Jan. 27, 2025

PHYSICS 2b – Quiz 1

This exam covers all of the readings in Griffiths text for the first three weeks of material, through HW 3.

This is an **OPEN BOOK** exam with the following limitations: You may consult only your textbook (Griffiths & Schroeter, Intro to QM), notes that you have taken in recitation or lecture, online lecture notes, and HW solns (including your own). No other references are allowed. You may use a calculator and symbolic manipulation programs (eg. Mathematica) for integrals or algebra, although they are not required.

In your studies, do not look at quizzes or exams from previous years of Phys 2 or 12.

The time limit is **2.5 HOURS**, in one continuous sitting. A 15 minute break beyond the 2.5 hours is permitted during this period, as long as you are not working on the exam during this time. No credit for overtime work.

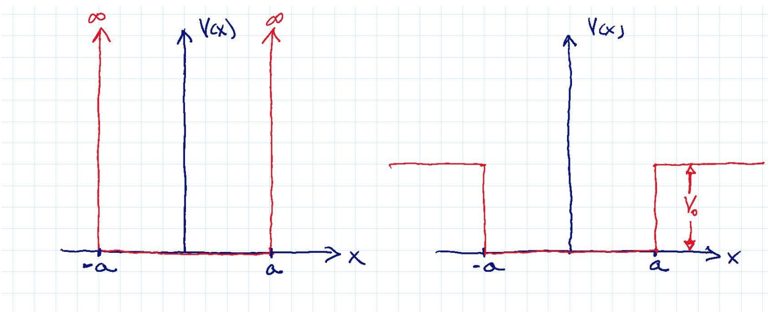
This Quiz is **DUE** Thursday, Jan. 30 at 9:00PM and should be turned in via Gradescope.
Late Quizzes will not be accepted for credit except by prior arrangement with the Head TA.

There are 3 problems on pages 2–4 for a total of 58 points

To get partial credit, show as much work as you can!

Problem 1 - Short"ish" Problems

(a) Consider a particle of mass m in a bound state (i.e. $E < V_0$) for each of the potentials sketched below. Potential A is an infinite square well of width $2a$. Potential B is a finite square well of the same width, but with walls of height V_0. In both cases $V(x) = 0$ for $-a < x < a$.



(i) (3 points) Explain which of these wells has a lower ground state energy and give a qualitative reason for your choice (no formulas needed)

(ii) (3 points) Sketch the wave function - $\psi(x)$ vs x - for the first excited state of both wells (assume that it exists for well B). Clearly label the points 0 and a on your x-axis.

(b) (4 points) In nuclear beta-decay an energetic electron can be emitted with typical energies of 1 - 10 MeV (1 Mev $= 1.602 \times 10^{-13}$ Joules). Given that the typical size of an atomic nucleus is 1×10^{-14} m, use the uncertainty principle to prove that the electron (mass $= 9.109 \times 10^{-31}$ kg) could not have been confined inside the nucleus before the decay.

(c) (4 points) Consider a Ping-Pong ball as a free particle gaussian wave packet. An MIT Sophomore wants to observe wave packet broadening to *Prove* Quantum Mechanics. Use the results from the HW3 set BP3 Prob. 2.21 to prove the idiocy of this by estimating the time for a regular Ping-Pong ball, $m = 2.7$ g, to "double in size" due to wave packet broadening by assuming that the ball's radius, $r = 2$ cm is equal to Δx at $t = 0$.

(d) (4 points) For a particle of mass m in a 1D harmonic oscillator potential use the Heisenberg Uncertainty Principle (HUP) to approximately estimate the ground state energy for the 1D oscillator by assuming that the particle's momentum and position are related by the HUP and then minimizing the total energy:

$$E = \frac{p_x^2}{2m} + m\omega_0^2 x^2/2$$

Problem 2 - Exponential Wave Function

A particle is described by the wave function

$$\psi(x) = 0, \text{ for } x < 0$$

$$\psi(x) = Cxe^{-x/x_0}, \text{ for } x \geq 0$$

where x_0 and C are constants.

(a) (3 points) Draw a sketch of the wave function, indicating the approximate location of x_0

(b) (3 points) Determine the value of C, in terms of x_0, that normalizes the wave function.

(c) (3 points) Calculate the average position $< x >$ of the particle.

(d) (3 points) Where is the particle most likely to be found? That is, for what value of x (expressed in terms of x_0) is the probability of finding the particle the largest?

(e) (3 points) Calculate the average momentum $< p >$ of the particle.

(f) (4 points) Now calculate σ_x and σ_p for the particle and confirm that it satisfies the Heisenberg Uncertainty Principle.

Compare this result with $< x >$ and comment on the difference.

The following integrals may be useful and may save some keystrokes wrt Mathematica:

$$\int_0^\infty e^{-ax}dx = \frac{1}{a}$$

$$\int_0^\infty xe^{-ax}dx = \frac{1}{a^2}$$

$$\int_0^\infty x^2e^{-ax}dx = \frac{2}{a^3}$$

$$\int_0^\infty x^3e^{-ax}dx = \frac{6}{a^4}$$

$$\int_0^\infty x^4e^{-ax}dx = \frac{24}{a^5}$$

$$\int_0^\infty x^5e^{-ax}dx = \frac{120}{a^6}$$

Problem 3 - Truth or Consequences

First, for the infinite square well of width a with $V = 0$ for $0 < x < a$ and $V = \infty$ elsewhere, consider a particle of mass m in the ground state of this potential. Label each of the following statements as true or false and justify your answer.

(a) (3 points) We cannot predict with certainty the result of a single measurement of the particle's momentum.

(b) (3 points) We cannot make a measurement of the particle's momentum with arbitrarily small uncertainty.

(c) (3 points) We cannot make a measurement of the particle's position with arbitrarily small uncertainty.

(d) (3 points) We cannot predict with certainty the result of a single measurement of the particle's energy.

(e) (3 points) Since the particle is in a stationary state, repeated measurements of the particle's position will always yield the same answer.

Now, consider a localized free particle wave packet [$\psi(x) \to 0$ as $x \to \pm\infty$]. Label each of the following statements as true or false and justify your answer.

(f) (3 points) The wave function $\psi(x, t)$ can never be purely real for all t.

(g) (3 points) It is possible to make such a wave packet with $< T >= 0$, where $< T >$ is the expectation value of the kinetic energy.