PH 203 HW 8

1. Bhaduri 3.1

2. Bertulani 12.1

3. Bertulani 12.3

4. Bertulani 12.8

5. Bertulanti 12.9

Exercise 3.1

Use the nonrelativitic quark-model to calculate the magnetic susceptibility of a proton. Use perturbation theory. With a uniform magnetic field B in the z-direction, the perturbing Hamiltonian is $\mathcal{H}_{\text{int}} = -\mu_z B + (e^2/8m_c)(x^2+y^2)B^2$, and $\mu_z = \sum_q (e_q/2m_c)\sigma_z$, where m_c is the constituent quark mass. We have taken $\mathbf{A} = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$. The first term, taken to second order, gives a paramagnetic contribution. The diagonal matrix-element of the second term is diamagnetic. Show that the proton magnetic susceptibility χ_p is given by

$$4\pi\chi_P = \frac{2\mu_{P\Delta}^2}{(E_\Delta - E_N)} - \frac{e^2}{6m_c}\langle r^2\rangle_P \,.$$

Here $\mu_{P\Delta} = \langle P|\mu_z|\Delta\rangle = 2\sqrt{2/3}\,\mu_P$, μ_P being the magnetic moment of the proton. Transitions to states other than $\Delta(1230)$ are ignored because of negligible spatial overlap with the ground state. Take $m_c = 336\,\text{MeV}$, $\langle r^2\rangle_P^{1/2} = 0.86\,\text{fm}$, and $E_\Delta - E_N = 300\,\text{MeV}$. Show that $\chi_P \approx 2\times10^{-4}\,\text{fm}^{+3}$. How do you think it can be measured? In the literature, χ_P is also called the magnetic polarizability and denoted by $\bar{\beta}$. The corresponding response function for the electric field is termed electric polarizability and denoted by $\bar{\alpha}$ (see Exercise 5.11).

1. (a) What is the most probable kinetic energy of a hydrogen atom at the interior of the sun ($T = 1.5\times10^7$ K)? (b) What fraction of these particles would have kinetic energy in excess of 100 keV?

3. About 3 s after the onset of the Big Bang, the neutron-proton ratio became frozen when the temperature was still as high as 10^{10} K ($kT \simeq 0.8$ MeV). About 250 s later, fusion reactions took place converting neutrons and protons into ^{4}He nuclei. Show that the resulting ratio of the masses of hydrogen and helium in the universe was close to 3. The neutron half-life = 10.24 min and the neutron-proton mass difference is 1.29 MeV.

8. Given that the sun was originally composed of 71% hydrogen by weight and assuming it has generated energy at its present rate (3.86×10^{26} W) for about 5×10^9 years by converting hydrogen into helium, estimate the time it will take to burn 10% of its remaining hydrogen. Take the energy release per helium nucleus created to be 26 MeV.

9. The CNO cycle that may contribute to energy production in stars similar to the sun begins with the reaction $p + {}^{12}C \rightarrow {}^{13}N + \gamma$. Assuming the temperature near the center of the sun to be 15×10^6 K, find the peak energy and width of the reaction rate.