Ph 203 L5

Effective Field Theory (Weinberg)

→ For context note:

- Fermi theory of β-decay is also an EFT, since Fermi didn't know about $W^\pm$, Z^0

→ For NN int. based on QCD EFT start with:

$$\mathcal{L}_{QCD} = \sum_{f=u,d,\ldots} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu}$$

$\mu\nu = 0-3$ space time

$a = 1-8 \Rightarrow$ color charge

w $\not{D} = \gamma[\partial_\mu - i g \sum_a \frac{\lambda_a}{2} A_{\mu,a}]$

coupling → g

$A_{\mu,a}$ ↳ QCD vector potential c.f. QED

λ_a → generators of SU(3)

↳ Gell-Mann matrices for color triplet r, b, g

3×3 e.g. $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots$

&

$$G_{\mu\nu,a} = \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} + g f_{abc} A_{\mu,b} A_{\nu,c}$$

this is gluon self-interaction

f_{abc} ↑ like ϵ_{ijk} SU(3) structure constants

For N-N force generally only need $q_f = u, d$

w $m_u \simeq$ 2.2 MeV

$m_d \simeq$ 4.7 MeV

$m_s \simeq 100$ MeV

$m_c \simeq 1.2$ GeV

$m_b \simeq 4.2$ GeV

$m_t \simeq 173$ GeV

called "current" quark masses undressed

focussing first on u, d quarks,
@ $E >> \text{GeV}$ $m_u \simeq m_d \simeq 0$ gives

$$\mathcal{L}_{QCD} \simeq \sum_f \bar{q}_f i \not{D} q_f - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu}$$

& in this limit helicity ($\vec{\sigma} \cdot \vec{p}$) is conserved
↳ right-handed $\vec{\sigma} \cdot \vec{p} > 0$
left-handed $\vec{\sigma} \cdot \vec{p} < 0$

& get an SU(2) symmetry ⇒ chiral sym.

&

$$\mathcal{L}_{QCD} = \sum_f \left(\bar{q}_{R,f} i \not{D} q_{R,f} + \bar{q}_{L,f} i \not{D} q_{L,f} \right) - \frac{1}{4} GG$$

note: for massless particles (or $m << E$) helicity = chirality
but chirality is actually a unique SU(2) sym.
$\begin{pmatrix} u \\ d \end{pmatrix}$ = isospin, $\begin{pmatrix} u_L \\ u_R \end{pmatrix}$ = chirality
w/ projection operator = $\frac{1}{2}(1 \pm \gamma_5)$

Note:

① Together isospin & chiral symmetry yield 2 strong conserved currents (for $E >> m_{u,d}$)

$$V_i^\mu = R_i^\mu + L_i^\mu = \bar{q} \gamma^\mu \frac{\hat{\tau}_i}{2} q \Rightarrow \text{isospin}$$

$$A_i^\mu = R_i^\mu - L_i^\mu = \bar{q} \gamma^\mu \gamma_5 \frac{\hat{\tau}_i}{2} q \Rightarrow \text{chiral}$$

② If $E \simeq m_{u,d}$ both isospin & chirality are broken
→ "small" violation of isospin
since $m_d - m_u << m_{p,n}$

→ but large "spontaneous" breakdown of chiral symmetry

→ chiral sym. requires existence of degenerate states ($m_1 = m_2$) with opposite intrinsic parity

but $m_p \; (I=\frac{1}{2}, J^\pi = \frac{1}{2}^+) = 938$ MeV

$m_{N^*} \; (I=\frac{1}{2}, J^\pi=\frac{1}{2}^-) = 1535$ MeV

also $m_\rho \; (1, 1^-) = 770$ MeV

$m_a \; (1 \; 1^+) = 1260$ MeV

→ Such spont. broken symmetries lead to appearance of a massive "Goldstone" boson

↳ pion!

$m_\pi \neq 0$ since $m_u, m_d \neq 0$

↳ leads to Modern EFT:

Chiral Perturbation Theory (χEFT)

① Construct general $\mathcal{L}$ using effective d.o.f. π, N w sym + ~~sym~~ of QCD

② Develop scheme for identifying key Feynman diags using expansion in $\frac{Q}{\Lambda_\chi}$

