⇒ Finished w Review now "Real" NP... Consider:

Deuteron = simplest nucleus

Properties:

(I) Binding Energy: 2.22 MeV = E_b ← slow n capture on p

⇒ only 1 bound state ($\frac{E_b}{V_0} \sim 5\%$!) ↪ see HW 2

$m_p = 938.27$ MeV, $m_n = 939.56$ MeV ($c \equiv 1$)

(II) Intrinsic Spin/Parity: $J^\pi = 1^+$

(III) Isospin: $t = 0$, $t_3 = 0$

(IV) Mag. Moment $\mu_d = 0.857\,\mu_N$

↪ μ_N = nuclear magneton

$\equiv \frac{e\hbar}{2m_p c}$ (Dirac moment)

$\ll \mu_B$

⇒ note $\mu_p = 2.79\,\mu_N$; $\mu_n = -1.91\,\mu_N$

(V) Quadrupole Moment: $Q_d = 0.286 \pm .002\ e\text{fm}^2$

recall $\hat{Q} = e(3z^2 - r^2)$

fm = 10^{-15} m

(VI) Charge Radius $r_d^8 = 2.128 \pm 0.001$ fm

↪ note $r_p^8 = 0.842 \pm 0.001$ fm (Maybe!) see later

$r_n^8 = -0.106 \pm 0.006$ fm

Discuss each of above...

(I) $E_b = 2.22$ MeV ≃ γ-ray energy from $n + p \rightarrow d + \gamma$ (slow n)

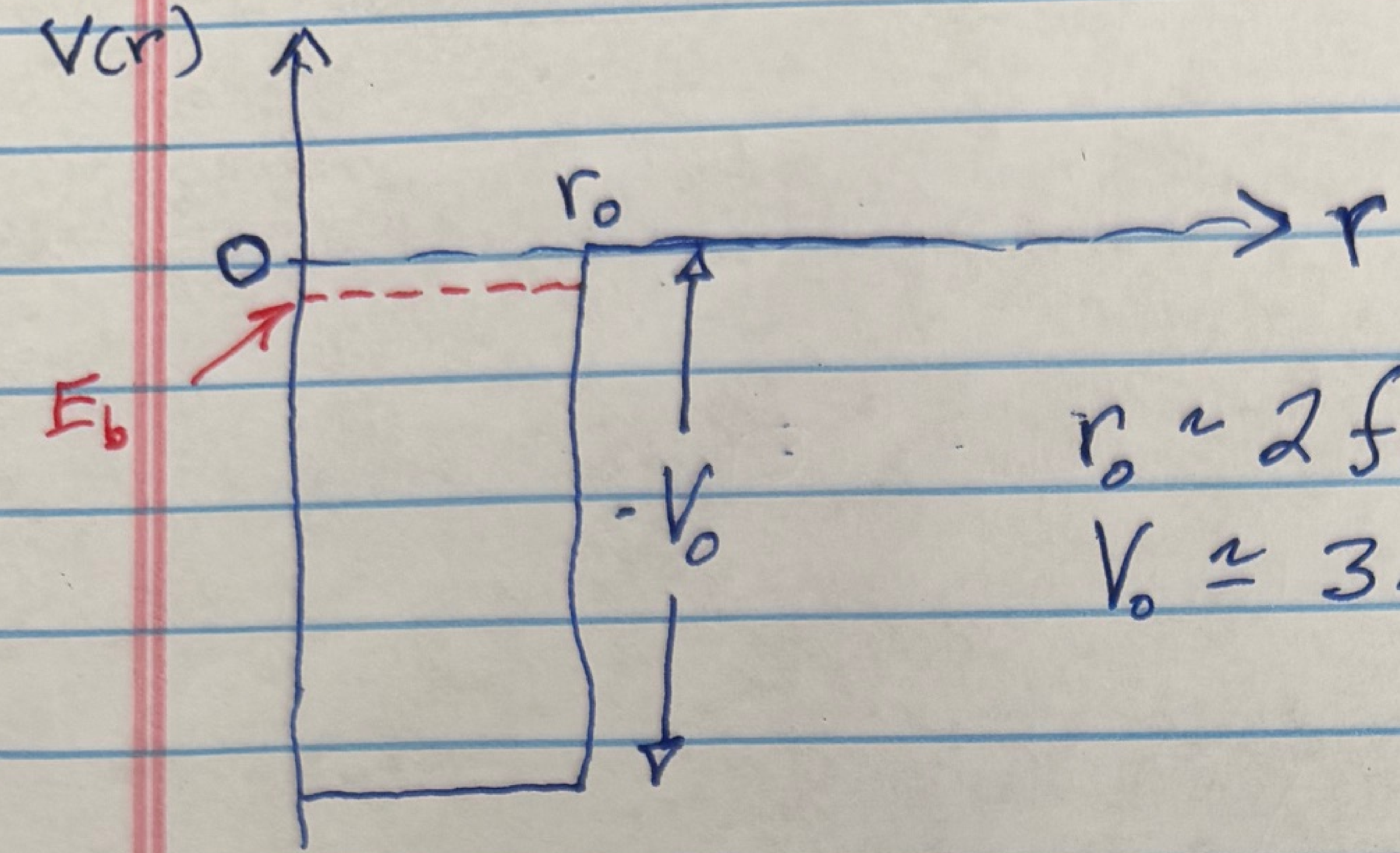
NP lingo:

↪ $p(n, \gamma)d$

↗ target ↑ beam ↖ detected

(2)

Can reproduce this via finite Square Well (HW 2).



$r_0 \sim 2\,fm$

$V_0 \simeq 35\,MeV$

(II) $J^\pi = 1^+$ ⇒ consistent w/ 2 spin $\frac{1}{2}$ particles coupled to $J=1$ in $l = 0, 2, \ldots$ state (see later ⇒ no $l=1$)

$J^\pi_{n,p} = \frac{1}{2}^+$ & $l = 0, 2$ ⇒ $\pi = +$

(III) $|t\, t_3\rangle = |00\rangle$ why not $|10\rangle$?

$t_3 = t_3^p + t_3^n = 0$

Assuming d is composed of 2 "identical" fermions

$$\psi_d = \psi_{space} \times \psi_{spin} \times \psi_{isospin}$$

need ψ_d anti sym. under $n \leftrightarrow p$ exchange

∴ if ψ_{space} is $l = 0, 2$ ⇒ sym!

ψ_{spin} is triplet $(S=1)$ ⇒ sym!

∴ $\psi_{isospin}$ is singlet $t=0$ ⇒ antisym.!

(IV) Mag. Moment ⇒ due to $\mu_n + \mu_p$ + orbital motion of p

thus if d is pure $l=0$ $\mu_d = \mu_n + \mu_p$

$= 0.88\,\mu_N$

& $\mu_d^{exp} = 0.857\,\mu_N$ => ~3% difference → Why?

(VI) Quad Moment (deduced from hyperfine splitting in HD & D_2)

$$Q_d \equiv \langle JM=J|\hat{Q}|JM=J\rangle$$

↳ likewise for $\underline{\underline{\mu_d}}$

$Q_d > 0$ requires non-spherical → spin can't do this

∴ need $l > 0$ => $l=1$ has wrong parity ∴

try $\psi_d = a\varphi_S + b\varphi_D$

w/ $\varphi_S = R_S(r)\,\mathcal{Y}^1_{110}$; $\varphi_D = R_D(r)\,\mathcal{Y}^1_{112}$

↑ space (θ,φ) & spin via

$$\mathcal{Y}^{M_J}_{JSL}(\theta,\varphi) = |JM\rangle_{LS} = \sum_{m_L m_S} Y_{Lm_L}(\theta,\varphi)\,(C.G.)\,|S m_S\rangle$$

then $\mathcal{Y}^1_{110} = Y_{00}|11\rangle$; $\mathcal{Y}^1_{112}$ = HW

then if $|b|^2 \approx 4\%$

can get $Q_d^{thy} = Q_d^{exp} = 0.286\ e\cdot fm^2$

<u>Note!</u>

① Pure D-state $|b|^2=1$ gives $Q_d < 0$ oblate
$(3z^2 - r^2)$

② Why D-state in gnd state? => "Tensor" Force = next week

<u>Info on N-N Force</u>: from low E N-N scattering

@ low Energy only $l=0$ important

How low?

