Ph. 203 - L1 (1)

No Exams → 9 HW sets 50% to pass P/F

See Canvas for details

Parts from 2 texts see Bertulani
Bhaduri

Coverage: N-N Force — Nucleon Models + QCD

Nuclei Props. — Nucl. Astro.

Nuclear Models

NP resources: PDG (web) + NNDC (web)

<u>QM prelims:</u> Ang. Mom. & Scattering

<u>Generalized Ang. Mom.</u> defined by

$$[\hat{J}_i, \hat{J}_j] = i\,\epsilon_{ijk}\,\hat{J}_k \quad ; \quad i,j,k = 1,2,3$$

$\hat{A}$ = operator

↑ anti-sym. unit tensor

$\epsilon_{123} = \epsilon_{312} = \epsilon_{231} = 1$

$\epsilon_{321} = \epsilon_{132} = \epsilon_{213} = -1$

all others $= 0$

P

Class

Eigenstates of $\hat{J}^2$ & $\hat{J}_3$ are $|jm\rangle$, j = int. or $\frac{1}{2}$ integ. ≥ 0

$\hat{J}_\pm |jm\rangle \propto |j\,m\pm1\rangle$ if $|j\,m\pm1\rangle$ exist

$= 0$ otherwise

<u>Addition of Ang. Mom.</u>

can combine $\vec{j}_1$ & $\vec{j}_2$ (e.g. spin + orbit, spin + spin, ...)

to form eigenstates of J_{total} $\hat{\vec{J}} = \hat{\vec{j}}_1 + \hat{\vec{j}}_2$

$$\Leftrightarrow |JM\rangle = \sum_{m_1, m_2} \underbrace{\langle j_1 j_2 m_1 m_2 | JM\rangle}_{\text{C.G. coefficient}} |j_1 m_1\rangle |j_2 m_2\rangle$$