Q → $Q = p - p'$ mom. xfer

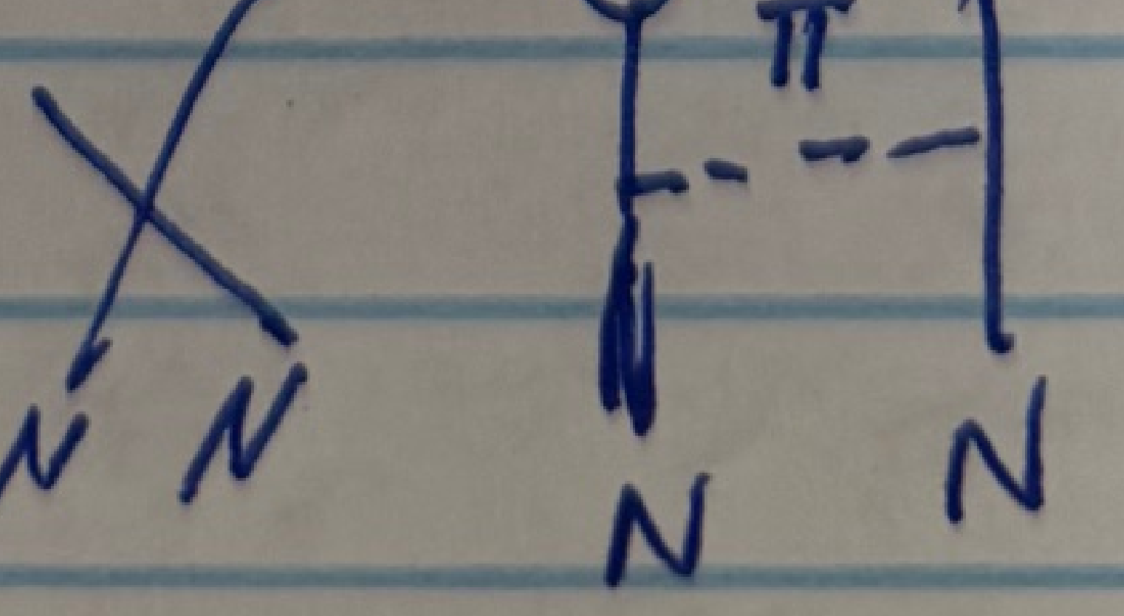
Λ_χ → Cutoff mass

w $\Lambda_\chi > m_\pi$ but $\Lambda_\chi < Q_{\text{perturb. QCD}}$ → $\simeq 2$ GeV

↳ $\Lambda_\chi \simeq 500 - 1000$ MeV

& use "power counting" to identify imp. diags

e.g.

Leading order 

$\Rightarrow \left(\frac{Q}{\Lambda_\chi}\right)^0$ ← power then scale diags by power × low energy constant

$$\hookrightarrow \left(\frac{Q}{\Lambda_\chi}\right)^n \times \quad c_{n_i}$$

fit these to data @ low E

Show Pic of Diags:

frontiers in Physics

REVIEW
published: 18 March 2020
doi: 10.3389/fphy.2020.00057

Nucleon-Nucleon Scattering Up to N^{5}LO in Chiral Effective Field Theory

David Rodriguez Entem[1,2]*, *Ruprecht Machleidt*[3] *and Yevgen Nosyk*[3]

[1] Department of Fundamental Physics, Faculty of Science, University of Salamanca, Salamanca, Spain, [2] Institute on Fundamental Physics and Mathematics (IUFFyM), University of Salamanca, Salamanca, Spain, [3] Department of Physics, University of Idaho, Moscow, ID, United States

