

# Homework Set #2

**Problem 2.14** In the ground state of the harmonic oscillator, what is the probability (correct to three significant digits) of finding the particle outside the classically allowed region? *Hint:* Classically, the energy of an oscillator is  $E = (1/2)ka^2 = (1/2)m\omega^2a^2$ , where  $a$  is the amplitude. So the “classically allowed region” for an oscillator of energy  $E$  extends from  $-\sqrt{2E/m\omega^2}$  to  $+\sqrt{2E/m\omega^2}$ . Look in a math table under “Normal Distribution” or “Error Function” for the numerical value of the integral, or evaluate it by computer.

**\*Problem 2.5** A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)].$$

- (a) Normalize  $\Psi(x, 0)$ . (That is, find  $A$ . This is very easy, if you exploit the orthonormality of  $\psi_1$  and  $\psi_2$ . Recall that, having normalized  $\Psi$  at  $t = 0$ , you can rest assured that it *stays* normalized—if you doubt this, check it explicitly after doing part (b).)
- (b) Find  $\Psi(x, t)$  and  $|\Psi(x, t)|^2$ . Express the latter as a sinusoidal function of time, as in Example 2.1. To simplify the result, let  $\omega \equiv \pi^2\hbar/2ma^2$ .
- (c) Compute  $\langle x \rangle$ . Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (If your amplitude is greater than  $a/2$ , go directly to jail.)
- (d) Compute  $\langle p \rangle$ . (As Peter Lorre would say, “Do it ze *kveek* vay, Johnny!”)
- (e) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of  $H$ . How does it compare with  $E_1$  and  $E_2$ ?