Phys. 2b 2025, Lecture Notes (Lectures 1 & 2) (1/7-9/2025)

Great time to be taking Q.M. - UN has declared 2025 as International Year of Quantum Science and Technology - https://quantum2025.org/

Course details

Most discussed on course webpage on Canvas
HW=50%, 2xQuiz=25%, F=25%
HW: 3-4 problems/wk; SP (Special Probs) $\Rightarrow$ from previous Exams: BP = Book Probs.
Text: Griffiths 3nd edition
This class covers first 4 chapters of Griffiths (Most of NRQM = Non-Relativistic Quantum Mechanics) + some of chap. 5

Key Concepts:

1. What is QM?
2. Why QM? (why do we need it?)

What is QM?

QM is : Description of free particle in motion when energies and/or distances are **very** small.
How small is small? $\Rightarrow h$ Planck's Constant, $h = 6.6 \cdot 10^{-34}$Joule-sec

$\hookrightarrow$ Note: dropping Ping-Pong Ball from 2 m give ball ~ 1 Joule in ~ 1 sec

What happens at these small scales? $\Rightarrow$ **very** bizarre things, not deterministic ...

Example:
Consider: Particle with Force $\Rightarrow$ measure $\vec{v}, \vec{x}$
Classically we can predict $\vec{v}(t), \vec{x}(t)$ if given initial conditions and forces $\rightarrow$ QM says no way!!
Instead it says: we can predict probability of obtaining a given value from measurement of $\vec{v}$ or $\vec{x} \rightarrow$ Huh??
or we say it predicts the distribution of measurements of an **ensemble of identically prepared systems.**

Goals of 2b

- Learn how to apply basic math of QM: Probability Concepts & PDE = Schrodinger Eq.
- Particles as waves leads to "Entanglement" & Quantum Computing ...

$\hookrightarrow$ Entanglement!

See NOVA epispode video clip: Einstein's Quantum Riddle 1:40 - 3:55 (first aired 1/17/2019) (posted on CANVAS)

Why QM:

Because Classical Physics has some key failures that were identified ~ 125 years ago
And ... QM works! $\rightarrow$ agrees with measurement & $\rightarrow$ has survived 100 yrs of study

Classical Failures:

Three key failures of Classical physics led to the development of Quantum Theory:
I. Black-Body Radiation - BBR
II. Photoelectric Effect
III. Atomic spectra

Classical Failure I. BBR:

• Ideal BB is a perfect absorber (i.e. no reflections) and emitter of radiation.
Approximate examples:
hot coals in a wood fire
The sun, human body (what about the moon? - No! - mostly reflection)

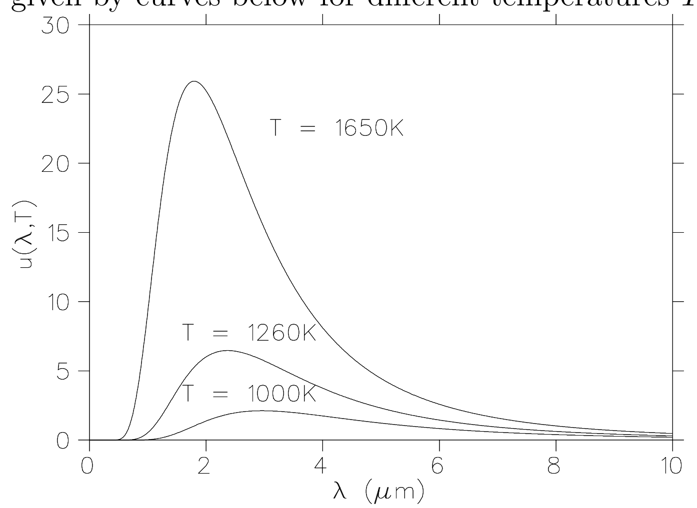
Demo: Three boxes with small hole ...

→ Experimental information at the beginning of the 20th Century:

Spectral distribution of radiation from BB depends only on temperature

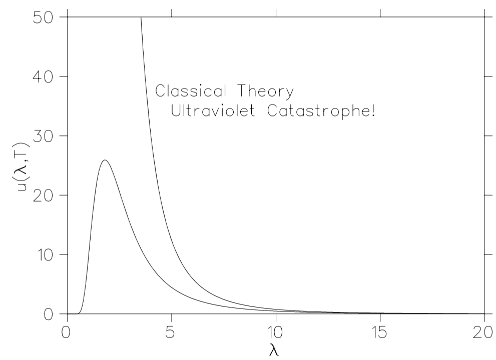
Spectral Energy density $\Rightarrow u(\lambda, T) \Rightarrow \frac{\text{Energy}}{\text{Unit-volume} \quad \text{unit-wavelength}}$

Experimental info is given by curves below for different temperatures T:



Demos: Big Light Bulb has varying colors
You can also play with BBR via PHET webapp:
https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html