$pr_0 << h$

$\Rightarrow$ need $kr_0 << 1$, $r_0 \sim 2\,fm$, $E_{cm} = \frac{\hbar^2 k^2}{2(\frac{M_N}{2})}$ $\quad M_N = \frac{m_n + m_p}{2}$

note $\frac{m_n}{m_p} \doteq 1.001$

$\therefore E_{cm} << 10\ MeV$

$l = 0$ Scattering

$$f_{l=0} = \frac{e^{i\delta_0} \sin\delta_0}{k} \quad ; \quad \sigma_0^{tot} = \frac{4\pi}{k^2} \sin^2\delta_0$$

Expect $\sigma_{tot}(k \to 0) \simeq$ const. (not $= \infty$)

$\hookrightarrow \therefore \delta_0 \propto k$ as $k \to 0$

& $\lim_{k \to 0} \sigma^{TOT} = 4\pi a^2$

$\hookrightarrow$ scattering length

w/ $f_0 = \frac{\sin\delta_0}{k(\cos\delta_0 - i\sin\delta_0)}$

define $a \equiv \lim_{k \to 0} \left(\frac{-1}{k\cot\delta_0}\right) = -\lim_{k \to 0}(f_0)$

$\hookrightarrow$ since $\sin\delta_0 << \cos\delta_0$ for $k \to 0$

Pics of a see next page

Note

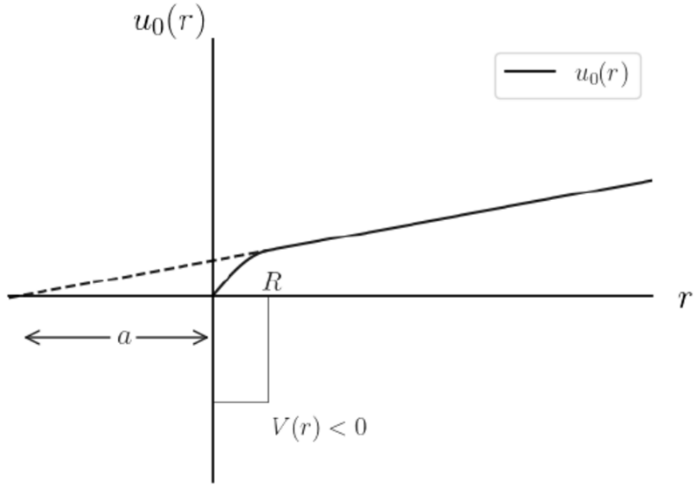
① For attractive potential

- if $a < 0 \Rightarrow$ no bound states
- if $a > 0 \Rightarrow$ bound states possible

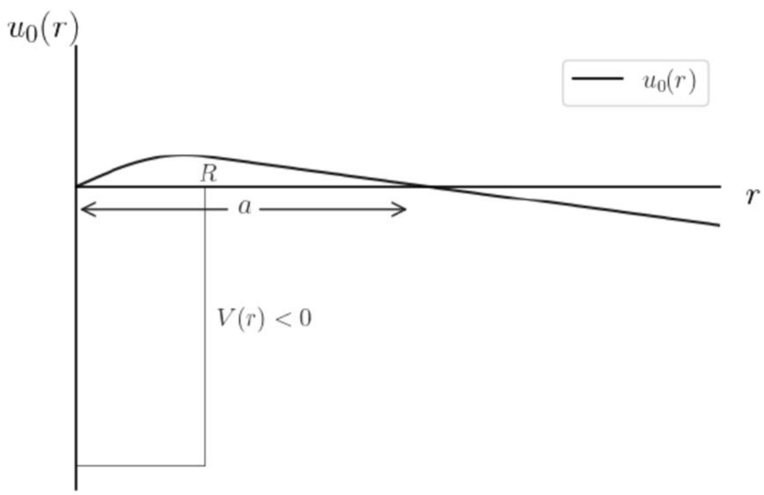
② For $k \gtrsim 0$, but still dominated by $l = 0$, can expand $k\cot\delta_0$ in powers of k:

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \ldots$$

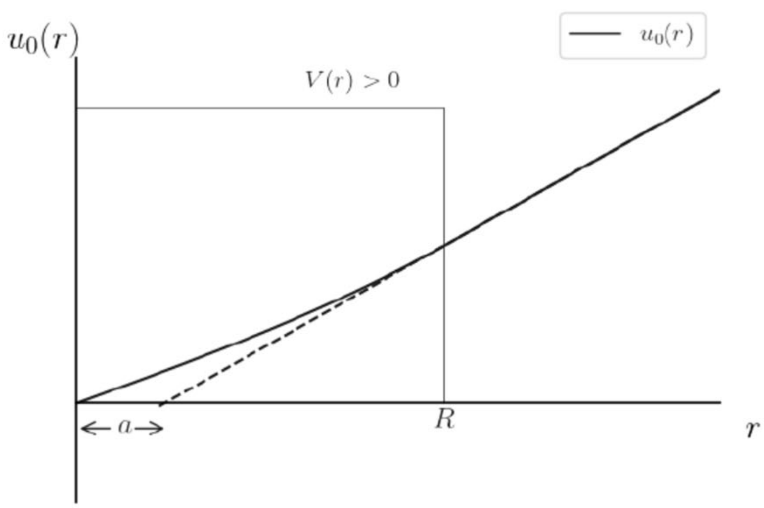
$\hookrightarrow$ called effective range for localized potential



(a) An attractive potential that is not strong enough to produce a bound state. In this case, $a < 0$ because we need to extrapolate the radial function to negative values to intercept the r-axis.



