March 16, 2026

PHYSICS 2b – Final

This exam covers all of the readings in the text and the lecture notes for the term.

This is an **OPEN BOOK** exam with the following limitations: You may consult only your textbook (Griffiths & Schroeter, Intro to QM), notes that you have taken in recitation or lecture, online lecture notes, and HW solns (posted as well as your own). No other references are allowed. You may use a calculator and symbolic manipulation programs (eg. Mathematica) for integrals or algebra, although they are not required.

In your studies, do not look at quizzes or exams from previous years of Phys 2 or 12.

The time limit is **4 HOURS**, in one continuous sitting. A 30 minute break beyond the 4 hours is permitted during this period, as long as you are not working on the exam during this time. No credit for overtime work.

This Exam is **DUE** Wed., March 18 at 11:59 PM and should be turned in via Gradescope.
Late Exams will not be accepted for credit except by prior arrangement with the Head TA.

There are 4 problems on pages 2–5 for a total of 70 points

To get partial credit, show as much work as you can!

FYI!
If you fill out the TQFR before March 20,
you get 5 (that's right 5)
extra credit points on the final.

Problem 1 - Short"ish" Problems

(a) (4 points) An electron in the second excited state of the hydrogen atom drops down to the first excited state by emitting a photon. Determine the wavelength of this photon and identify it as being either infrared, visible or ultraviolet. (IR = 1mm - 780 nm, Vis = 780 nm - 380 nm, UV = 380 - 80 nm).

(b) Using the eigenstates of $\hat{S}_z$ for a spin $\frac{1}{2}$ particle we can form superposition states from the two-particle eigenstates $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$ and $\downarrow\downarrow\rangle$ where $|m_1 m_2\rangle$ are the spin $\frac{1}{2}$ m quantum numbers for each particle. Label the states as E for entangled and N for not entangled and provide a brief explanation for your choices.

(i) (2 points) $\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$

(ii) (2 points) $\frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

(iii) (2 points) $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

(iv) (2 points) $\frac{1}{\sqrt{3}}(|\uparrow\uparrow\rangle - |\downarrow\uparrow\rangle)$

(c) An electron is in the following spin state:

$$\chi = A \begin{pmatrix} 1/\sqrt{2} \\ (1+i)/\sqrt{2} \end{pmatrix}$$

(i) (3 points) Determine the normalization constant A.

(ii) (3 points) Find the expectation values of S_x, S_y, and S_z for this state.

(d) "Lies, Damned Lies, and Statistics" (a quote often attributed to Mark Twain)
This statement was intended to indicate that Statistics is worse than a Damned Lie, but QM is basically Statistics, so please label the following statements as L, DL or S where L/DL are not true for QM, while S is valid for QM and give a brief explanation for your choice.

(i) (2 points) For a spin $= \frac{1}{2}$ particle we cannot make a simultaneous measurement of S_z and S_x.

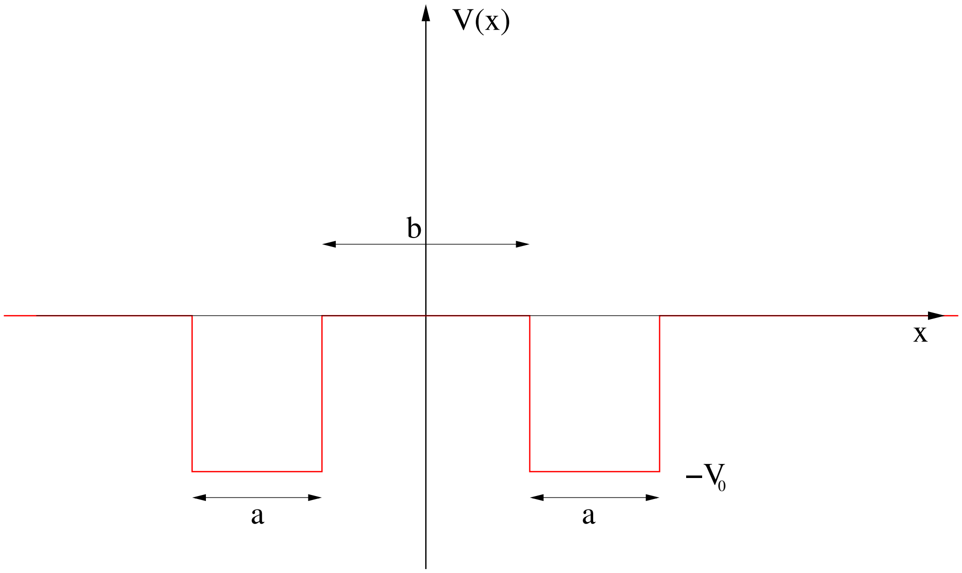
(ii) (2 points) For a spin $= \frac{1}{2}$ particle we cannot make a simultaneous measurement of S^2 and S_x.