46. Clebsch-Gordan Coefficients, Spherical Harmonics, and d Functions

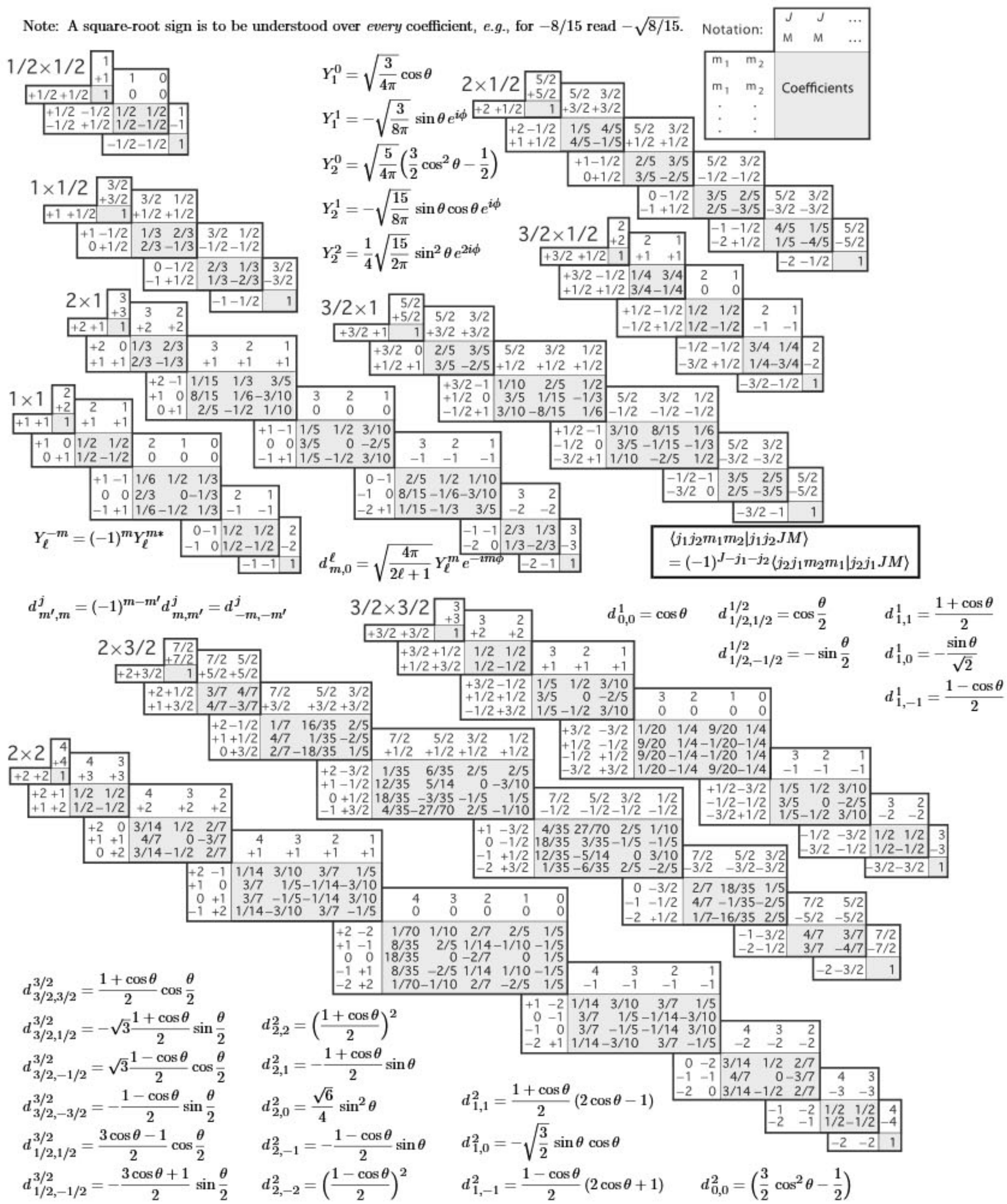

Figure 46.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

include C.G pic

2

Example 1 combine 2 spin $\frac{1}{2}$ nucleons (n or p)

$$\vec{J} = \underbrace{\underbrace{\vec{s}_1 + \vec{s}_2}_{S} + \vec{\ell}}_{J}$$ → relat. orb. ang. mom.

∴

$\ell = 0$		NP Lingo	$\ell = 1$	
$\vert s\, m_s\rangle$	$\vert J\, M_J\rangle$		$\vert s\, m_s\rangle$	$\vert J\, m_J\rangle$
$\vert 11\rangle$	$\vert 11\rangle$		$\vert 11\rangle$	$\vert 2, \begin{smallmatrix}2\\ \vdots \\ -2\end{smallmatrix}\rangle \Rightarrow {}^3P_2$
$\vert 10\rangle$	$\vert 10\rangle$	${}^3S_1$	$\vert 10\rangle$	$\vert 1 \begin{smallmatrix}1\\ \vdots \\ -1\end{smallmatrix}\rangle \Rightarrow {}^3P_1$
$\vert 1{-1}\rangle$	$\vert 1{-1}\rangle$		$\vert 1{-1}\rangle$	$\vert 00\rangle \Rightarrow {}^3P_0$
$\vert 00\rangle \rightarrow$	$\vert 00\rangle \rightarrow$	${}^1S_0$	$\vert 00\rangle \rightarrow$	$\vert 1 \begin{smallmatrix}1\\ \vdots \\ -1\end{smallmatrix}\rangle \Rightarrow {}^1P_1$

where ↔ ${}^{2S+1}\ell_J$

& for $\ell = 2 \Rightarrow$ ${}^3D_3,\ {}^3D_2,\ {}^3D_1,\ {}^1D_2$

then 2N W.F.:

e.g. $|{}^3P_0\rangle_{M=0} = \sum_i (C.G.)\,|s\,m_s\rangle|\ell\, m_\ell\rangle = |JM\rangle = |00\rangle$

$$= \frac{1}{\sqrt{3}}\left(|11\rangle Y_{1-1} - |10\rangle Y_{10} + |1{-1}\rangle Y_{11}\right)$$

see C.G.

or

$$|{}^3P_2\rangle_{M=2} = \underset{JM}{|22\rangle} = \underset{sm}{|11\rangle} Y_{11}$$

Ex. 2: Isospin as Gen. Ang. Mom.

Assume p & n are 2 states of same particle → nucleon

↳ see later

then isospin W.F. ⇒ $|t\, t_3\rangle$ eigenstates of $\hat{T}^2, \hat{T}_3$

w $|\tfrac{1}{2}\tfrac{1}{2}\rangle = p$

$|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle = n$

also works for u, d quarks

③

w $\hat{\vec{T}} = \frac{1}{2}\hat{\vec{\tau}}$, $\hat{\vec{\tau}} \equiv \hat{\vec{\sigma}}$ & W.F. is spinor

&

$\hat{T}^2 |t\, t_3\rangle = t(t+1)|t\, t_3\rangle$; $\hat{T}_3 |t\, t_3\rangle = t_3 |t\, t_3\rangle$

$\therefore$ $\hat{T}^2 |p\rangle = \frac{3}{4}|p\rangle$, $|p\rangle$ = proton

also have $\hat{T}_{\pm}$ w $\underbrace{\hat{T}_- |p\rangle = |n\rangle}_{\beta\text{-decay}}$, $\hat{T}_- = \hat{T}_1 - i\hat{T}_2$

also works for $t > \frac{1}{2}$

$\Leftrightarrow$ $\pi^{+,0,-}$ $|\pi^+\rangle = |11\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$|\pi^0\rangle = |10\rangle$

$|\pi^-\rangle = |1-1\rangle$

G-parity

$G = (-1)^I C$

Ques: what's $|00\rangle$? $\Rightarrow \eta^0$

$C = (-1)^{l+s}$ for $f\bar{f}$

here 3x3 rep. of SU(2):

$\hat{T}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$; $\hat{T}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}$; $\hat{T}_3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$