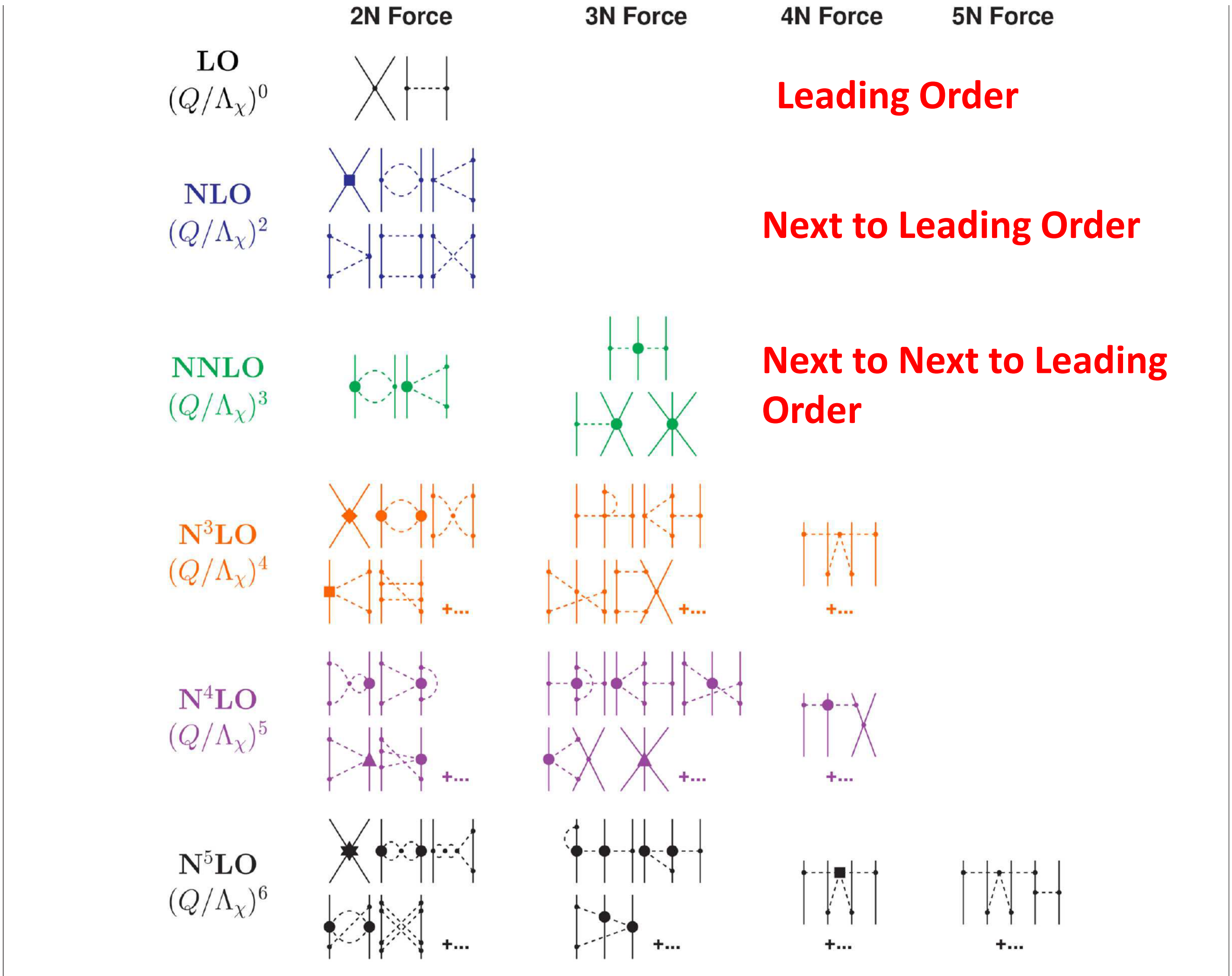


FIGURE 1 | Hierarchy of nuclear forces up to N^{5}LO or sixth order of the chiral expansion. Only some representative diagrams are included. Small dots, large solid dots, solid squares, triangles, diamonds, and stars denote vertexes of index $\Delta_i = 0, 1, 2, 3, 4$, and 6, respectively. Reprinted figure with permission from Entem et al. [37], copyright (2017) by the American Physical Society.

Note: ① @ each order several LEC's extracted via fits

② @ higher order can use lower order LEC's (sometimes)

③ If higher order terms are smaller ↪ convergence (Maybe?!)

Results for N-N Phase Shifts:

n-p phase shifts (from LO to N4LO)

