Homework #6

* **Problem 3.14**

(a) Prove the following commutator identities:

$$\left[\hat{A}+\hat{B},\hat{C}\right]=\left[\hat{A},\hat{C}\right]+\left[\hat{B},\hat{C}\right], \tag{3.64}$$

$$\left[\hat{A}\hat{B},\hat{C}\right]=\hat{A}\left[\hat{B},\hat{C}\right]+\left[\hat{A},\hat{C}\right]\hat{B}. \tag{3.65}$$

(b) Show that

$$\left[x^n,\hat{p}\right]=i\hbar n x^{n-1}.$$

(c) Show more generally that

$$\left[f(x),\hat{p}\right]=i\hbar\frac{df}{dx}, \tag{3.66}$$

for any function $f(x)$ that admits a Taylor series expansion.

Problem 3.33 Sequential measurements. An operator $\hat{A}$, representing observable A, has two (normalized) eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two (normalized) eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1=(3\phi_1+4\phi_2)/5, \quad \psi_2=(4\phi_1-3\phi_2)/5.$$

(a) Observable A is measured, and the value a_1 is obtained. What is the state of the system (immediately) after this measurement?

(b) If B is now measured, what are the possible results, and what are their probabilities?

(c) Right after the measurement of B, A is measured again. What is the probability of getting a_1? (Note that the answer would be quite different if I had told you the outcome of the B measurement.)

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* **Problem 3.37 Virial theorem.** Use Equation 3.73 to show that

$$\frac{d}{dt}\langle xp\rangle = 2\langle T\rangle - \left\langle x\frac{\partial V}{\partial x}\right\rangle, \tag{3.112}$$

where T is the kinetic energy $(H = T + V)$. In a *stationary* state the left side is zero (why?) so

$$2\langle T\rangle = \left\langle x\frac{dV}{dx}\right\rangle. \tag{3.113}$$

This is called the **virial theorem**. Use it to prove that $\langle T\rangle = \langle V\rangle$ for stationary states of the harmonic oscillator.