$\Longrightarrow$ But classical "prediction", based on thermodynamics and Classical E&M is a disaster ...
In particular at short wavelength, compared with experiment (see figure below) the theory goes to infinity = Ultraviolet Catastrophe



Classical Failure II. Atomic Spectra

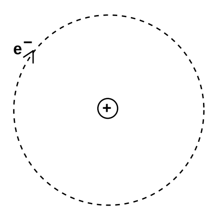
→ Experimental Observations:

1. Tube of gas with electric discharge inside produces light at well-defined frequencies (quantized frequencies, not continuum)

2. Different gases give different frequencies ⇒ all quantized

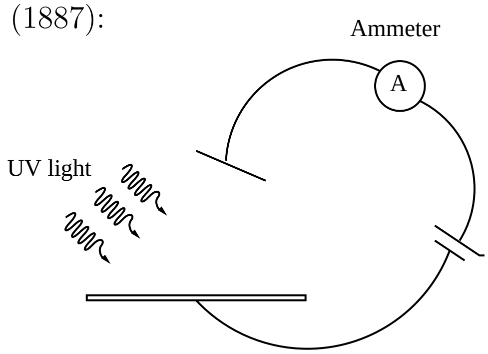
Demo: Light bulbs with grating for Hg & Ne

But atoms have e^- and positive nucleus (e.g., proton for H)
if e^- "orbits" proton, all orbits are possible
∴ no discrete lines



Classical Failure III. Photoelectric Effect

Experiment of Hertz (1887):



Polished metal plates emit electrons when irradiated by UV light.
Energy of EM wave apparently liberates electrons.

Recall that Classical EM theory says that energy density in EM wave $E/V \propto |\vec{E}|^2$, which is independent of wavelength
But classical theory FAILS
→ since we see that electron emission depends on wavelength.
↪ see posted handout for details if interested.

How to fix these failures??

Quantized Repair I. BBR

$\rightarrow$ **Enter Max Planck (1900)**:

Planck "guessed" a high frequency (or low wavelength) cut-off (á là $e^{-\beta\nu}$) for $\langle$Energy$\rangle$/mode could fix the problem:

Planck's Postulate:

- For each mode, energy is absorbed and emitted only in quantized amounts: $E = h\nu$,
  i.e. harmonic oscillations (of field? or walls?) occupy only discrete states

$$E_n = nh\nu, \; n = 0, 1, 2, 3, \ldots$$

$\Rightarrow$ Planck distribution is a resounding success
$\hookrightarrow$ (see BBR handout on CANVAS if you want details)
but...
What is this $E_n = nh\nu$? Is it a mathematical artifact? Are the walls of BB quantized oscillators or is there something else? (see below)

Quantized Repair II. Atomic Spectra

$\rightarrow$ Bohr guessed a Quantized Model (1913)
Coulomb force provides centripetal acceleration for "orbit"
But! ... Orbits are quantized with discrete values
$\hookrightarrow$ (See SP1 in HW set 1)

Quantized Repair III. Photoelectric Effect

Enter Einstein ...

Einstein explained this effect in 1905, by using $E = nh\nu$ for the EM field (i.e. photons)
This explains source of BBR quantization = photons!

But all of above repairs indicate that "quantization" is important. But this is only a hint
Not a Theory! $\rightarrow$ see next time ...

Key Concepts

1. What is the Wave Function (WF)?
2. Matter Waves
3. Heisenberg's Uncertainty Principle (HUP)

From Early Quantized Models to Modern Quantum Theory

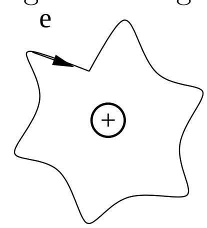
This required four major Breakthroughs

To discuss the Wave Function we will start with the first two breakthroughs:

Breakthrough I: - Matter Waves

In 1923 PhD thesis, deBroglie tried to explain Atomic spectra quantization by postulating "Matter Waves"'

$\rightarrow$ de Broglie asked: what if $p = \frac{h}{\lambda}$ held for e^-, then can get standing waves around proton:



standing wave has $n\lambda = 2\pi r$

or $\frac{nh}{p} = 2\pi r \Rightarrow pr = \frac{nh}{2\pi} \Rightarrow |\vec{L}| = n\hbar \Rightarrow$ Consistent with Bohr's Postulate!!

Thus he proposed all matter possesses wave-like properties:

$$\boxed{\lambda = \frac{h}{p}} \Rightarrow \text{ de Broglie wavelength}$$

$\rightarrow$ 1927 Davisson and Germer confirmed $\lambda = \frac{h}{p}$ for electrons via diffraction from crystals.

