SP1. Hydrogen as a structure of minimum energy. The de Broglie hypothesis leads to the conclusion that the Hydrogen atom has its observed size because it minimizes the total energy of the system: larger size means longer de Broglie wavelength, and therefore smaller momentum and kinetic energy, while smaller size means smaller radius and thus lower potential energy, since the potential well is deepest near the nucleus. The observed size is a compromise which minimizes the sum of kinetic and potential energies.

a) Write down an expression for the total energy in terms of the momentum p and position relative to the nucleus r, keeping kinetic and potential energy terms separate. Then demand that the orbit *circumference* be one de Broglie wavelength. Obtain an expression for the total energy as a function of radius.

b) Find the radius that minimizes the total energy, and the energy at that radius. Compare with the Bohr model ground state energy (this is Eq. 4.70 in Griffiths' 3rd edition with n = 1).

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SP1.5

Flat "Wave Function"

Consider the following wave function

$$\psi(x) = A \ , \ \frac{-a}{2} \leq x \leq \frac{+a}{2}$$
$$\psi(x) = 0 \qquad , \qquad \text{elsewhere}$$

The potential, $V(x)$, is equal to zero everywhere.

(a) Technically this is not a physically allowed wave function. Explain why this is so. In spite of this issue we can still calculate some of its properties.

(b) Determine the normalization constant A.

(c) Compute Δx for this wave function.