Ph203 (L4)

From last time:

"Most" general V_{NN} consistent w symmetries & invariance:

$$V_{NN} = \underbrace{V_0(r) + V_\sigma(r)(\hat{\vec{\sigma}}_1 \cdot \hat{\vec{\sigma}}_2) + V_\tau(r)(\hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2) + V_{\sigma\tau}(r)(\hat{\vec{\sigma}}_1 \cdot \hat{\vec{\sigma}}_2)(\hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2)}_{\text{Central Force}}$$

$$+ \underbrace{V_T(r)\,\hat{S}_{12} + V_{T\tau}\,\hat{S}_{12}(\hat{\tau}_1 \cdot \hat{\vec{\tau}}_2)}_{\text{Tensor Force}}$$

$$+ \underbrace{V_{LS}(r)\,\hat{\vec{L}} \cdot \hat{\vec{S}} + \ldots \text{ quadratic } LS,\ \hat{\vec{L}} \cdot \hat{\vec{S}}(\hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2),\ (\sigma_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p}), \ldots}_{\text{Velocity dependent Force}}$$

Can we build nuclei using V_{NN}? $\Rightarrow$ Maybe

Why do we care?

Astrophysics

1. Neutron Star properties/structure
   - → depends on Nuclear Eq. of State (NEOS)
   - → Nuclear matter properties vs ρ_n, ρ_p
     from subnuclear to supranuclear
     "pasta" $\rho > \rho_{Pb}$

2. Production of Elemental Abundances
   - → Nucleosynthesis → Why more O than F ?
   - → Big Bang Nucl. (BBN) — Fe " Co ?
   - → depends on Masses, Lifetimes, ... — Pb " Bi ?

3. Supernovae (SN)
   - → Why do they explode?

4. Neutrinos

→ Solar ν, SN ν, BB ν ← Big Bang

Physics Beyond Standard Model

1. Electroweak Interactions
   - → CKM matrix : Nuclear β-decay gives V_{ud}
     - ↳ "Nuclear corrections"
   - → CP Violation : Electric Dipole Moments (EDM)
     - ↳ "Nucl. corr." for n, p, nuclei to get
       - ↳ quark EDM
   - → New particles (M > TeV) modify nuclear matrix elements in precision measurements
2. Neutrino Properties
   - → Oscillation of ν flavors
     - ↳ need cross sections $\sigma_{\nu+A\to x}$ where $A = N+Z$ of nuclear target
   - → Is $\nu = \bar{\nu}$ (Majorana neutrino) violates Lepton Number Conservation 0ν Double β decay
     - ↳ m_ν depends on "Nucl. corr."
3. Dark Matter detection
   - → Nuclear targets need σ_{DM+A}

Solving ~~the~~ Quantum Many-Body Problem

1. Techniques for Nuclear Force / Reactions used in Condensed Matter & AMO many-body problem

Need:

Approaches to Nuclear Force (How to calculate nuclear W.F.?)

We will discuss:

1. Phenomenological $V(\vec{r})$
2. Boson Exchange Potentials (OBEP) — one Boson exch. potential
3. Effective Field Theory (EFT) ⇒ use QCD

1. Phenom. Potential

Start with V_{NN} above based on symmetries / Invariance (rotation, parity, Time Reversal, Isospin, ...)

→ Must add 3 nucleon force (3NF) + maybe 4NF, ...

→ Most successful is: Argonne $\underbrace{V18}_{18 \text{ params}} + \underbrace{3NF}_{22 \text{ params}}$

note: without 3NF Binding Energy of ^3He, ^3H off by ~ 1 MeV

↖ previous V_{NN} (see page 1)

↳ Fit the params to existing data & predict...

↳ works well for N-N + few-nucleon properties

→ How far can we go? ⇒ heaviest stable nucleus is $^{208}_{82}Pb_{126}$ ($^{A}_{Z}E_{N}$)

→ In practice $A \simeq 20$ is possible. Why?

e.g. ^{16}O w 8 basis states

↳ 4 energy states + 2 spin states

must evaluate: $N_{basis}^{A} = 8^{16} \simeq 3 \cdot 10^{14}$

but ^{76}Ge w 20 basis states

must consider $20^{76} \simeq 10^{99}$ (Ouch!)