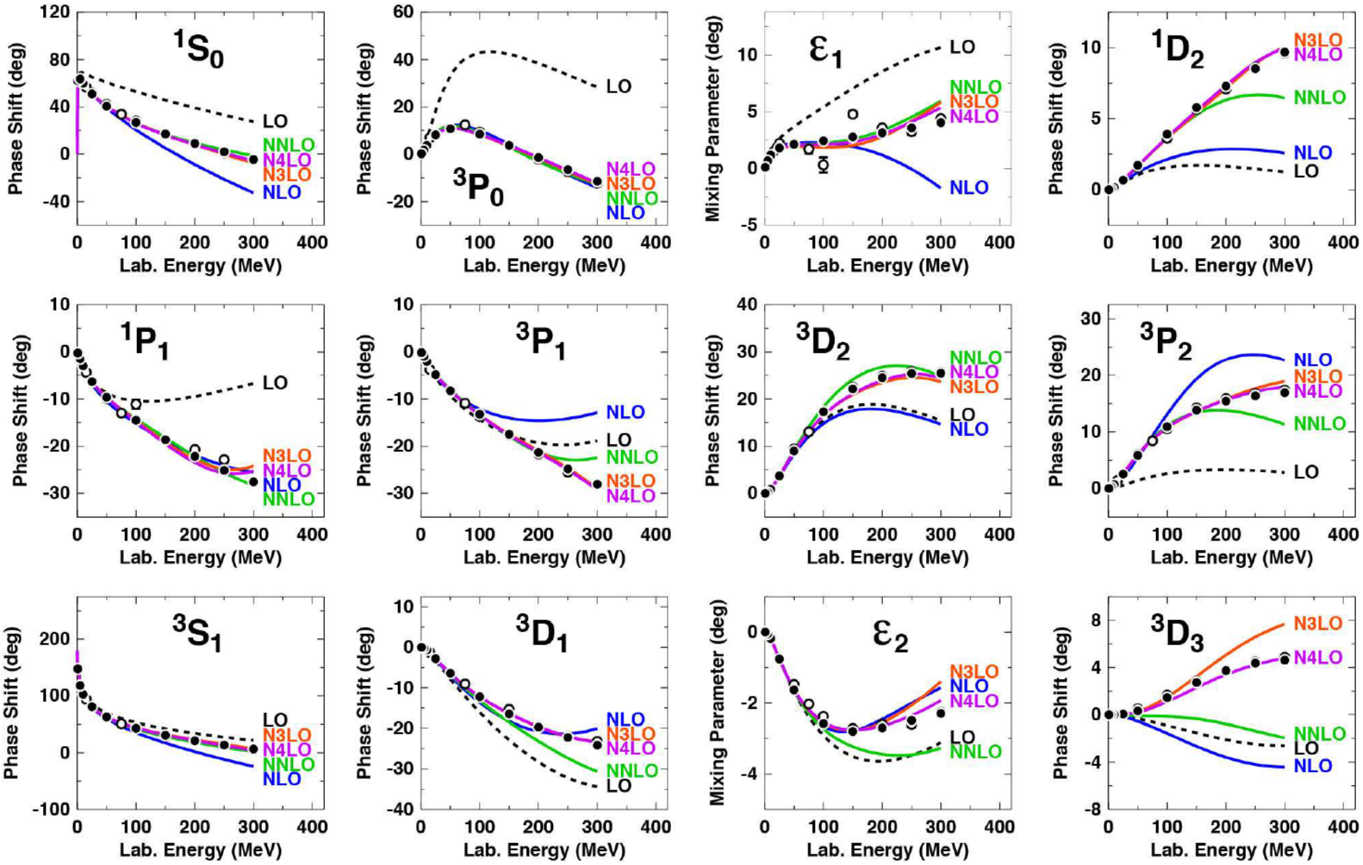


FIGURE 15 | Chiral expansion of neutron-proton scattering as represented by the phase shifts in *S*, *P*, and *D* waves and mixing parameters ϵ_1 and ϵ_2. Five orders ranging from LO to N^{4}LO are shown as denoted. A cutoff $\Lambda = 500$ MeV is applied in all cases. The filled and open circles represent the results from the Nijmegen multi-energy *np* phase-shift analysis [93] and the GWU single-energy *np* analysis SP07 [95], respectively. Reprinted figure with permission from Entem et al. [37], copyright (2017) by the American Physical Society.

TABLE 4 | χ^2/datum for the fit of the 2016 *NN* data base by *NN* potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases).

T_{lab} bin (MeV)	No. of data	LO	NLO	NNLO	N^{3}LO	N^{4}LO
			Proton-proton			
0–100	795	520	18.9	2.28	1.18	1.09
0–190	1206	430	43.6	4.64	1.69	1.12
0–290	2132	360	70.8	7.60	2.09	1.21
			Neutron-proton			
0–100	1180	114	7.2	1.38	0.93	0.94
0–190	1697	96	23.1	2.29	1.10	1.06
0–290	2721	94	36.7	5.28	1.27	1.10
			pp* plus *np			
0–100	1975	283	11.9	1.74	1.03	1.00
0–190	2903	235	31.6	3.27	1.35	1.08
0–290	4853	206	51.5	6.30	1.63	1.15

From Entem et al. [37].

Is χEFT better than Phenom. NN or OBE

Theorists say Yes!!

