Ph 203 L3

Last time:

Recall! $\delta_\ell > 0$ Attractive V, $a < 0$ no bound state
$\delta_\ell < 0$ Repulsive V ; $a > 0$ bound state OK

called "Charge Symmetry" $nn \equiv pp$

$$\left\{\begin{array}{l} pp: \ {}^1S_0,\ a^N_{pp} = -17.1 \pm 0.2\,\text{fm},\quad r_e = 2.79 \pm 0.02\,\text{fm} \\ nn: \ {}^1S_0,\ a^{nr}_{nn} = -18.7 \pm 0.6\,\text{fm},\quad r_e = 2.84 \pm 0.03\,\text{fm} \end{array}\right.$$

Info from Low E n-p Scattering

→ EM corrections are modest

→ Scattering can occur in both ${}^1S_0$ or ${}^3S_1$, since $t = 0$ or 1 are possible

↳ Thus could be different "a" for spin $S=0$ (singlet) "a_s" & spin $S=1$ (triplet) "a_t":

∴ for $l = 0$ define 4×4 matrix for f_0:

$$f_0 = -\begin{array}{c} \begin{array}{cccc} |11\rangle & |10\rangle & |1{-1}\rangle & |00\rangle \end{array} \\ \begin{pmatrix} a_t & & \phi & \\ & a_t & & \\ & \phi & a_t & \\ & & & a_s \end{pmatrix} \end{array} \begin{array}{l} |11\rangle \\ |10\rangle \\ |1{-1}\rangle \\ |00\rangle \end{array}$$

⇒ matrix must be diagonal since J^2 & J_z are conserved

⇒ Can't depend on S_z ($\hat{H}$ → Hamiltonian → is invariant under rotations

⇒ but $a_s \neq a_t$ is possible

~~thus~~ for unpolarized np scattering expect an incoherent sum over spin states

$$\sigma_{np}(E \to 0) = 4\pi\left(\frac{3a_t^2 + a_s^2}{4}\right)$$

E ~ MeV

↳ probabilities add, not amplitudes

$$\sigma^{Exp}_{np}(E \to 0) = 20.44 \pm 0.23\,\text{b}$$

↳ barn $= 10^{-28}\,\text{m}^2 = 100\,\text{fm}^2$

To separately determine a_s & a_t can scatter "cold" neutrons $\lambda_n \sim 10$ Å from H_2 molecules

↪ note "thermal" neutrons $\lambda \sim 1$ Å

MB dist. @ 300K

But... use para-H_2 (recall para H $\uparrow\downarrow$ (pp) gnd state

ortho H $\uparrow\uparrow$ (ortho) $l = 1$ state

then $\lambda_n > R_{H_2}$

Both protons contribute coherently (amplitudes add)

$\therefore$

$$\sigma_{H_2}^{para} = 4\pi\left(\frac{3a_t + a_s}{4}\right)^2 ; \quad \underbrace{\sigma_{H_2\,exp}^{para} = 1.73 \pm 0.01\,b}$$

$\therefore$ $3a_t = -a_s$, but which is negative?

⟺ since deuteron is bound ($S=1$, t) need $a_t > 0$

$\therefore$ $a_t^N = 5.423 \pm 0.005$ fm ; $r_e^t = 1.73 \pm 0.02$ fm

$\underline{a_s^N = -18.5 \pm 1.5 \text{ fm}}$; $r_e^s = 2.73 \pm 0.03$ fm

$\boxed{.72 \pm 0.02 \text{ fm}}$

↳ consistent w/ a_{nn}, a_{pp}^N

↪ suggest charge independence of NN force

<u>Conclusions on N-N interaction</u>

1. $[\hat{H}, \hat{T}_3] = 0 \Rightarrow$ isospin symmetry
2. Is spin and/or isospin dependent $a_s \neq a_t$
3. 2 nucleons are unbound in $^1S_0$, but bound in $^3S_1$ (np)

but something's missing ⇒ how to get $Q_d > 0$??

