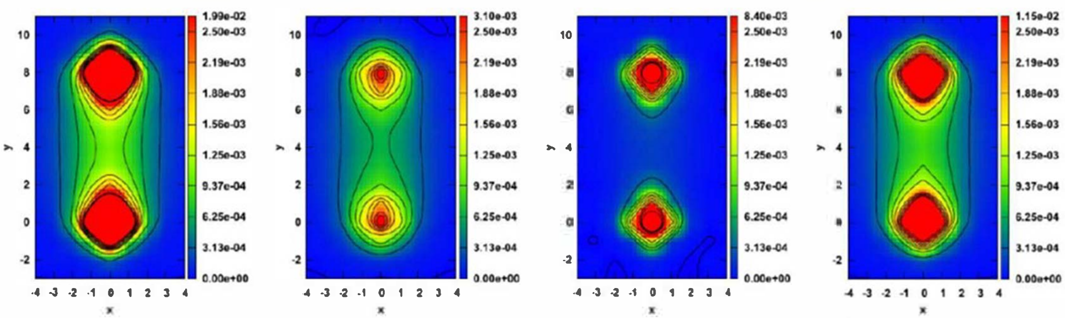
q

$\bar{q}$



(a) $\langle E^2 \rangle$ (b) $-\langle B^2 \rangle$ (c) Energy Density (d) Lagrangian Density

FIG. 5 (color online). Results for the static quark-antiquark system. To make the comparison between the values of different fields easier we have chosen a common scale for values between 0 and 2.5e − 3, but in each figure the deepest red represents the maximum value of the represented field. Because of this choice the flux tube, responsible for the string tension, that should appear in the energy density plot (c) is less visible. The axes and results are in lattice spacing units.

Results from LGT:

6 Standard Model Parameters

The Standard Model (with nonzero neutrino masses and mixing angles) has 28 free parameters:

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (M_W/v)^2/\pi$; CKM
- Quark sector: $m_u e^{i\bar\theta}$, m_d, m_s, m_c, m_b, m_t; $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, δ_{KM};
- Lepton sector: m_{ν_1}, m_{ν_2}, m_{ν_3}, m_e, m_μ, m_τ; θ_{12}, θ_{23}, θ_{13}, δ_{PMNS}, α_{21}, α_{31};
- Standard electroweak symmetry breaking: $v = 246$ GeV, $\lambda = (M_H/v)^2/2$.

Lattice QCD is essential or important in determining the values of eleven parameters (the first under gauge couplings and all but m_t under quark sector).

Table 2: Quark masses from lattice QCD converted to the $\overline{\mathrm{MS}}$ scheme and run to the scale indicated. Entries are in MeV.

Flavor (scale)	Ref. (28)	Ref. (53)	Ref. (54)	Ref. (55)	Ref. (56)
$\bar m_u(2\ \mathrm{GeV})$	1.9 ± 0.2	2.01 ± 0.14	2.24 ± 0.35	2.15 ± 0.11	MeV
$\bar m_d(2\ \mathrm{GeV})$	4.6 ± 0.3	4.79 ± 0.16	4.65 ± 0.35	4.79 ± 0.14	
$\bar m_s(2\ \mathrm{GeV})$	88 ± 5	92.4 ± 1.5	97.7 ± 6.2	95.5 ± 1.9	
$\bar m_c(3\ \mathrm{GeV})$					986 ± 10
$\bar m_b(10\ \mathrm{GeV})$					3617 ± 25

Note $M_{q\bar q}$

140 MeV

1.0 GeV

3.1 GeV

9.5 GeV

$M_{q\bar q} > 2m_q$!

Different LGT calcs.

QCD coupling constant from LGT:

the errors on almost all determinations are dominated by the perturbative truncation error. Instead, the error on the pre-range for α_s from the step-scaling method is taken, since perturbative truncation errors are sub-dominant in this method. The final FLAG 2021 average (rounded to four digits) is

$$\alpha_s(m_Z^2) = 0.1184 \pm 0.0008 \quad \text{(FLAG 2021 average)}, \quad \text{LGT} \tag{9.23}$$

which is fully compatible with the FLAG 2019 result of $\alpha_s(m_Z^2) = 0.1182 \pm 0.0008$.