1. Allows estimate of uncertainty @ each order
2. Automatically includes 3NF, 4NF, ...
3. Fewer Params:

AV18 + 3NF needs 50 parms

N^3LO needs 24 parms

PIC of ^2H & ^3H Results

Deuteron and Triton Properties

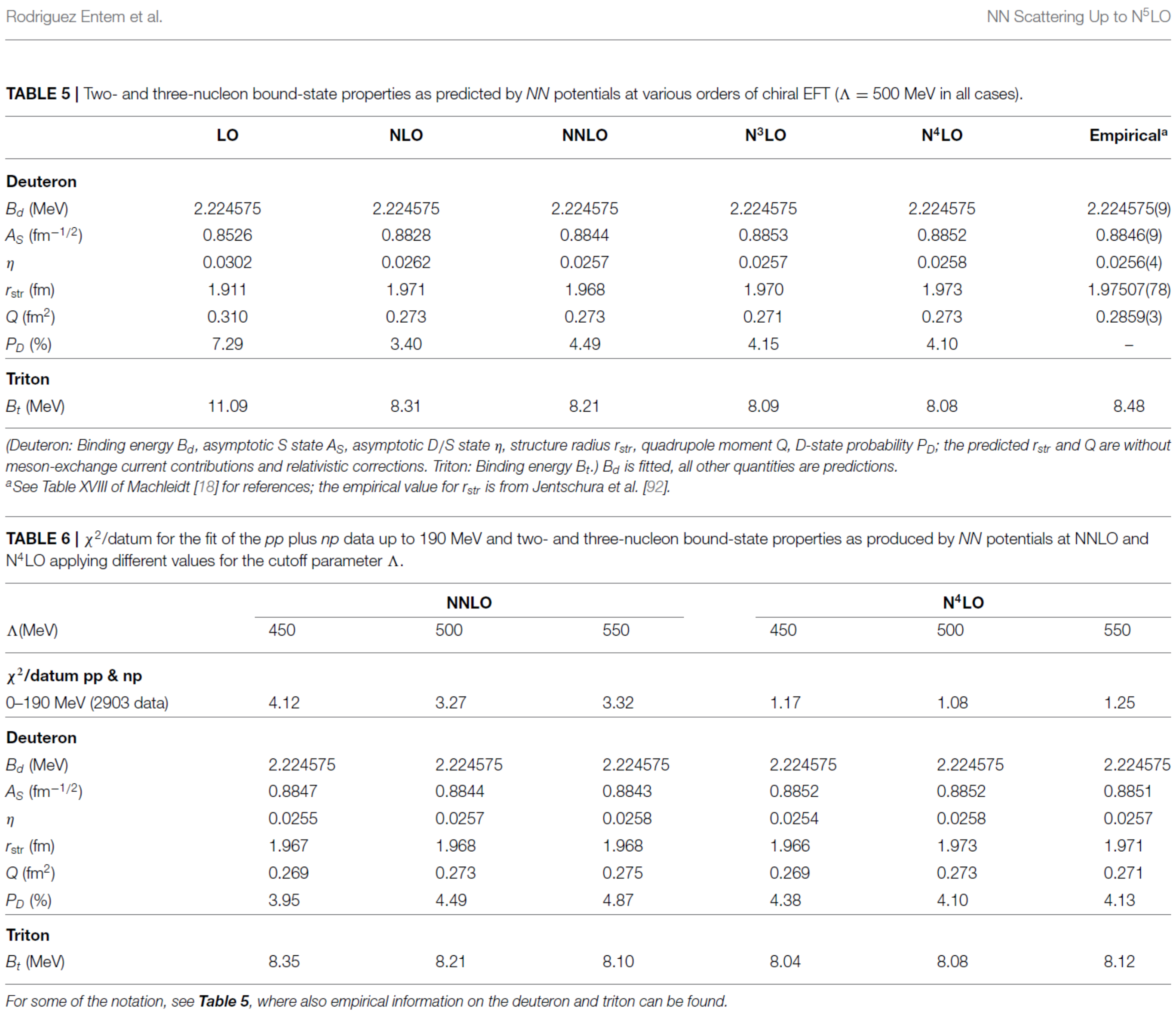


TABLE 5 | Two- and three-nucleon bound-state properties as predicted by *NN* potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases).

	LO	NLO	NNLO	N^3LO	N^4LO	Empirical[a]
Deuteron						
B_d (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575	2.224575(9)
A_S ($fm^{-1/2}$)	0.8526	0.8828	0.8844	0.8853	0.8852	0.8846(9)
η	0.0302	0.0262	0.0257	0.0257	0.0258	0.0256(4)
r_{str} (fm)	1.911	1.971	1.968	1.970	1.973	1.97507(78)
Q (fm^2)	0.310	0.273	0.273	0.271	0.273	0.2859(3)
P_D (%)	7.29	3.40	4.49	4.15	4.10	–
Triton						
B_t (MeV)	11.09	8.31	8.21	8.09	8.08	8.48

(Deuteron: Binding energy B_d, asymptotic S state A_S, asymptotic D/S state η, structure radius r_{str}, quadrupole moment Q, D-state probability P_D; the predicted r_{str} and Q are without meson-exchange current contributions and relativistic corrections. Triton: Binding energy B_t.) B_d is fitted, all other quantities are predictions.

[a] See Table XVIII of Machleidt [18] for references; the empirical value for r_{str} is from Jentschura et al. [92].

TABLE 6 | χ^2/datum for the fit of the *pp* plus *np* data up to 190 MeV and two- and three-nucleon bound-state properties as produced by *NN* potentials at NNLO and N^4LO applying different values for the cutoff parameter Λ.

	NNLO			**N^4LO**		
Λ(MeV)	450	500	550	450	500	550
χ^2/datum pp & np						
0–190 MeV (2903 data)	4.12	3.27	3.32	1.17	1.08	1.25
Deuteron						
B_d (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575	2.224575
A_S ($fm^{-1/2}$)	0.8847	0.8844	0.8843	0.8852	0.8852	0.8851
η	0.0255	0.0257	0.0258	0.0254	0.0258	0.0257
r_{str} (fm)	1.967	1.968	1.968	1.966	1.973	1.971
Q (fm^2)	0.269	0.273	0.275	0.269	0.273	0.271
P_D (%)	3.95	4.49	4.87	4.38	4.10	4.13
Triton						
B_t (MeV)	8.35	8.21	8.10	8.04	8.08	8.12

*For some of the notation, see **Table 5**, where also empirical information on the deuteron and triton can be found.*

Low E - Hi E connection:

Recall: g coupling in $\mathcal{L}_{QCD}$

$\alpha_s \propto g^2$

↑ strong interaction coupling constant

Asymptotic Freedom (- sign) causes

$\alpha_s \downarrow$ as $E \uparrow$ opposite of QED

Pic of α_s vs Q

for $\alpha_s \lesssim 0.3$ QCD is perturbat

but for $E < M_\pi$, $\alpha_s > 1$!

Non-Perturbative

Lastly: χEFT for Nuclei . . .