(b) A stronger attractive potential produces a bound state, and $a > 0$.



(c) For a repulsive potential, we always have $a > 0$.

Examples:

Gaussian: $V = -V_G \, e^{-r^2/r_G^2}$

Exponential: $V = -V_E \, e^{-r/r_E}$

Yukawa: $V = -V_Y \dfrac{e^{-r/r_Y}}{r/r_Y}$ → motivated by meson exchange (e.g. $\pi^{\pm}_{0}$)

"a" from low E N-N Scattering

① pp scattering

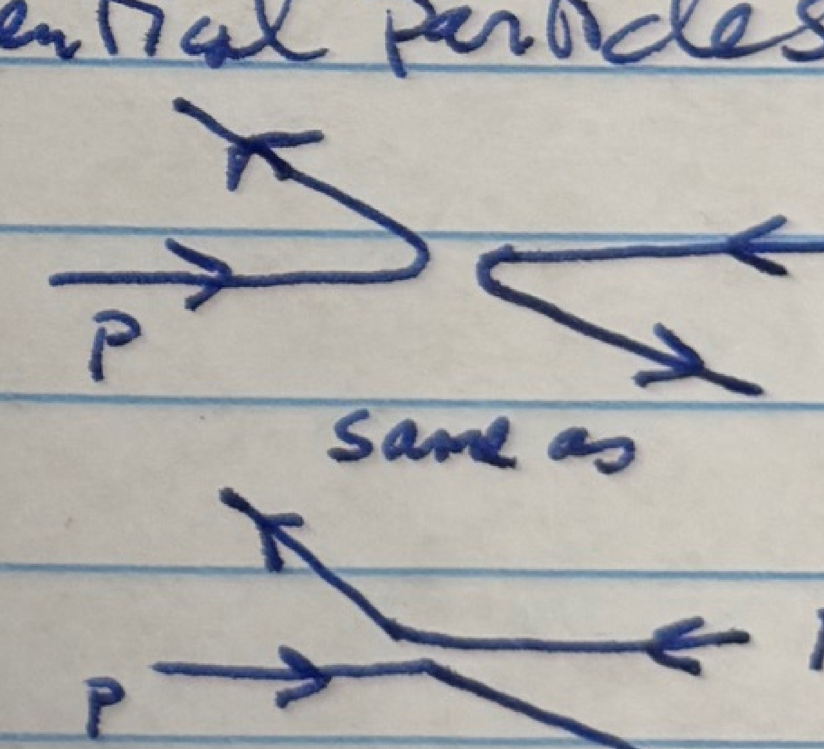
for $l=0$, proton spins must be in singlet state ($^1S_0$), since $t = t_3 = 1$ (antisym.)
↑ sym.