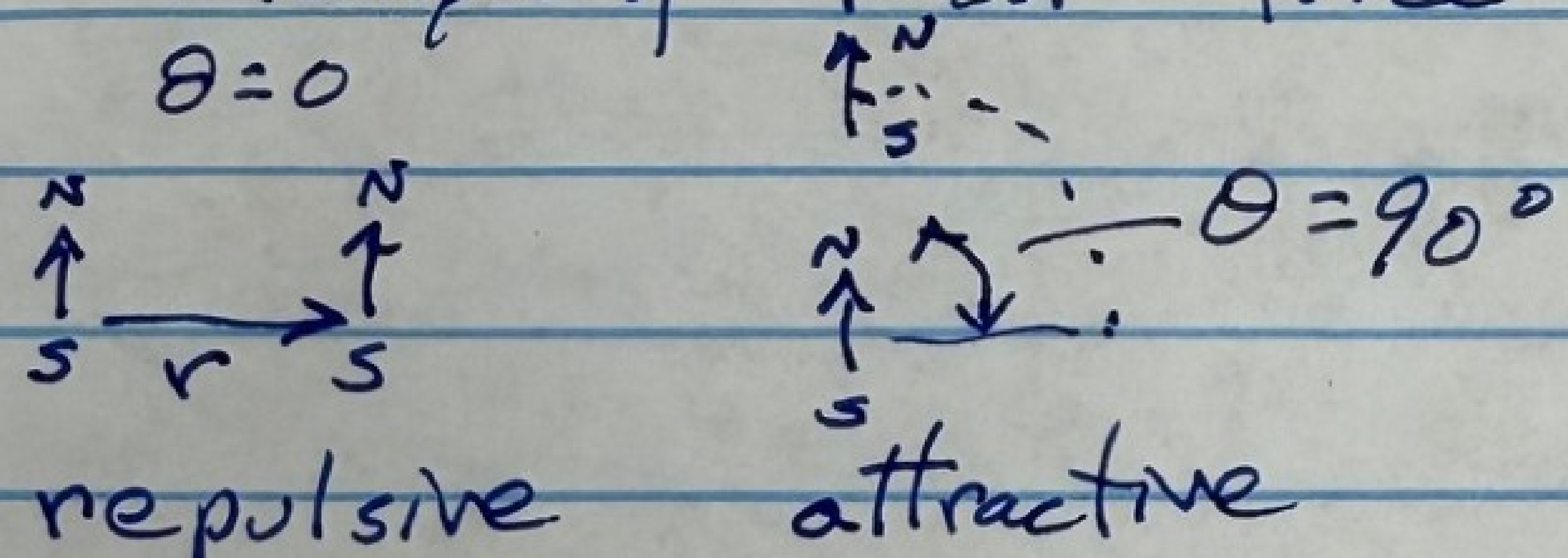
⟺ must have $V(\vec{r})$ non-central : $V(\vec{r}) \neq V(r)$

⟺ Consider "Tensor" Force : $V_{Tensor} = V_T(r)\,\hat{S}_{12}$

with $$\hat{S}_{12} = \underbrace{\frac{3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2}$$

This is scalar product of 2 rank 2 tensors (see HW 3)

⟺ Example of "Tensor" force : 2 magnetic dipoles



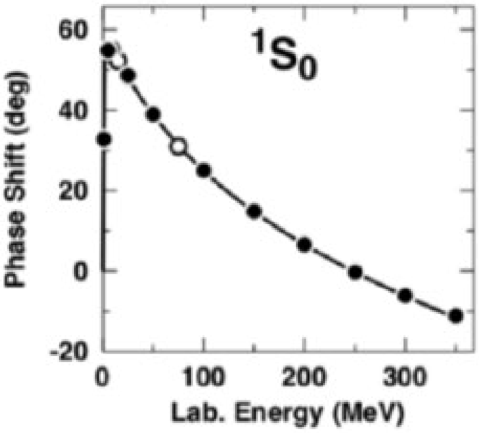
What other terms in N-N Force are allowed/required
Consider energy dependence of phase shifts