↳ too much for today's computers

↳ maybe Quantum Computer?! see CANVAS

② OBEP (One Boson Exchange Potential)

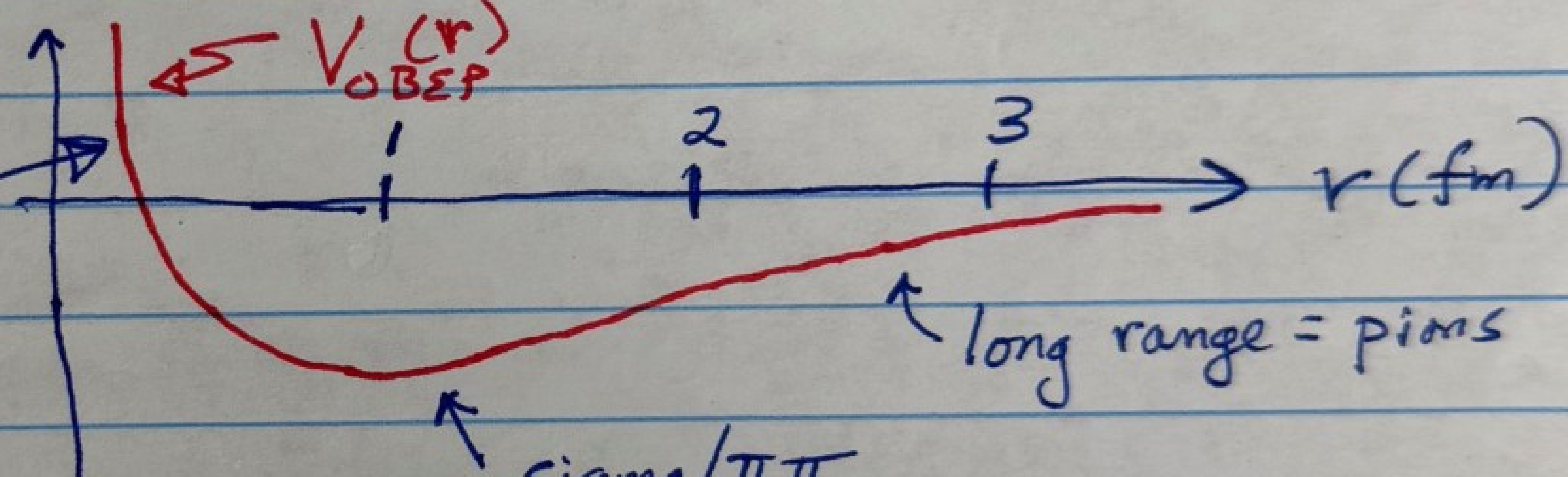
⟺ Yukawa-like force from massive particle exchange

↳ gives short range $V(\vec{r})$

Consider "light" mesons $M \lesssim 1$ GeV

Meson	M (MeV)	$I\,J^{\pi}$	
π	138	$1\,0^-$	Pseudoscalar
η	548	$0\,0^-$	
a_0	980	$1\,0^+$	Scalar
$f_0/\sigma/\pi\text{-}\pi$	600 ↳ but $\Gamma \sim 400$	$0\,0^+$	
ρ	770	$1\,1^-$	Vector
ω	782	$0\,1^-$	

then anticipate:



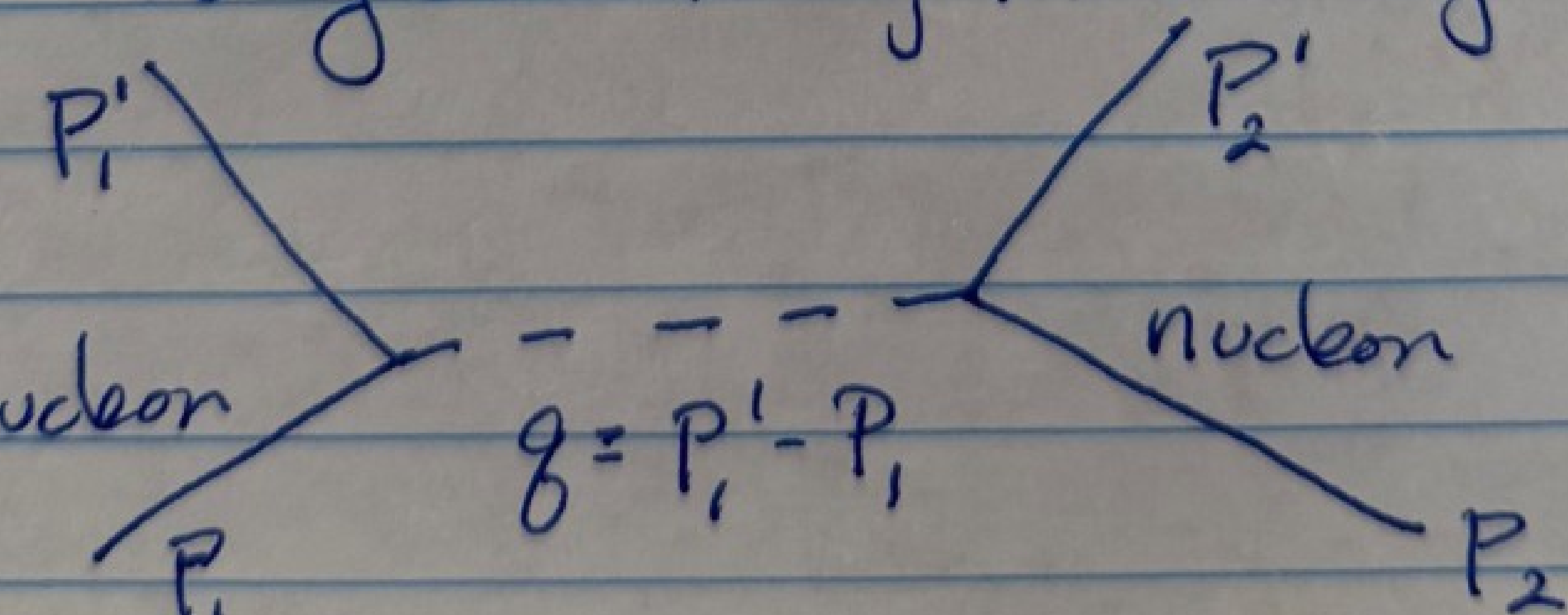
OBEP field theory

$$V_{NN}^{OBEP} = \sum_{i=\pi,\rho,\sigma,\ldots} V_i(q)$$

momentum space

Use leading order Feynman diagrams:

@ tree-level



see App. D in Bert.

Long Range:

then for pion exchange

pure pseudoscalar current gives wrong $a_{\pi N}$, but pseudovector works & gives $\hat{S}_{12}$!

$$\Leftrightarrow \mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^\mu \gamma_5 \hat{\vec{\tau}} \psi \cdot \partial_\mu \vec{\phi}^\pi$$

which yields

$$V_{\pi NN}(q) \simeq -\frac{f_{\pi NN}^2}{3m_\pi^2} \underbrace{\left(\frac{\vec{q}^{\,2}}{q^2 + m_\pi^2} \right)}_{\text{gives via F.T. } \frac{e^{-\frac{m_\pi c r}{\hbar}}}{r}} \left[\underbrace{\hat{S}_{12}(q)} - \hat{\vec{\sigma}}_1 \cdot \hat{\vec{\sigma}}_2 \right] \times \hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2$$

F.T. = Fourier transform

$$\hat{\vec{S}}_{12}(\vec{q}) = \frac{3}{q^2} (\hat{\vec{\sigma}}_1 \cdot \vec{q})(\hat{\vec{\sigma}}_2 \cdot \vec{q}) - \hat{\vec{\sigma}}_1 \cdot \hat{\vec{\sigma}}_2$$