(slides)

Strong coupling constant: $\alpha_s(Q)$

Q is momentum transfer to system

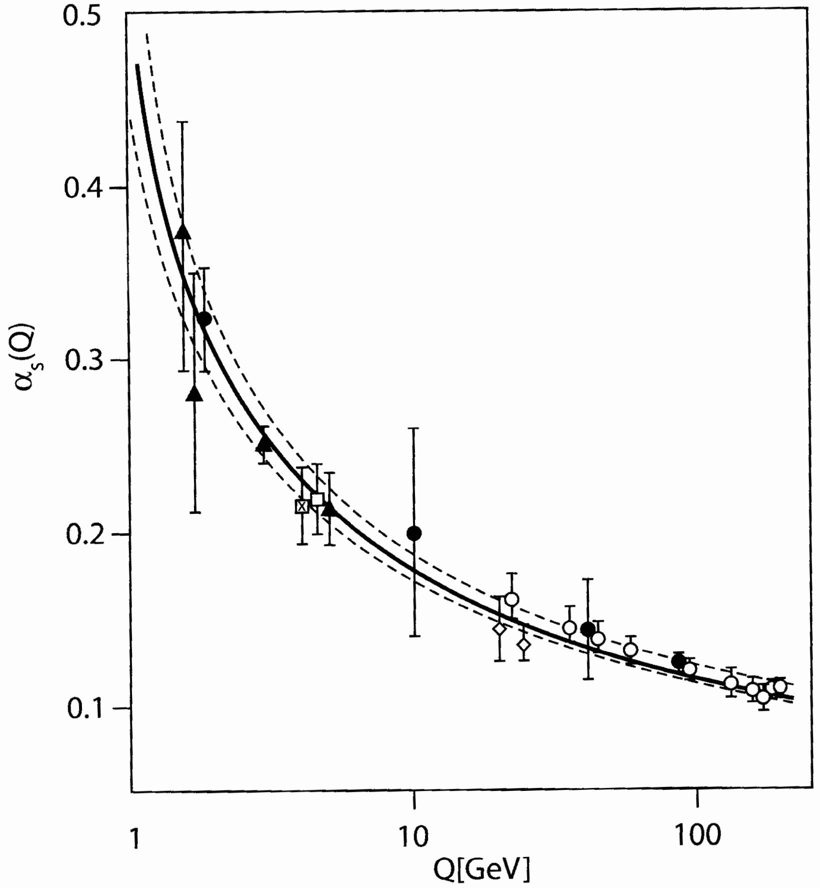
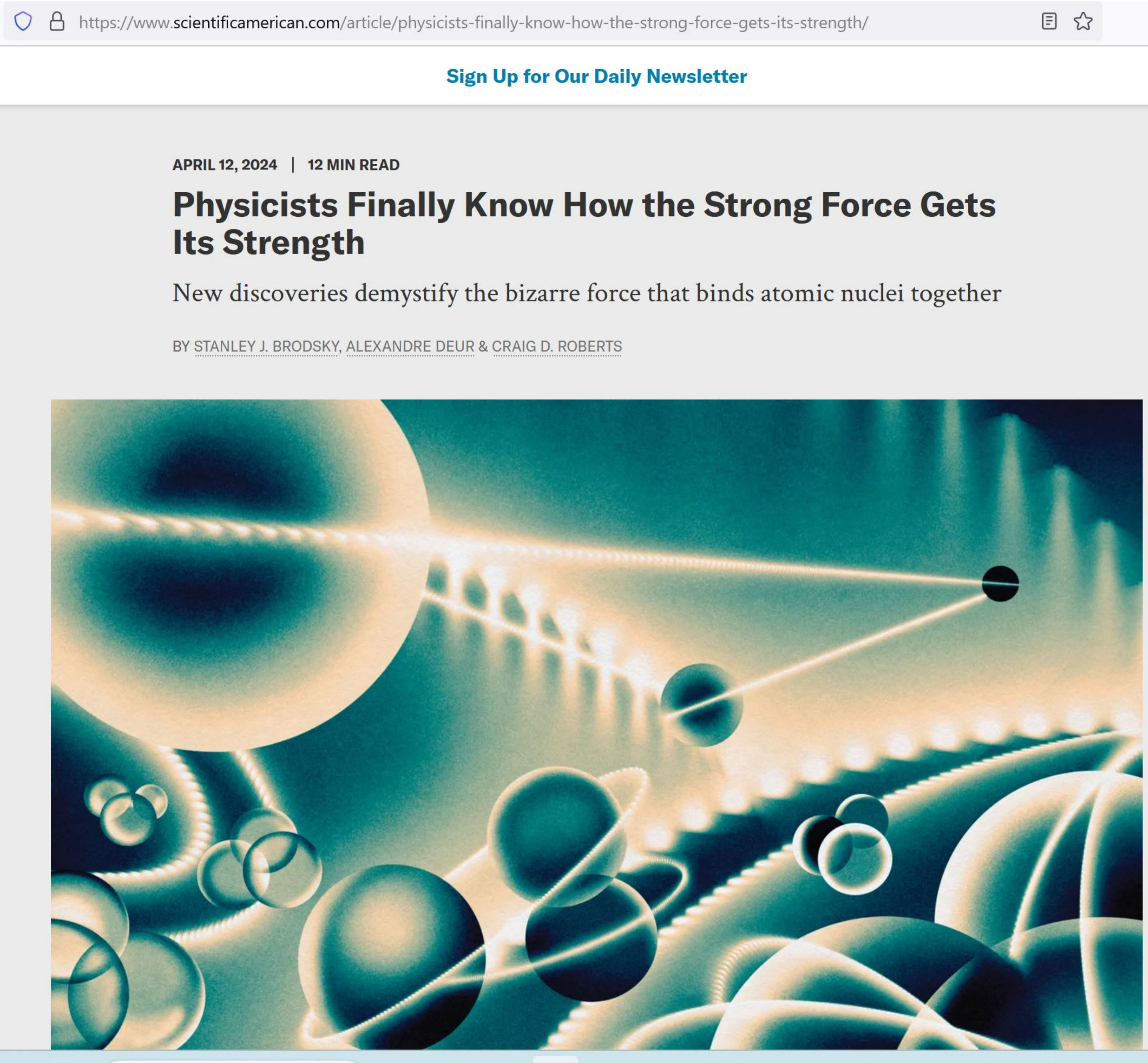