Rotations & Tensor Operators

Can change basis (e.g. rotate by 90°) of Ang. Mom. eigenstates via rotation operator:

$$\hat{R}(\vec{\theta}) = e^{-i\hat{\vec{J}}\cdot\vec{\theta}}$$ $\rightarrow$ ($\hbar = 1$ here)

clearly $[\hat{R}, \hat{J}^2] = 0$ $\therefore$ j in $|jm\rangle$ unchanged

but, generally, m will change via

$$|jm\rangle' = \hat{R}(\vec{\theta})|jm\rangle = \sum_{m'=-j}^{+j} |j\,m'\rangle \underbrace{\mathcal{D}^{j}_{mm'}(\vec{\theta})}_{\text{Rotation Matrix}}$$

(4)

for arbitrary $\vec{\theta}$ can use Euler angles α, θ, γ

w $\hat{\mathbb{R}}(\vec{\theta}) = \mathbb{R}(\alpha, \theta, \gamma) \equiv e^{-i\gamma \hat{J}_3''} e^{-i\theta \hat{J}_2'} e^{-i\alpha \hat{J}_3}$

↳ $\hat{J}_2'$ is in sys. rotated by α

OR!!

$= e^{-i\alpha \hat{J}_3} e^{-i\theta \hat{J}_2} e^{-i\gamma \hat{J}_3}$

↳ all in fixed coord. sys.! (↳ lab)

$\therefore \mathcal{D}^j_{m'm}(\alpha, \theta, \gamma) = \langle j m' | \mathbb{R}(\alpha, \theta, \gamma) | j m \rangle$

$= e^{-im\gamma} e^{+im'\alpha} \underbrace{\langle j m' | e^{-i\theta \hat{J}_2} | j m \rangle}$

reduced rotation matrix element $\equiv d^j_{m'm}(\theta)$

e.g. $d^{1/2}_{\frac{1}{2}\,-\frac{1}{2}} = -\sin(\frac{\theta}{2})$

$d^2_{21} = -\frac{1}{2}\sin\theta(1 + \cos\theta)$

see P.D.G.

Spherical Tensors:

An irreducible spherical tensor of rank n $[T_{n\mu}(c); \mu = -n, \ldots, +n]$ (coordin.) transforms like Ang. Mom. under coord. rotation:

$$T_{n\mu}(c) = \sum_{\mu'} T_{n\mu'}(c') \mathcal{D}^n_{\mu'\mu}(\vec{\theta})$$

Note:

① Scalar is sph. Ten. of rank ϕ $\Rightarrow T_{00} = a$

② Vector w components A_1, A_2, A_3 can be a spherical tens. of rank 1 via

$T_{10} = A_3$, $T_{1\pm1} = \mp\frac{1}{\sqrt{2}}(A_1 - iA_2)$

3 $T_{n\mu}$ can be quantum States or operators

leads to (no proof) <u>Wigner - Eckart Thm</u>

Matrix element of $T_{n\mu}$:

$$\langle \sigma j' m' | \hat{T}_{n\mu} | \sigma j m \rangle = \langle \sigma j' \| \hat{T}_n \| \sigma j \rangle \times \left[\frac{\langle j n m \mu | j' m' \rangle}{\sqrt{2j'+1}} (-1)^{2n} \right]$$

other quant. #'s

C.G.

⇒ Can lead to selection rules in γ or β decay if $C.G = \phi$

<u>Ex</u>: spin $\frac{1}{2}$ particle cannot have Quad. Moment

$$\hat{Q} = e(3z^2 - r^2) = \left(\frac{16\pi}{5}\right)^{1/2} e r^2 Y_{20}$$

(see HW for $\hat{Q}$)

N.R. Scattering

Consider upolarized, elastic scattering

$$\psi_{tot}(\vec{r}) = \underbrace{e^{ikz}}_{\text{incident plane wave}} + \underbrace{f(\theta)}_{\text{scattering amplitude}} \overbrace{\frac{e^{ikr}}{r}}^{\text{scattered wave}}$$

Due to finite range of NN force [($r_0 \sim$ few fm) $\Rightarrow V = 0$ if $r > r_0$]
partial wave expansion of $f(\theta)$ is <u>very</u> <u>useful</u>

Semi-classically w $|\vec{L}_{max}| \simeq r_0 P$
$$\simeq \sqrt{\ell(\ell+1)}\,\hbar$$
$$\ell_{max} \simeq \frac{r_0 P}{\hbar} = k r_0$$

$\therefore$ if $k r_0$ is small, only few ℓ values needed for $f(\theta)$

see link to 3D scattering

$$f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} Y_{\ell 0}(\theta) \sqrt{4\pi(2\ell+1)}\, e^{i\delta_\ell} \sin \delta_\ell$$

phase shift

Note:

① Since $\frac{d\sigma}{d\Omega} \equiv |f(\theta)|^2$

$$\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) \sin^2 \delta_\ell$$

since cross terms vanish b/c $Y_{\ell m}$ are orthog.

② δ_ℓ completely specifies elastic scattering

w $\delta_\ell > 0$ for attractive V

$\delta_\ell < 0$ " repulsive "

③ Since $f(0) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1)(\cos\delta_\ell + i \sin\delta_\ell) \sin\delta_\ell$

b/c $Y_{\ell 0}(0) = \sqrt{\frac{2\ell+1}{4\pi}}$

then $\underbrace{\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im} f(0)}_{}$

special case of Optical Thm