Physics 203

Homework 3

1.) Show that two particle scattering at 90° in the center-of-mass can only take place through partial waves of even angular momenta. Use this result and assume charge independence of the nuclear force to show, at any energy, that:

$$\frac{d\sigma}{d\Omega}_{pp}(\theta_{cm} = 90^\circ) \leq 4\frac{d\sigma}{d\Omega}_{np}(\theta_{cm} = 90^\circ)$$

2.) What are the possible values of orbital angular momentum, total intrinsic spin, and isospin for the ground state of the deuteron if it had turned out to have $J^\pi = 0^-$ instead of the measured value of $J^\pi = 1^+$. Discuss the implications for the form of the nuclear force if the deuteron had $J^\pi = 0^-$

3.) Consider the following two strong interaction processes:

$$(1)\ \pi^- + p \to K^0 + \Lambda^0$$

$$(2)\ \pi^0 + n \to K^0 + \Lambda^0.$$

Assuming charge independence and isospin conservation in these reactions, determine the isospin of the K^0 given that the Λ^0 has isospin 0 and that the ratio of the above two cross sections is $\sigma(1)/\sigma(2) = 2$.

4.) Express the operator

$$\hat{S}_{12} = (\frac{3}{r^2})(\hat{\boldsymbol{\sigma}}_1 \cdot \hat{\mathbf{r}})(\hat{\boldsymbol{\sigma}}_2 \cdot \hat{\mathbf{r}}) - \hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2$$

in terms of Y_{lm}'s and the spherical tensor components of the vector operators $\hat{\boldsymbol{\sigma}}_1$ and $\hat{\boldsymbol{\sigma}}_2$. Also show that you can reduce your result to a scalar product of a rank 2 operator in spin space and a rank 2 operator in coordinate space. (Hint: show that

$$\hat{S}_{12} = \sqrt{24\pi}\sum_m \langle 2,2;m,-m|0,0\rangle(\hat{\boldsymbol{\sigma}}_1 \mathrm{x} \hat{\boldsymbol{\sigma}}_2)_{2m} Y_{2-m}$$

5.) Supplemental Problem SP1 (see adjacent pdf).

6.) Bertulani Text Problem 3-11.

SP1

3-7. In classical electrodynamics, the scalar field $\phi(\boldsymbol{r})$ produced by an electron located at the origin is given by the Poisson equation

$$\nabla^2\phi(\boldsymbol{r}) = -4\pi e\delta(\boldsymbol{r})$$

Show that the radial dependence of the field is given by

$$\phi(r) = \frac{e}{r}$$

For a nucleon, the scalar field satisfies the Klein-Gordon equation

$$\left(\nabla^2 - \frac{1}{r_0^2}\right)\phi(\boldsymbol{r}) = 4\pi g\delta(\boldsymbol{r})$$

Show that the radial dependence of the field is given by

$$\phi(r) = -g\frac{e^{-r/r_0}}{r}$$

Derive that the range r_0 is given by the relation $r_0 = \hbar/mc$ using the fact that the boson, with mass m, is a virtual particle and can therefore exist only for a time Δt given by the Heisenberg uncertainty relation.