Figure 1.6 The running coupling constant $\alpha_S(Q^2)$ as a function of momentum transfer Q^2 determined from a variety of processes. The figure is from [Bet00].

Strong coupling constant: $\alpha_s(Q)$



APRIL 12, 2024 | 12 MIN READ

Physicists Finally Know How the Strong Force Gets Its Strength

New discoveries demystify the bizarre force that binds atomic nuclei together

BY STANLEY J. BRODSKY, ALEXANDRE DEUR & CRAIG D. ROBERTS

Experimental determination of the QCD effective charge $\alpha_{g_1}(Q)$

A. Deur,[1,2] V. Burkert,[1] J.P. Chen,[1] W. Korsch[3]

[1] *Thomas Jefferson National Accelerator Facility, Newport News, VA 23606. USA*

[2] *University of Virginia, Charlottesville, VA 22904. USA*

[3] *University of Kentucky, Lexington, KY 40506. USA*

Abstract

The QCD effective charge $\alpha_{g_1}(Q)$ is an observable that characterizes the magnitude of the strong interaction. At high momentum Q, it coincides with the QCD running coupling $\alpha_s(Q)$. At low Q, it offers a nonperturbative definition of the running coupling. We have extracted $\alpha_{g_1}(Q)$ from measurements carried out at Jefferson Lab that span the very low to moderately high Q domain, $0.14 \leq Q \leq 2.18$ GeV. The precision of the new results is much improved over the previous extractions and the reach in Q at the lower end is significantly expanded. The data show that $\alpha_{g_1}(Q)$ becomes Q-independent at very low Q. They compare well with two recent predictions of the QCD effective charge based on Dyson-Schwinger equations and on the AdS/CFT duality.

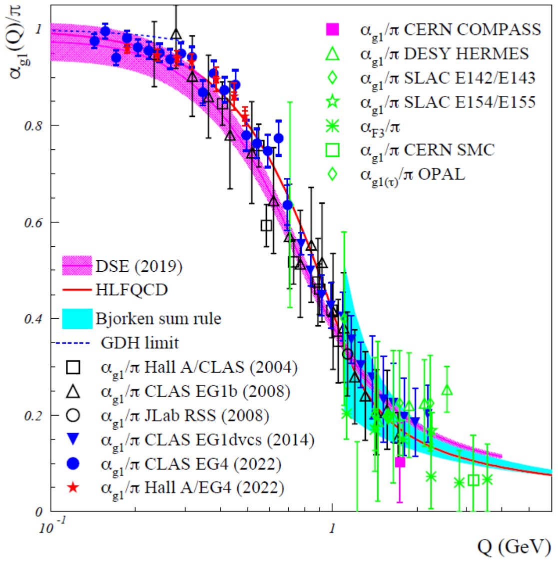
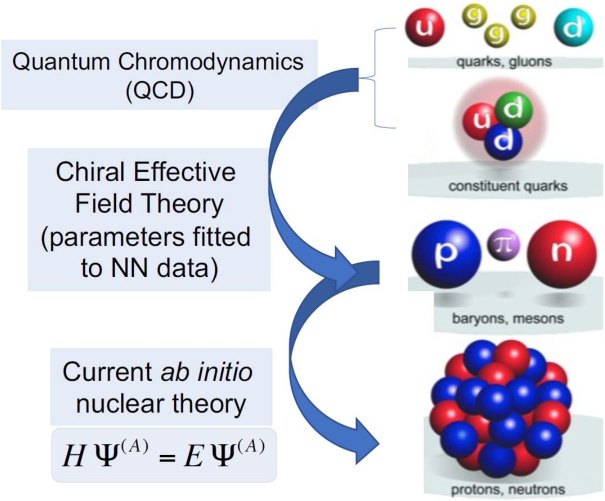