We believe that this result expresses to a large extent the consensus of the lattice community and that the imposed criteria and the rigorous assessment of systematic uncertainties qualify for a direct inclusion of this FLAG average here. As in the previous review, we therefore adopt the FLAG average with its uncertainty as our value of α_s for the lattice category. Moreover, this lattice result will not be directly combined with any other sub-field average, but with our non-lattice average to give our final world average value for α_s.

9.4.8 *Determination of the world average value of $\alpha_s(m_Z^2)$:*
Obtaining a world average value for $\alpha_s(m_Z^2)$ is a non-trivial exercise. A certain arbitrariness and subjective component is inevitable because of the choice of measurements to be included in the average, the treatment of (non-Gaussian) systematic uncertainties of mostly theoretical nature, as well as the treatment of correlations among the various inputs, of theoretical as well as experimental origin.

We have chosen to determine pre-averages for sub-fields of measurements that are considered to exhibit a maximum degree of independence among each other, considering experimental as well as theoretical issues. The seven pre-averages, illustrated also in Fig. 9.2, are listed in column two of Table 9.1. We recall that these are exclusively obtained from extractions that are based on (at least) NNLO QCD predictions, and are published in peer-reviewed journals at the time of completing this *Review*. To obtain our final world average, we first combine six pre-averages, excluding the lattice result, using a χ^2 *averaging method*. This gives

— a.k.a. Experiment

$$\alpha_s(m_Z^2) = 0.1175 \pm 0.0010 \quad \text{(PDG 2023 without lattice)}. \tag{9.24}$$

This result is fully compatible with the lattice pre-average Eq. (9.23) and has a comparable error. To avoid a possible over-reduction, we combine these two numbers using an unweighted average and take as an uncertainty the average between these two uncertainties. This gives our final world average value

$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009 \quad \text{(PDG 2023 average)}. \tag{9.25}$$

If for the sub-field of hadron colliders we are more restrictive and instead only accept results from a simultaneous fit of PDFs, we arrive at 0.1157 ± 0.0021 for this sub-field leading to 0.1172 ± 0.0010

Now has equal footing to experiment!
→ took 45 yrs + ExaCPU

Hadron Masses:

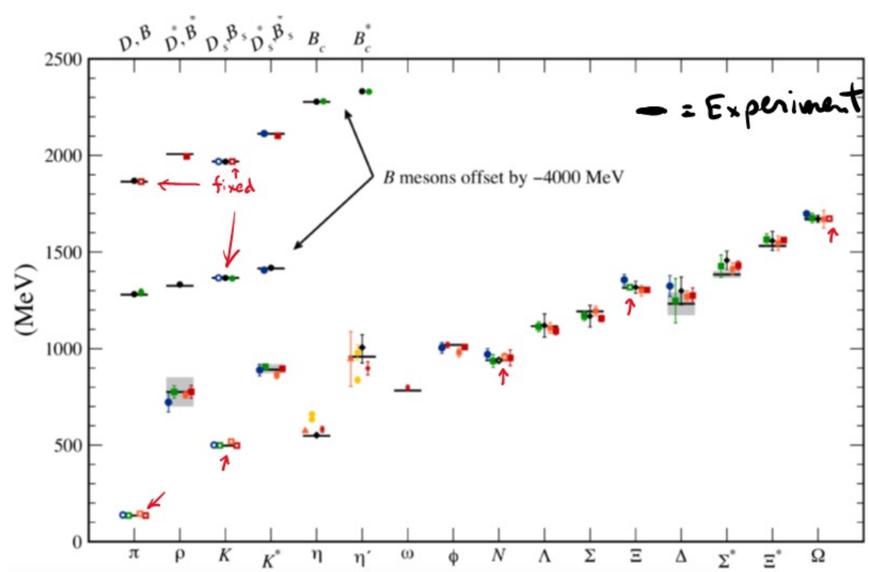


Figure 15.9: Hadron spectrum from lattice QCD. Comprehensive results for mesons and baryons are from MILC [111,112], PACS-CS [113], BMW [114], QCDSF [115], and ETM [116]. Results for η and η' are from RBC & UKQCD [21], Hadron Spectrum [117] (also the only ω mass), UKQCD [118], and Michael, Ottnad, and Urbach [119]. Results for heavy-light hadrons from Fermilab-MILC [120], HPQCD [121,122], and Mohler and Woloshyn [123]. Circles, squares, diamonds, and triangles stand for staggered, Wilson, twisted-mass Wilson, and chiral sea quarks, respectively. Asterisks represent anisotropic lattices. Open symbols denote the masses used to fix parameters. Filled symbols (and asterisks) denote results. Red, orange, yellow, green, and blue stand for increasing numbers of ensembles (i.e., lattice spacing and sea quark mass) Black symbols stand for results with 2+1+1 flavors of sea quarks. Horizontal bars (gray boxes) denote experimentally measured masses (widths). b-flavored meson masses are offset by -4000 MeV.

Glueball Masses (?!?):

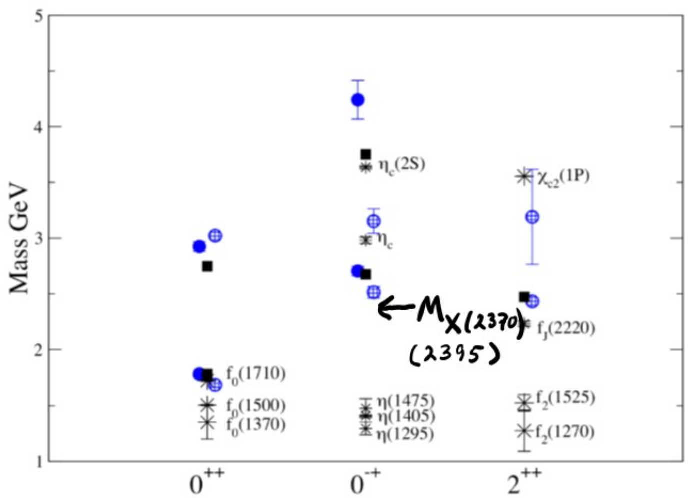


Figure 15.15: Lattice QCD predictions for glueball masses. The open and closed circles are the larger and smaller lattice spacing data of the full QCD calculation of glueball masses of Ref. [140], at pion masses of 280 and 360 MeV. Squares are the quenched data for glueball masses of Ref. [33]. The bursts labeled by particle names are experimental states with the appropriate quantum numbers.

$n \to pe^- \bar{\nu}_e$ DECAY PARAMETERS

See the above "Note on Baryon Decay Parameters." For discussions of recent results, see the references cited at the beginning of the section on the neutron mean life. For discussions of the values of the weak coupling constants g_A and g_V obtained using the neutron lifetime and asymmetry parameter A, comparisons with other methods of obtaining these constants, and implications for particle physics and for astrophysics, see DUBBERS 91 and WOOLCOCK 91. For tests of the $V-A$ theory of neutron decay, see EROZOLIMSKII 91B, MOSTOVOI 96, NICO 05, SEVERIJNS 06, and ABELE 08.

$\lambda \equiv g_A / g_V$

VALUE			DOCUMENT ID		TECN	COMMENT
−1.2754	**±0.0013**	**OUR AVERAGE**	Error includes scale factor of 2.7. See the ideogram below.			
−1.2796	±0.0062		[1] HASSAN	21	SPEC	Proton recoil spectrum
−1.2677	±0.0028		[2] BECK	20	SPEC	Proton recoil spectrum
−1.27641	±0.00045	±0.00033	[3] MAERKISCH	19	SPEC	pulsed cold n, polarized
−1.2772	±0.0020		[4] BROWN	18	UCNA	Ultracold n, polarized
−1.2748	±0.0008	$^{+0.0010}_{-0.0011}$	[5] MUND	13	SPEC	Cold n, polarized
−1.275	±0.006	±0.015	SCHUMANN	08	CNTR	Cold n, polarized
−1.2686	±0.0046	±0.0007	[6] MOSTOVOI	01	CNTR	A and B × polarizations

