Phys. 2b 2026 Week 10 (Lecture Notes 19) (3/10/2026)

Final will focus on the basics and the last 3 weeks. 5 points extra for TQFR.

QM for Ph2b

A. Solving 1D Schrödinger Equation

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &= \hat{H}\psi \\ H &= \frac{\hat{p}^2}{2m} + V \end{aligned}$$

For $\hat{H} \neq \hat{H}(t)$, Easy to solve **the** QM question:

Given arbitrary $\psi(x, t=0)$, find $\psi(t)$ (important since ψ contains **all** information possible)

Then to find $\psi(t)$

1. Solve TISE: $\hat{H}\psi_n = E_n\psi$ (to find the energy eigenstates - ψ_n and eigenvalues - E_n.)
2. Expand $\psi(x, t=0) = \sum_{n=1}^{\text{complete set}} c_n\psi_n$ with

$$c_m = \int \psi_m^*\psi(x, t=0)dx = \langle\psi_m|\psi\rangle \text{ is amplitude to be in } \psi_m$$

$$|c_m|^2 = |\langle\psi_m|\psi\rangle|^2 \text{ is probability that measuring energy leaves state in } \psi_m$$

3. $\psi(x,t) = \sum c_n\psi_n e^{-iE_nt/\hbar}$ Done!

B. H.U.P. $\Delta x\Delta p_x = \sigma_x\sigma_{p_x} \geq \frac{\hbar}{2} \Rightarrow$ can't make an ideal measurement of **both** x and p_x **at the same time**.

C. Example Solutions: Infinite Square Well, Free Particle, QSHO, $\delta(x)$ potential, Finite Well, Scattering…

D. **Math**: Observables are Hermitian operators and $\hat{Q}$ is Hermitian if

$$\int \phi^*\hat{Q}\psi dx = \int (\hat{Q}\phi)^*\psi dx = \langle\psi|\hat{Q}\psi\rangle = \langle\hat{Q}\psi|\psi\rangle$$

Commutators $[\hat{x}, \hat{p}_x] = i\hbar \rightarrow$ can an evaluate arbitrary commutator $[\hat{A}, \hat{B}]$ by operating on W.F., e.g. $[\hat{A}, \hat{B}]\psi$. See week 5 notes.

E. 3DQM

Hydrogen Atom: should know E_n, ψ_n (This is a Big Deal, we since can predict energy levels to 1 part per thousand!)

Angular Momentum

General Angular Momentum $\hat{\vec{J}}$ can be either orbital $\hat{\vec{L}}$ or spin $\hat{\vec{S}}$ or a sum of these.

All Angular momenum obey: $[\hat{J}_x, \hat{J}_y] = i\hbar\hat{J}_z$.

Also if $|jm\rangle$ is a simultaneous eigenstate of $\hat{J}^2$ and $\hat{J}_z$ then

$\hat{J}^2|jm\rangle = j(j+1)\hbar^2|jm\rangle$

$\hat{J}_z|jm\rangle = m\hbar|jm\rangle$

For orbital: $\hat{\vec{J}} = \hat{\vec{L}}$ and $|jm\rangle = Y_l^m$ - the Spherical Harmonics

For spin $\frac{1}{2}$, the eigenstates $|jm\rangle = |sm\rangle = \chi_\pm = \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

Above $\chi_\pm$ are eigenstates of $\hat{S}_z$. You can also construct eigenstates of $\hat{S}_x$ and $\hat{S}_y$: $\chi_\pm^{(x)}, \chi_\pm^{(y)} \rightarrow$ see Notes and Text. The matrix form for the spin 1/2 operators are:

$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}; \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}; \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

$\hat{S}^2 = \frac{3\hbar^2}{4}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

Then the total W.F.: $\psi_{Tot}(t) = \phi(x,y,z,t)\chi(t)$

F. Multiplarticle Wave Functions for Identical Particles & Entanglement

For $\frac{1}{2}$ integer spin particles = Fermions, the **total** W.F. must be antisymmetric:

$\psi(x_1, x_2) = -\psi(x_2, x_1)$.

For integer spin particles = Bosons, the **total** W.F. must be symmetric:

$\psi(x_1, x_2) = -\psi(x_2, x_1)$.

These Wave Functions can lead to the important concept of **Quantum Entanglement**

Consider the possible wavefunctions for two spin $\frac{1}{2}$ particles as discussed last week:

$$|s_{tot}m_{tot}\rangle = \sum_{m_1,m_2|} |s_1m_1\rangle|s_2m_2\rangle = \sum_{m_1,m_2|} |m_1m_2\rangle$$

Consider e.g. $|m_1m_2\rangle = |\uparrow\uparrow\rangle \leftarrow$ this state is clearly not entangled.

But $|m_1m_2\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ **IS** an entangled state.

A 2-particle state is entangled if the measurement of the state of one particle depends on the earlier result of a measurement of the other particle.

Clearly in the above entangled state, if the first particle is measured to be $|\downarrow\rangle$, the wave function for the other particle immediately collapses to the $|\uparrow\rangle$ state and vice versa.