FIG. 1: Effective charge $\alpha_{g_1}(Q)/\pi$ obtained from JLab experiments E03006/E97110 [33] (solid stars), E03006/E05111 [33] (solid circles) and EG1dvcs [32] (solid triangles) and from COMPASS [31] (solid square). Inner error bars represent the statistical uncertainties and outer ones the systematic and statistical uncertainties added quadratically. The open symbols show the older world data [27–30] with the error bars the quadratic sum of the systematic and statistical uncertainties. Also shown are the HLFQCD [24] (red line,

Can use χEFT to calculate nuclear properties (up to A ~ 20)

First principles or *ab initio* nuclear theory

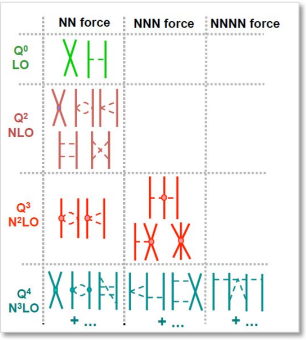


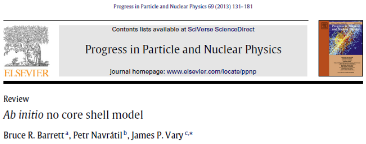
Quantum Chromodynamics (QCD)

Chiral Effective Field Theory (parameters fitted to NN data)

Current *ab initio* nuclear theory

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$





Progress in Particle and Nuclear Physics 69 (2013) 131–181

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Review

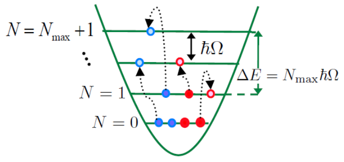
Ab initio no core shell model

Bruce R. Barrett[a], Petr Navrátil[b], James P. Vary[c,*]

Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

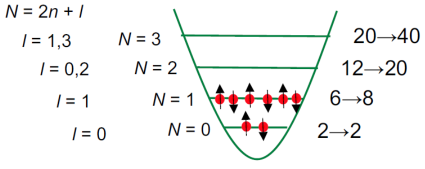
NCSM

- Basis expansion method
    - Harmonic oscillator (HO) basis truncated in a particular way ($N_{\max}$)
    - Why HO basis?
        - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^{4}He, ^{16}O, ^{40}Ca)
        - Equivalent description in relative(Jacobi)-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni}\, \Phi^{HO}_{Ni}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

$$\Psi^A_{\mathrm{SD}} = \sum_{N=0}^{N_{\max}} \sum_j c^{\mathrm{SD}}_{Nj}\, \Phi^{HO}_{\mathrm{SD}\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A\, \varphi_{000}(\vec{R}_{CM})$$



$$E = (2n + l + \tfrac{3}{2})\hbar\Omega$$

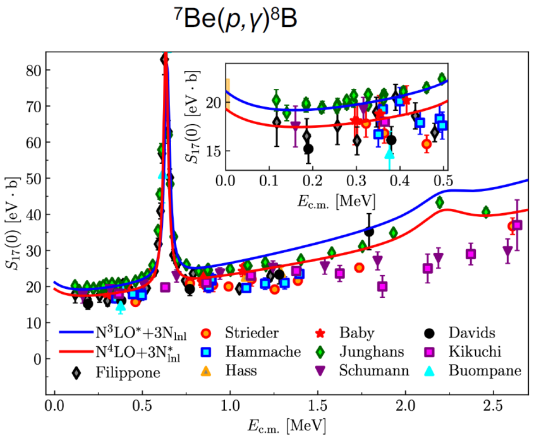
Example of real NP calculation

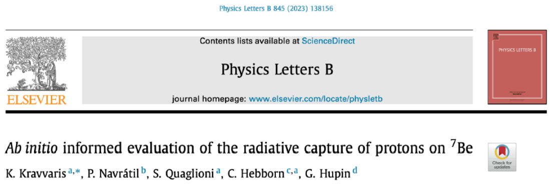
Radiative capture of protons on ^{7}Be

- Solar pp chain reaction, solar ^{8}B neutrinos
- NCSMC calculations with a set of chiral NN+3N interactions as input
  - Radiative capture S-factor
    - Dominated by *E1* non-resonant
    - *M1/E2* significant at 1^+ and 3^+ resonances
  - Correlations between results obtained by different chiral interactions and experimental data → evaluation of the S-factor at *E*=0 energy relevant for the solar physics

Recommended value $S_{17}(0)$ ~ 19.8(3) eV b

Latest evaluation in *Rev. Mod. Phys.* **83,**195–245 (2011):
$S_{17}(0)$ = 20.8 ± 0.7(expt) ± 1.4(theory) eV b

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Ab initio informed evaluation of the radiative capture of protons on ^{7}Be

K. Kravvaris [a,*], P. Navrátil [b], S. Quaglioni [a], C. Hebborn [c,a], G. Hupin [d]