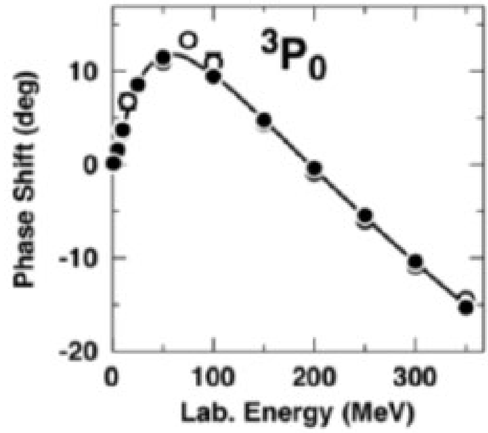
⟺ see Next page

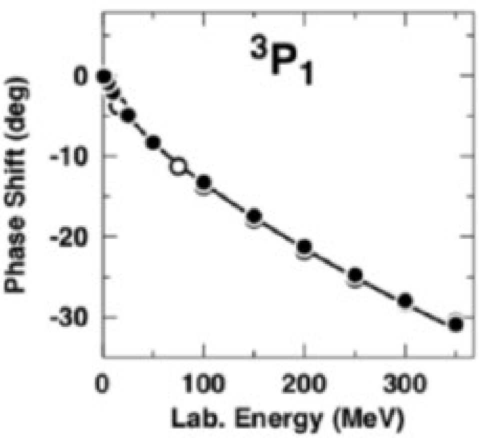
Note deBroglie λ for proton/neutron

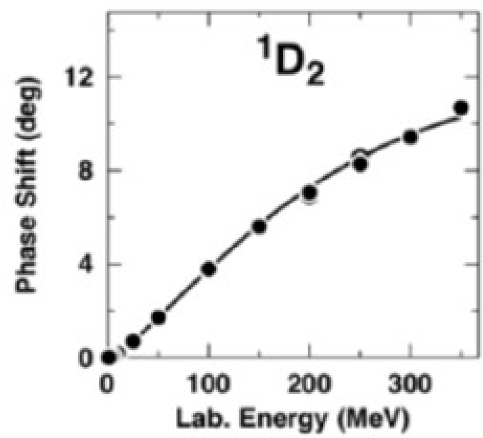
$$\lambda_N = \underline{2.8\ \text{fm}} \quad @\ 100\ \text{MeV}$$

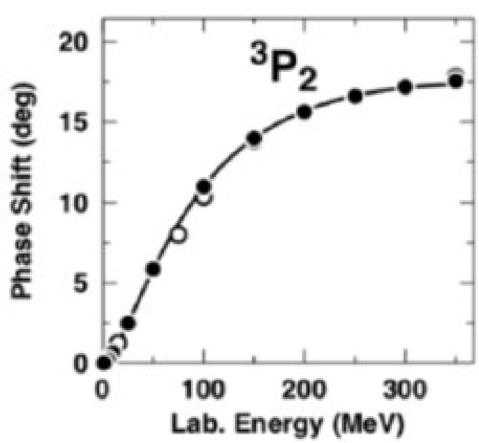
n-p Phase Shifts

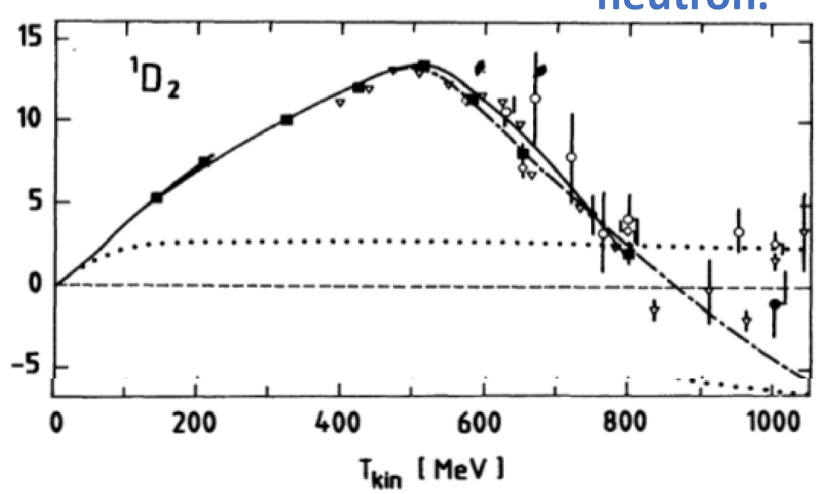












Notes:

1. Large energy scale obscures k -> 0 limit for L = 0, since need $\delta \rightarrow 0$ as k-> 0.

2. Both $^3P_0$ and $^1D_2$ δ go negative at high energy –> repulsive core.

3. $^3S_1 > {}^1S_0$ consistent with bound deuteron, but unbound di-neutron.

Summary of Required terms in V_{NN} (2-nucleon interaction)

1. Nuclear Force (N.F.) is spin +/or isospin dependent

   $\hookrightarrow a_s \neq a_t$

   $^1S_0 \quad ^3S_1$

   $T=1 \quad T=0$

2. N.F. has a "Tensor" component ($Q_d \neq 0$)
3. N.F. is short range ($r \lesssim 2$ fm)
4. N.F. is attractive @ $r \simeq 1-2$ fm

   why: a/ $^1S_0$ & $^3S_1$ have $\delta > 0$

   b/ Deuteron is bound

5. N.F. has repulsive core $\quad p \simeq 650$ MeV/c

   why? a/ $^3P_0$ δ_ℓ becomes < 0 for $E_k > 200$ MeV

   $|\vec{L}| = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2}\hbar = R_c p \Rightarrow R_c \simeq 0.4$ fm

   b/ $^1D_2$ $\delta_\ell < 0$ for $E > 850$ MeV

   $|\vec{L}| = \sqrt{6}\hbar = R_c p \quad \hookrightarrow p = 1500$ MeV/c

   $\hookrightarrow R_c \sim 0.3$ fm

6. N.F. has spin-orbit component

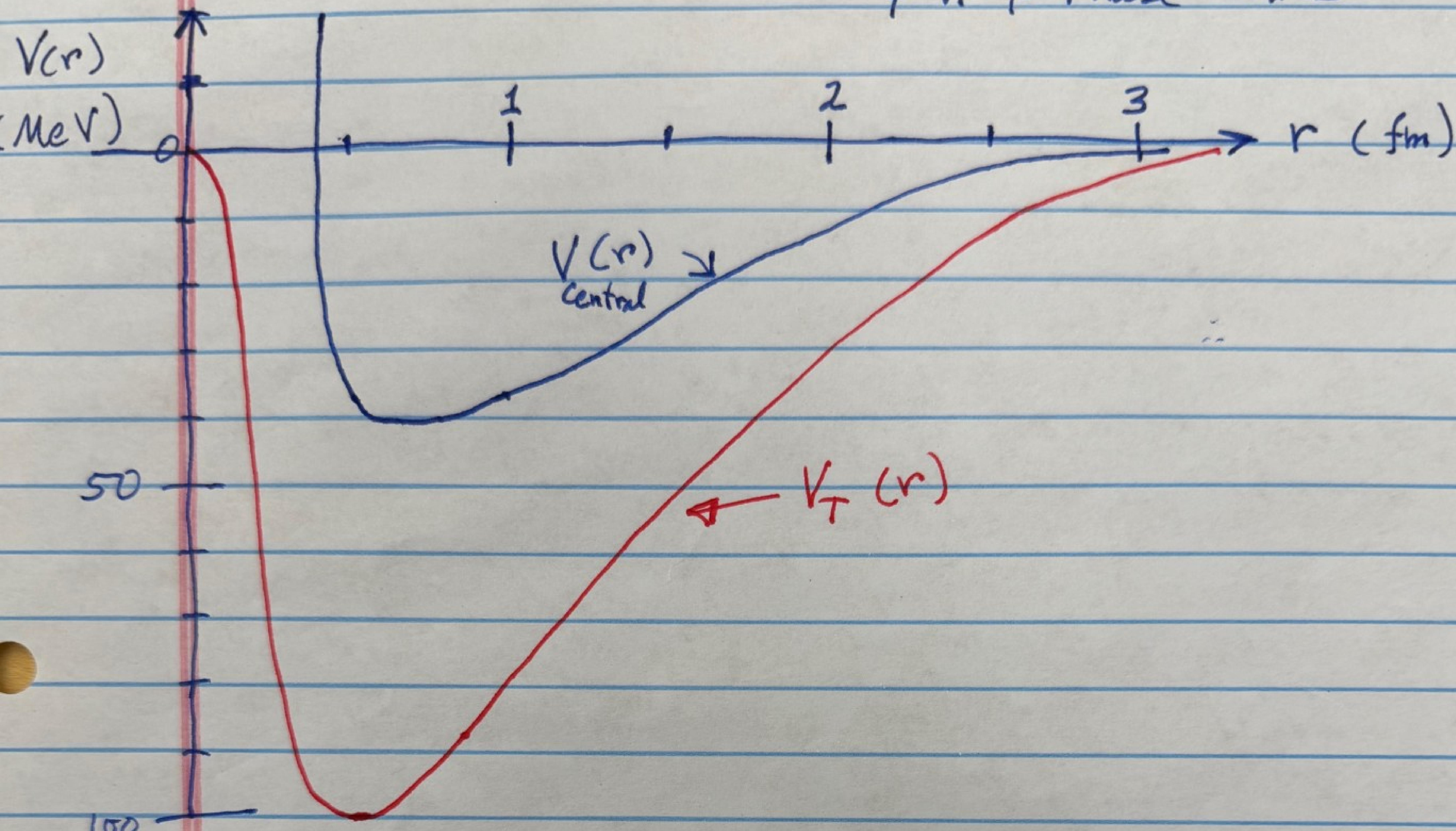
   why? a/ ^3P phase-shifts depend on $|\vec{J}| = |\vec{L}+\vec{S}|$