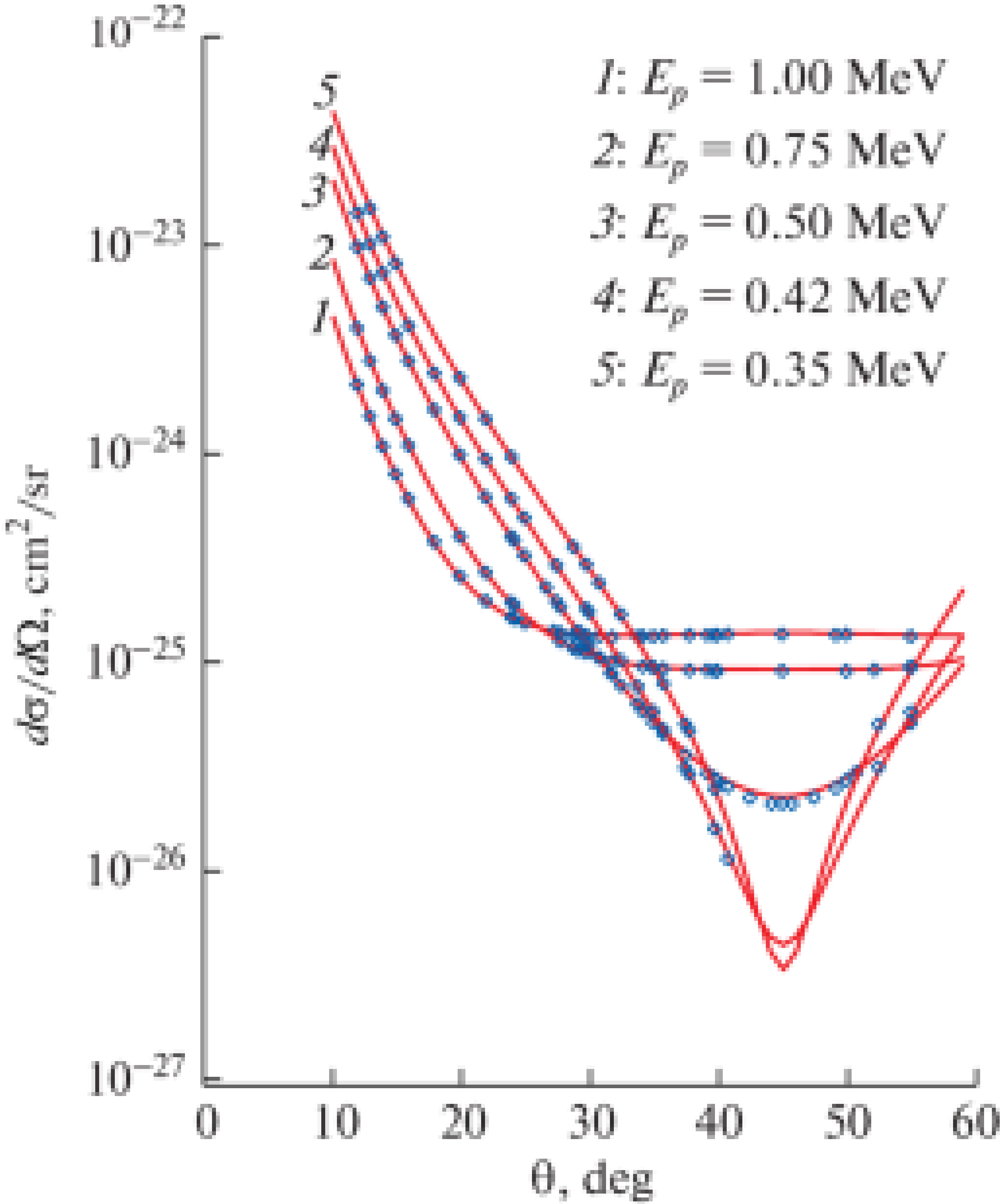
Coulomb interaction dominates for $E \lesssim 0.2$ MeV

Data for 0.3 MeV → 1 MeV

↳ see next page

Note: $\frac{d\sigma}{d\Omega}$ must be sym. wrt 90° since identical particles





10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
$d\sigma/d\Omega$, cm^2/sr
0
10
20
30
40
50
60
θ, deg
1
2
3
4
5
1: E_p = 1.00 MeV
2: E_p = 0.75 MeV
3: E_p = 0.50 MeV
4: E_p = 0.42 MeV
5: E_p = 0.35 MeV

Extract a_{pp} from Coulomb-Nuclear interference
also gives sign ←

w EM corrections:

$a_{pp}^{N} = -17.1 \pm 0.2$ fm, also $r_e^{pp} = 2.794 \pm 0.015$ fm

↑ implies no bound state for attract. V

② nn Scattering

→ Again only $^1S_0$ possible

→ EM corrections are small @ low E
(only $\vec{\mu}_n$ dipole-dipole)

→ Use indirect exps. via nn final states

e.g. $n + d \to n n p$
or
$\pi^- + d \to n n \gamma$

} p/γ gives info on nn distrib. ↳ $\frac{d\sigma_{nn}}{d\Omega}$

Results: $a_{nn} = -18.7 \pm 0.6$ fm

& $r_e^{nn} = 2.84 \pm 0.03$ fm,

∴ $a_{pp}^{N} \simeq a_{nn}$ ∴ N-N interaction is "charge" symmetric

↳ small charge-symmetry breaking also possible

This is consistent w isospin symmetry but isospin symmetry also requires charge independence

⟺ $a_{nn} = a_{pp}^{N} = a_{pn}^{^1S_0}$

↳ see Next Time