

Homework Set #2

Problem 2.14 In the ground state of the harmonic oscillator, what is the probability (correct to three significant digits) of finding the particle outside the classically allowed region? *Hint:* Classically, the energy of an oscillator is $E = (1/2)ka^2 = (1/2)m\omega^2a^2$, where a is the amplitude. So the “classically allowed region” for an oscillator of energy E extends from $-\sqrt{2E/m\omega^2}$ to $+\sqrt{2E/m\omega^2}$. Look in a math table under “Normal Distribution” or “Error Function” for the numerical value of the integral, or evaluate it by computer.

***Problem 2.38** A particle of mass m is in the ground state of the infinite square well (Equation 2.19). Suddenly the well expands to twice its original size—the right wall moving from a to $2a$ —leaving the wave function (momentarily) undisturbed. The energy of the particle is now measured.

- What is the most probable result? What is the probability of getting that result?
- What is the next most probable result, and what is its probability?
- What is the expectation value of the energy? *Hint:* If you find yourself confronted with an infinite series, try another method.

***Problem 2.5** A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)].$$

- Normalize $\Psi(x, 0)$. (That is, find A . This is very easy, if you exploit the orthonormality of ψ_1 and ψ_2 . Recall that, having normalized Ψ at $t = 0$, you can rest assured that it *stays* normalized—if you doubt this, check it explicitly after doing part (b).)
- Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$. Express the latter as a sinusoidal function of time, as in Example 2.1. To simplify the result, let $\omega \equiv \pi^2\hbar/2ma^2$.
- Compute $\langle x \rangle$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (If your amplitude is greater than $a/2$, go directly to jail.)
- Compute $\langle p \rangle$. (As Peter Lorre would say, “Do it *ze kveek* vay, Johnny!”)
- If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of H . How does it compare with E_1 and E_2 ?