(iii) (2 points) We cannot put more than two spin $\frac{3}{2}$ particles into the same $\psi(x, y, z)$ state.

(iv) (2 points) We can put four spin 1 particles into the same $\psi(x, y, z)$ state.

Problem 2: The Double Finite Square Well

Consider the double finite square-well potential shown below for a single particle of mass m.



For this problem assume that the well depth (V_0) and the width a are fixed and large enough so that several bound states exist. Also all of the following questions are qualitative questions - **no detailed calculations needed!**.

(a) (4 points) Sketch the ground state ψ_1 and first excited state ψ_2 wave functions for a barrier separation $b >> a$. This is a little tricky. We would expect to find the particle equally likely in either well and the math should give a solution that looks the same for both wells (up to a minus sign). If you're stuck or unsure, just move on to the next part and come back to this one later - you might get an "Ah Ha" moment after doing the later parts.

(b) (3 points) Sketch the ground state ψ_1 and first excited state ψ_2 wave functions for a barrier separation $b = 0$ (this is essentially a single well).

(c) (3 points) Is the ground state energy lower or higher for the case in part (a) compared to the case in part (b)? [Hint: remember that there's only one particle].

(d) (3 points) Is the energy gap between the ground state and first excited state lower or higher for the case in part (a) compared to the case in part (b)?

(e) (3 points) Sketch the ground state ψ_1 and first excited state ψ_2 wave functions for a barrier separation $b \sim a$. This case should be between the cases for parts (a) and (b).

Problem 3: The Exchange Operator

We can define an "exchange operator" that exchanges two identical particles in a two-particle wave function:

$$\hat{X}_{12}\psi(p_1, p_2) = \psi(p_2, p_1)$$

where $\psi(p_1, p_2)$ is an arbitrary two-particle wavefunction in one dimension: $\psi(p_1, p_2) = \phi_1\phi_2\chi_1\chi_2$ with ϕ a spatial state and χ a spin state.

(a) (3 points) What are the normalized eigenstates of $\hat{X}_{12}$ expressed in terms of superpositions of $\psi(p_1, p_2)$ and $\psi(p_2, p_1)$? and what are the corresponding eigenvalues?

(b) (4 points) Assume there is a certain class of particles that must be in an eigenstate of $\hat{X}_{12}$ with eigenvalue $= -1$. What happens when you put two such particles in the same spatial and spin state?

(c) (3 points) Repeat part (b), for particles that must be in an eigenstate of $\hat{X}_{12}$ with eigenvalue $= +1$.

(d) (4 points) Show that $\hat{X}_{12}$ is Hermitian.
[Hint: remember variables inside an integration are just "dummy" variables]

Problem 4: Simple Model for Quarks in a Proton

Years ago your Prof scattered high energy electrons from the quarks inside the proton. We're going to model the short-range "Strong Force" that holds the quarks in the proton as a simple 3D harmonic oscillator inside the proton and $V(r) = 0$ outside the radius of the proton ($r = r_p$).

(a) (4 points) Consider a 3D harmonic oscillator for which the potential is

$$V(r) = \frac{1}{2}m\omega^2 r^2, \quad \text{where } r^2 = x^2 + y^2 + z^2$$

Show that by assuming the energy eigenstates are a simple product wave function in cartesian coordinates: $\psi(x, y, z) = X(x)Y(y)Z(z)$, where X, Y, Z are only functions of the indicated variable, we get three one-dimensional harmonic oscillators with allowed total energy:

$$E_n = \left(n + \frac{3}{2}\right)\hbar\omega, \quad \text{where } n = n_x + n_y + n_z, \text{ with } n_x, n_y, n_z \text{ are positive integers } 0, 1, 2, ...$$

(b) (4 points) It takes a certain energy ($E_q \sim 1.6 \times 10^{-10}$ joules) to liberate a quark from inside the proton (much like electrons in atoms).
Use the Heisenberg Uncertainty Principle (HUP) to show that it's possible for the quark (with $m_q = m_p/3$) to be confined inside the proton by setting $\sigma_x = r_p$ and using the HUP to find σ_{p_x} to show that the corresponding kinetic energy $\sigma_p^2/2m_q$ is less than E_q. Note that the proton has a mass of $m_p = 1.67 \times 10^{-27}$ kg and a radius of $r_p = 0.84 \times 10^{-15}$ m.

(c) (3 points) Use E_q and r_p to determine the quark ground state energy, i.e. when $n = 0$ in the result from part (a). You can find ω using $E_q = \frac{1}{2}m_q\omega^2 r_p^2$

(d) (3 points) Now you can determine the energy gap between the ground state ($n = 0$) and first excited state of the proton ($n = 1$) and compare it to the experimental energy $E_1 - E_0$ of 8×10^{-11} joules.