   $\hookrightarrow \vec{L}\cdot\vec{S} = \frac{1}{2}(J^2 - L^2 - S^2)$

   b/ "Discovered" by Marie-~~Goeppert~~ Mayer to explain "Magic" nuclei

   (see next week)

Realistic n-p Potential to explain Deuteron properties
& n-p Phase Shifts:



Given what the data requires for N-N Force, we can ask, given the symmetries & Invariance of Strong Int. what operators are possible?

Assumptions: (Basic phys. + exp)

I) $\hat{H}$ = Hamiltonian is invariant w.r.t. rotations

$\hookrightarrow$ J is conserved

$\therefore$ can't have e.g. S_z.

II) $\hat{H}$ invariant w.r.t. exchange of identical particles

III) $\hat{H}$ invariant w.r.t. Parity : $\hat{P}\vec{r} \to -\vec{r}$

then since $\hat{H}$ has $\vec{p}^{\,2}$ terms (even under parity) all terms must be even under parity

Recall: $\hat{P}\vec{p} = -\vec{p}$

$$\left.\begin{aligned} \hat{P}\vec{\sigma} &= +\vec{\sigma} \\ \hat{P}\vec{L} &= +\vec{L} \end{aligned}\right\} \text{Axial Vectors}$$

$\therefore$ $\hat{\vec{\sigma}}_1 \cdot \hat{\vec{r}}$ not permitted (P-odd)

IV) $\hat{H}$ appears to be charge independent

$[\hat{H}, \hat{T}_3] = 0$

$[\hat{H}, \hat{T}^2] = 0$

but this allows $\hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2$, $(\hat{\vec{\sigma}}_1 \cdot \hat{\vec{\sigma}}_2)(\hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2)$

but $\hat{\vec{\sigma}}_1 \cdot \hat{\vec{\tau}}_1$ & $\hat{\vec{\tau}}_1 \cdot \vec{r}$ are meaningless

V) Velocity-dependent terms needed

e.g. $\vec{L}\cdot\vec{S}$; $\vec{S} = \frac{\hbar}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$

other terms are possible:

$(\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})$; $\vec{p} = \vec{p}_1 - \vec{p}_2$

$(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L})$ usually ignored

Next time: most general V_{NN} consistent w symmetries, invariance & data.