PH 203 HW9

1. Bertulani 12.2
2. Bertulani 12.4
3. Bertulani 12.16
4. SP4
5. SP5

2. Suppose the iron core of a supernova star has a mass of $1.4\ M_\odot$ (the sun's mass is $M_\odot = 1.99 \times 10^{30}$ kg) and a radius of 100 km, and that it collapses to a uniform sphere of neutrons of radius 10 km. Assume that the virial theorem

$$2\langle T\rangle + \langle V\rangle = 0$$

holds, where $\langle T\rangle$ is the average of the internal kinetic energy and $\langle V\rangle$ is the average of the gravitational potential energy. $E = \langle T\rangle + \langle V\rangle$ is the total mechanical energy of the system. Calculate the energy consumed in neutronization and the number of electron neutrinos produced. Given that the remaining energy is radiated as neutrino-antineutrino pairs of all kinds of average energy 12 + 12 MeV, calculate the total number of neutrinos radiated.

4. Given that the supernova of exercise 2 is at a distance of 163,000 light years, calculate the total number of neutrinos of all types arriving at each square meter of the earth. Also estimate the number of reactions

$$\bar{\nu}_e + \mathrm{p} \longrightarrow \mathrm{n} + e^+$$

that will occur in 1000 metric tons of water. Assume that the cross section is given by

$$\sigma = \frac{4 p_e E_e G_F^2}{\pi \hbar^4 c^3}$$

where p_e and E_e are the positron momentum and energy, respectively, and G_F is the Fermi coupling constant. Assume that only one-sixth of the neutrinos are electron neutrinos.

16. The nonexistence of a bound nucleus with $A = 8$ was one of the major puzzles in nuclear astrophysics. How could heavier elements than $A = 8$ be formed? Using typical values of concentration of α-particles in the core of a heavy star, $n_\alpha \sim 1.5 \cdot 10^{28}/\mathrm{cm}^3$ (corresponding to $\rho_\alpha \sim 10^5\ \mathrm{g/cm}^3$) and $T_8 \sim 1$, one obtains

$$\frac{n(^8\mathrm{Be})}{n(\alpha)} \sim 3.2 \times 10^{-10}.$$

Salpeter suggested that this concentration would then allow $\alpha + {}^8\mathrm{Be}(\alpha + \alpha) \rightarrow {}^{12}\mathrm{C}$ to take place. Hoyle then argued that this reaction would not be fast enough to produce significant burning unless it was also resonant. Now the mass of $^8\mathrm{Be} + \alpha$ is 7.366 MeV, and each nucleus has $J^\pi = 0^+$. Thus s wave capture would require a 0^+ resonance in $^{12}\mathrm{C}$ at ~ 7.4 MeV. No such state was then known, but an experimental search revealed a 0^+ level at 7.644 MeV, with decay channels $^8\mathrm{Be} + \alpha$ and γ-decay to the 2^+ 4.433 level in $^{12}\mathrm{C}$. The parameters are

$$\Gamma_\alpha \sim 8.9\mathrm{eV},$$

$$\Gamma_\gamma \sim 3.6 \cdot 10^{-3}\mathrm{eV}.$$

Show that the maximum amount of energy released in the form of electromagnetic radiation from converting four protons to a ^{4}He is given by the binding energy of ^{4}He less twice the sum of the neutron-proton mass difference and the mass of positrons. Ignore any rest mass the neutrino may have.

Using the fact that three-fourths of the solar mass of 1.99×10^{30} kg consists of protons, calculate the length of time that fusion energy can be generated at the present rate of 1.4 kW/m^2 at a distance of 1.50×10^{11} m in converting four protons to a ^{4}He nucleus.

A neutron star is a compact, dense object made of degenerate neutrons having a density similar to that in the central part of a heavy nucleus.

(a) If the density of nuclear matter is 0.17 nucleons/fm^3 or 2.8×10^{17} kg/m^3, what is the radius of a neutron star having a mass one and a half times that of the sun? (One solar mass $= 2.0 \times 10^{30}$ kg.)

(b) A neutron star is one of the possible remanents of a supernova explosion such as SN 1987a, the one which took place in the Large Magellanic Cloud 160,000 light years away and was first observed on earth on February 24, 1987. When the core of a large star exhausts its nuclear fuel, there is no longer the thermal pressure to counterbalance the gravitational force, and the core of the star collapses. For simplicity, we can consider that all the material in the core of the collapsing star is in the form of ^{56}Ni made of 28 neutrons and 28 protons. Because of the tremendous gravitational force, the protons in ^{56}Ni change into neutrons by capturing atomic electrons through the reaction

$$p + e^- \to n + \nu_e$$

Calculate the number of neutrinos released in converting 1.5 solar mass of ^{56}Ni atoms into neutrons during the gravitational collapse.

(c) If the total cross section for a neutrino to interact with each nucleon is 10^{-48} m^2, how many reactions due to the neutrinos from such a gravitational collapse can one expect in a detector on earth made of 3000 tons of water? Compare this with the number of events (12) observed with such a detector at Kamioka due to supernova SN 1987a.

(d) Assuming that the average energy of each neutrino is 10 MeV in such an event, calculate the total amount of energy carried away by the neutrinos from the gravitational collapse. Compare this value with the rest-mass energy of the sun.