PROCEEDINGS OF SCIENCE

Lattice QCD Determination of g_A

Andre Walker-Loud*, Lawrence Berkeley National Laboratory
Evan Berkowitz, University of Maryland
David A. Brantley, Arjun Gambhir, Pavlos Vranas, Lawrence Livermore National Laboratory
Chris Bouchard, University of Glasgow
Chia Cheng Chang, RIKEN-iTHEMS
M.A. Clark, NVIDIA Corporation
Nicolas Garron, Liverpool Hope University
Balint Joó, Thomas Jefferson National Accelerator Facility
Thorsten Kurth NERSC, Lawrence Berkeley National Laboratory
Henry Monge-Camacho, Amy Nicholson, University of North Carolina Chapel Hill
Christopher J Monahan, Kostas Orginos, The College of William & Mary
Enrico Rinaldi, Arithmer Inc. & RIKEN iTHEMS

The nucleon axial coupling, g_A, is a fundamental property of protons and neutrons, dictating the strength with which the weak axial current of the Standard Model couples to nucleons, and hence, the lifetime of a free neutron. The prominence of g_A in nuclear physics has made it a benchmark quantity with which to calibrate lattice QCD calculations of nucleon structure and more complex calculations of electroweak matrix elements in one and few nucleon systems. There were a number of significant challenges in determining g_A, notably the notorious exponentially-bad signal-to-noise problem and the requirement for hundreds of thousands of stochastic samples, that rendered this goal more difficult to obtain than originally thought.

3321v1 [hep-lat] 17 Dec 2019

2. A percent-level determination of g_A from QCD

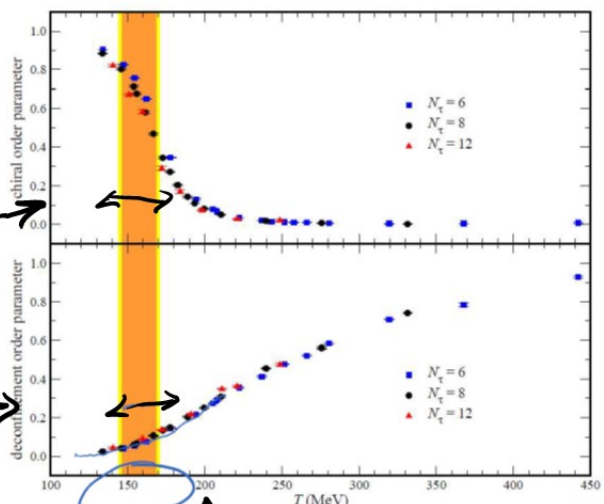
We have recently determined g_A with an unprecedented percent-level of uncertainty [5]

$$g_A = 1.2711(103)^s(39)^\chi(15)^a(04)^V(55)^M. \tag{2.1}$$

The sources of uncertainty are statistical (s), extrapolation to the physical pion mass (χ), continuum extrapolation (a), infinite volume extrapolation (V) and a model average uncertainty (M). Prior to this result, it was estimated that a 2% uncertainty could be achieved with near-exascale computing (such as Summit at OLCF) by 2020 [6]. There were several key features of our calculation that enabled a determination with 1% uncertainty with the previous generation of supercomputers:

LGT @ Finite Temperature:
QCD Phase Transition:

Soft Phase Transition



Λ_{QCD}

Quark Gluon Plasma

LGT calc.

Figure 5: Order parameters for deconfinement (*bottom*) and chiral symmetry restoration (*top*), as a function of temperature. The physical temperature $T = (N_\tau a)^{-1}$, where a is the lattice spacing and $N_\tau = N_4$. Agreement for several values of N_τ thus indicates that discretization effects from the lattice are under control. Data are from Reference 126.