$\hookrightarrow$ See Demo next lecture

Breakthrough II: - Born Postulate (1925-26)

If, assuming matter waves, particle is a wave with wave amplitude $\psi(x,t)$

$\therefore$ Postulate:

$\rightarrow$The probability of finding a particle between x and $x + dx$ at time t, aka its probability distribution, is the squared modulus of the wave amplitude $|\psi|^2 = \psi^*\psi$dx

Note: ψ^* is complex conjugate of ψ. Why complex? see next week

This suggests that probability is key to Quantum Mechanics

Particle described by Wave Function ... What does this mean?

1. If particle is constrained to exist within some bounds (Volume $= V$) then

$$\int_V |\psi(\vec{r})|^2 dxdydz = 1 \ ; \ |\psi|^2 = \psi^*\psi$$

Using the above integral, $\psi(\vec{r})$ is said to be *normalized*

2. Wave Function contains all that can be known about the particle, but ...
3. Wave Function itself **CAN'T** be measured, only $\psi^*\psi$ is measureable

Breakthrough III. Wave Function satisfies Schrodinger Equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

This equation governs the time evolution of the Wave Function and allows superposition (since it's a linear equation) such that if ψ_1 and ψ_2 are both solutions of Schrodinger Equation, then $\psi_1 + \psi_2$ is also a solution

$\rightarrow$ Superposition of Wave Functions implies that wave interference is possible

Experiment confirms this via electron double slit interference experiment
$\rightarrow$ see picture in Text Chap. 1 Fig. 1.4.

Works even when only one electron goes through slits at a time:
$\Rightarrow$ interference pattern is only visible after many e^- are detected $\rightarrow$ but what's interfering?
$\Rightarrow$ electron interferes with itself!

DEMO: Electron diffraction from crystal = next lecture

Breakthrough IV. Heisenberg's Uncertainty Principle (HUP)

(For now this is only a plausibility argument for HUP. Derivation of it is in Chap. 3)

Statistical nature of $\psi^*\psi$ as a probability density indicates that if we measure the position of an ensemble of identically-prepared systems, then we don't get the same answer each time. Instead we get a distribution of answers x_i
Then we can define a mean

$$\langle x\rangle \equiv \sum_{i=1}^{N} x_i P_i \quad \text{where } P_i \text{ is probability of getting } x_i$$

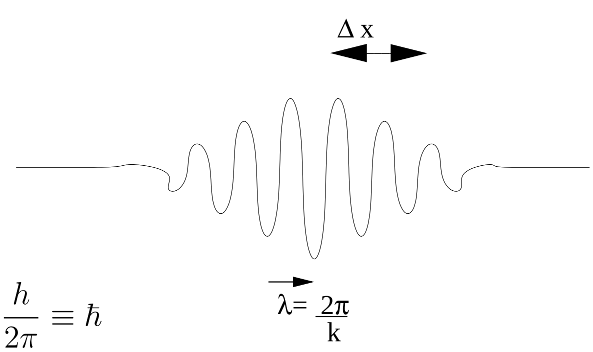
and a standard deviation

$$\Delta x = \sigma_x(\text{Griffiths}) = \text{ r.m.s. deviation or uncertainty} \equiv \sqrt{\langle (x-\langle x\rangle)^2\rangle}$$

Note $\Delta x^2 = \langle x^2 - 2x\langle x\rangle + \langle x\rangle^2\rangle = \langle x^2\rangle - \langle x\rangle^2$

$\Rightarrow$ then from Ph2a, for classical waves, we know that:

$$\Delta x\Delta k \gtrsim 1$$



Thus since $\lambda = 2\pi/k$ and $\lambda = h/p$ (from deBroglie)
we have $k = 2\pi p/h$, which suggests

$$\Delta x\frac{2\pi\Delta p}{h} \gtrsim 1, \text{ or } \Delta x\Delta p \gtrsim \frac{h}{2\pi} \equiv \hbar$$

This led Heisenberg to postulate the H.U.P.
But how do we calculate Δp?

Consider Mathematical Operators: if $\hat{A}$ is an operator, then $\hat{A}f(x) = g(x)$
e.g. if $\hat{A} = \beta\frac{\partial}{\partial x}$, then $\hat{A}\psi = \beta\frac{\partial\psi}{\partial x}$

$\hookrightarrow$ In fact, all quantum observables can be described by operators (see Ch 3)

Text shows that $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ = momentum operator, while position operator is $\hat{x} = x$.
And for quantum system we can define:
$\langle x\rangle = \int_{-\infty}^{\infty} \psi^* x\psi dx$
Likewise, perhaps we can associate a mean & uncertainty with any physical observable e.g.,

$$\langle p_x\rangle \text{ and } \Delta p_x = \sqrt{\langle(p_x - \langle p_x\rangle)^2\rangle}$$

and
$\langle p_x\rangle = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar\frac{\partial}{\partial x}\right)\psi dx$

Rigorous derivation (see Ch 3 in text) gives:

$$\Delta x\Delta p_x \geq \frac{\hbar}{2}$$

as well as ...

$$\Delta y\Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z\Delta p_z \geq \frac{\hbar}{2}$$

and there are even more (see later)