SP6 ("Fun" with Angular Momentum)

Consider a rigid rotor whereby two particles of mass M are attached to the ends of a massless rigid rod of length $2a$. The system is free to rotate in 3-D about the center, but with the center point fixed. In this case the classical energy is given by $E = \frac{L^2}{4Ma^2}$, where L is the total angular momentum. Using this we can write the quantum Hamiltonian as

$$\hat{H} = \frac{\hat{L}^2}{4Ma^2}$$

(i) What are the eigenstates for this Hamiltonian and what are the energy eigenvalues?

(ii) Using the proton mass $M_p = 1.67 \times 10^{-27}$ kg and $2a = 1 \times 10^{-10}$ m, calculate the energy of the first excited state. This is the typical energy of rotational states for diatomic molecules.