Problem 4.28

$$r_c = \frac{(1.6 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})(9.11 \times 10^{-31})(3.0 \times 10^8)^2} = 2.81 \times 10^{-15} \text{ m.}$$

$$L = \frac{1}{2}\hbar = I\omega = \left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right) = \frac{2}{5}mrv \quad \text{so}$$

$$v = \frac{5\hbar}{4mr} = \frac{(5)(1.055 \times 10^{-34})}{(4)(9.11 \times 10^{-31})(2.81 \times 10^{-15})} = \boxed{5.15 \times 10^{10} \text{ m/s.}}$$

Since the speed of light is 3×10^8 m/s, a point on the equator would be going more than 100 times the speed of light. $\boxed{\text{Nope :}}$ This doesn't look like a very realistic model for spin.

Problem 5.4

(a)

$$1 = \int |\psi_\pm|^2 d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= |A|^2 \int \left[\psi_a(\mathrm{r}_1)\psi_b(\mathrm{r}_2) \pm \psi_b(\mathrm{r}_1)\psi_a(\mathrm{r}_2)\right]^* \left[\psi_a(\mathrm{r}_1)\psi_b(\mathrm{r}_2) \pm \psi_b(\mathrm{r}_1)\psi_a(\mathrm{r}_2)\right] d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= |A|^2 \left[\int |\psi_a(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_b(\mathrm{r}_2)|^2 d^3\mathrm{r}_2 \pm \int \psi_a(\mathrm{r}_1)^*\psi_b(\mathrm{r}_1) d^3\mathrm{r}_1 \int \psi_b(\mathrm{r}_2)^*\psi_a(\mathrm{r}_2) d^3\mathrm{r}_2\right.$$

$$\left.\pm \int \psi_b(\mathrm{r}_1)^*\psi_a(\mathrm{r}_1) d^3\mathrm{r}_1 \int \psi_a(\mathrm{r}_2)^*\psi_b(\mathrm{r}_2) d^3\mathrm{r}_2 + \int |\psi_b(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_a(\mathrm{r}_2)|^2 d^3\mathrm{r}_2\right]$$

$$= |A|^2(1\cdot 1 \pm 0\cdot 0 \pm 0\cdot 0 + 1\cdot 1) = 2|A|^2 \Longrightarrow \boxed{A = 1/\sqrt{2}.}$$

(b)

$$1 = |A|^2 \int \left[2\psi_a(\mathrm{r}_1)\psi_a(\mathrm{r}_2)\right]^* \left[2\psi_a(\mathrm{r}_1)\psi_a(\mathrm{r}_2)\right] d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= 4|A|^2 \int |\psi_a(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_a(\mathrm{r}_2)|^2 d^3\mathrm{r}_2 = 4|A|^2. \qquad \boxed{A = 1/2.}$$

Problem 5.5

(a)

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x_2^2} = E\psi} \quad \text{(for } 0 \le x_1, x_2 \le a, \text{ otherwise } \psi = 0).$$

$$\psi = \frac{\sqrt{2}}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

$$\frac{d^2\psi}{dx_1^2} = \frac{\sqrt{2}}{a}\left[-\left(\frac{\pi}{a}\right)^2\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \left(\frac{2\pi}{a}\right)^2\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

$$\frac{d^2\psi}{dx_2^2} = \frac{\sqrt{2}}{a}\left[-\left(\frac{2\pi}{a}\right)^2\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \left(\frac{\pi}{a}\right)^2\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

$$\left(\frac{d^2\psi}{dx_1^2}+\frac{d^2\psi}{dx_2^2}\right)=-\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{2\pi}{a}\right)^2\right]\psi=-5\frac{\pi^2}{a^2}\psi,$$

$$-\frac{\hbar^2}{2m}\left(\frac{d^2\psi}{dx_1^2}+\frac{d^2\psi}{dx_2^2}\right)=\frac{5\pi^2\hbar^2}{2ma^2}\psi=E\psi,\quad \text{with}\quad E=\frac{5\pi^2\hbar^2}{2ma^2}=5K.\quad \checkmark$$

(b) Distinguishable:

$\boxed{\psi_{22}=(2/a)\sin(2\pi x_1/a)\sin(2\pi x_2/a)\,,\ \text{with } E_{22}=8K}$ (nondegenerate).

$\boxed{\left.\begin{aligned}\psi_{13}&=(2/a)\sin(\pi x_1/a)\sin(3\pi x_2/a)\\ \psi_{31}&=(2/a)\sin(3\pi x_1/a)\sin(\pi x_2/a)\end{aligned}\right\},\ \text{with } E_{13}=E_{31}=10K}$ (doubly degenerate).

Identical Bosons:

$\boxed{\psi_{22}=(2/a)\sin(2\pi x_1/a)\sin(2\pi x_2/a),\ E_{22}=8K}$ (nondegenerate).

$\boxed{\psi_{13}=(\sqrt{2}/a)\left[\sin(\pi x_1/a)\sin(3\pi x_2/a)+\sin(3\pi x_1/a)\sin(\pi x_2/a)\right],\ E_{13}=10K}$ (nondegenerate).

Identical Fermions:

$\boxed{\psi_{13}=(\sqrt{2}/a)\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right)-\sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right],\ E_{13}=10K}$ (nondegenerate).

$\boxed{\psi_{23}=(\sqrt{2}/a)\left[\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right)-\sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)\right],\ E_{23}=13K}$ (nondegenerate).