& FT of $\hat{S}_{12}(q) \Rightarrow \hat{S}_{12}(r)$

Medium Range $\Rightarrow$ σ-meson:

$$\mathcal{L}_{\sigma NN} = -g_\sigma \bar{\psi} \psi \phi^\sigma$$

which yields:

$$V_{\sigma NN}(q) \simeq \frac{g_\sigma^2 q^2}{q^2 + m_\sigma^2} \left(-1 - \frac{q^2}{2M_N^2} - \frac{\vec{L} \cdot \vec{S}}{2M_N^2} \right)$$

spin-orbit

Short Range! $\Rightarrow$ ω meson

$$\mathcal{L}_{\omega NN} = -g_\omega \bar{\psi} \gamma^\mu \psi$$

which yields:

$$V_{\omega NN}(q) \simeq \frac{g_\omega^2 q^2}{q^2 + m_\omega^2} \left(1 - \frac{3 \vec{L} \cdot \vec{S}}{M_N^2} \right)$$

repulsive core

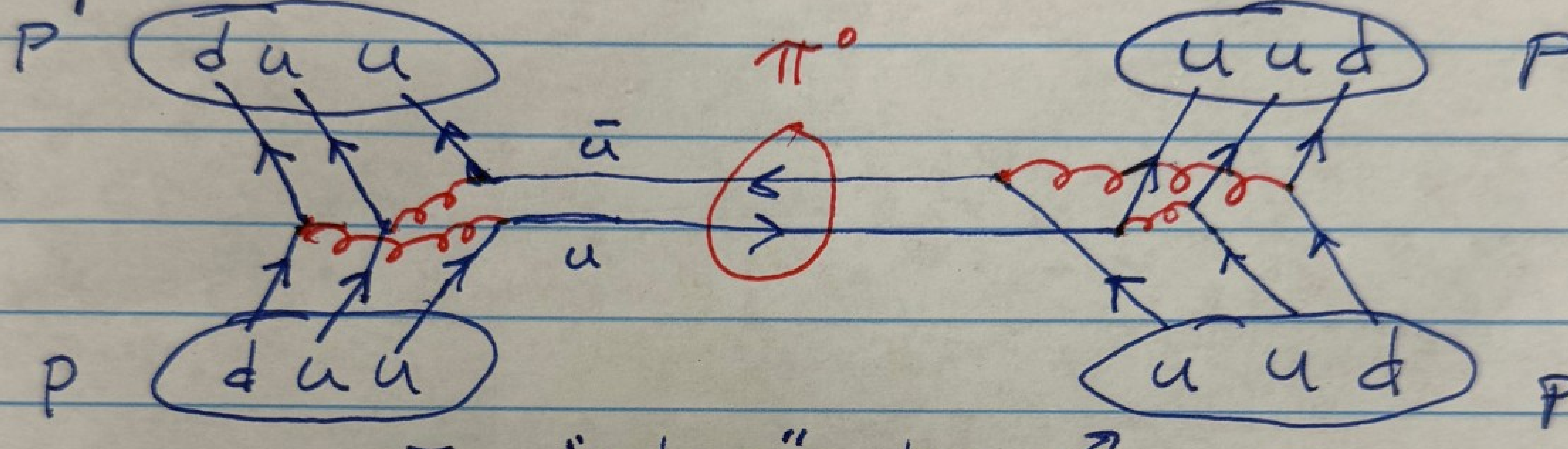
lots of params (g_i's) to fit to lots of data but what are "theory" uncertainties ??

But! OBEP & V_{NN} have Problems!

1. No handles for "theory" uncertainties
2. OBEP is a little ad hoc
   ↳ no real σ-meson
3. No QCD! the theory of strong int.

(3) N-N in QCD

Consider $p+p \rightarrow p+p$



"valence" quarks

Clearly this is tree-level (1-gluon), then for low E $p+p$ @ few MeV $\alpha_s \geq 1$ (see later)
↳ strong coupling constant

∴ many other diagrams needed
⟺ Not perturbation theory

What to do?

1. Quark-based "Models" using q/g (quark / gluon) degrees of freedom
   not a real theory ← "naive" q-models
2. Try non-perturbative calcs. via "Lattice Gauge Theory"
   ↳ more later
   ↳ only works "barely" for N+N
3. Use symmetries (or lack thereof) in QCD to develop an Effective Field Theory (EFT)

S. Weinberg ('91) said ...

"If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-Matrix consistent with analyticity, perturbative unitarity, cluster decomposition and assumed symmetry principles." - S. Weinberg