Ph203 (L6) ① ①

Physical properties of Nuclei :

1. Nuclear Size via charge radius
2. " Binding Energy
3. " Stability
4. " Excited States, decays, reactions, ...nucl. data

(I) Nuclear Size

Assuming $\rho_p \simeq \rho_n$, charge radius $\simeq$ matter radius

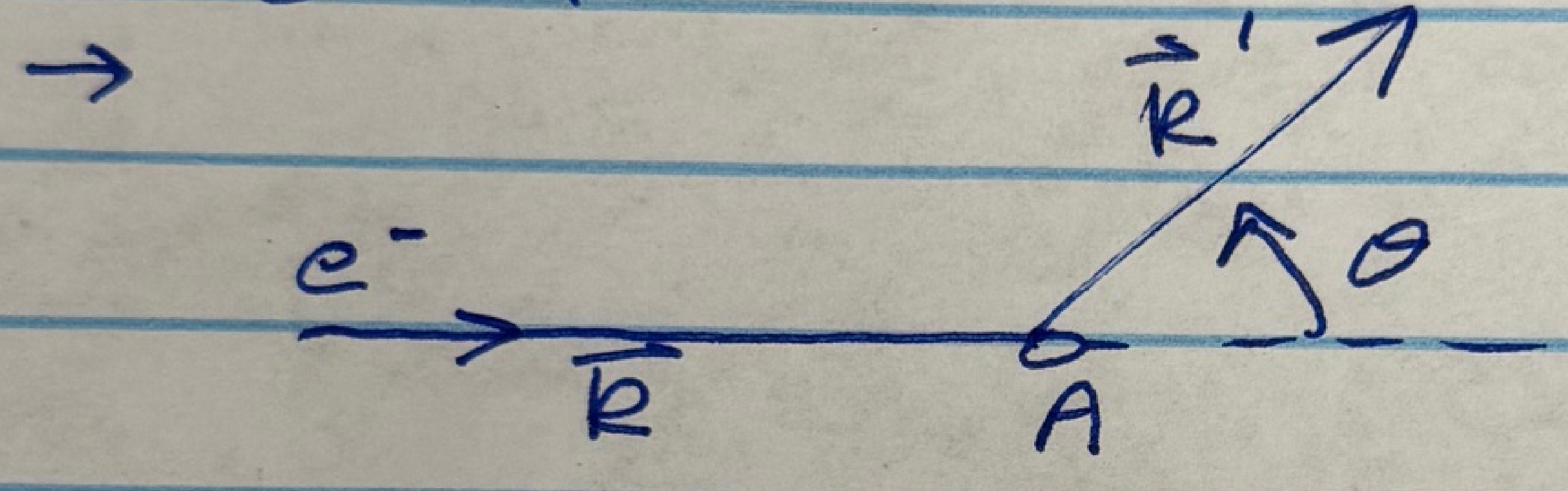
R_{ch} R_m

→ For precise R_{ch} can use $e^- + A$ scattering

(also μ^-A atoms spectroscopy)

more later { n+A scattering for R_m

$e^- + A$



in Born Approx (Non. Relativ. for now)

$$f(\theta) = \frac{-m_e}{2\pi\hbar^2}\int e^{-\vec{q}\cdot\vec{r}'}\, V(r')\, d^3r'$$

$$\vec{q} = \vec{k} - \vec{k}'$$

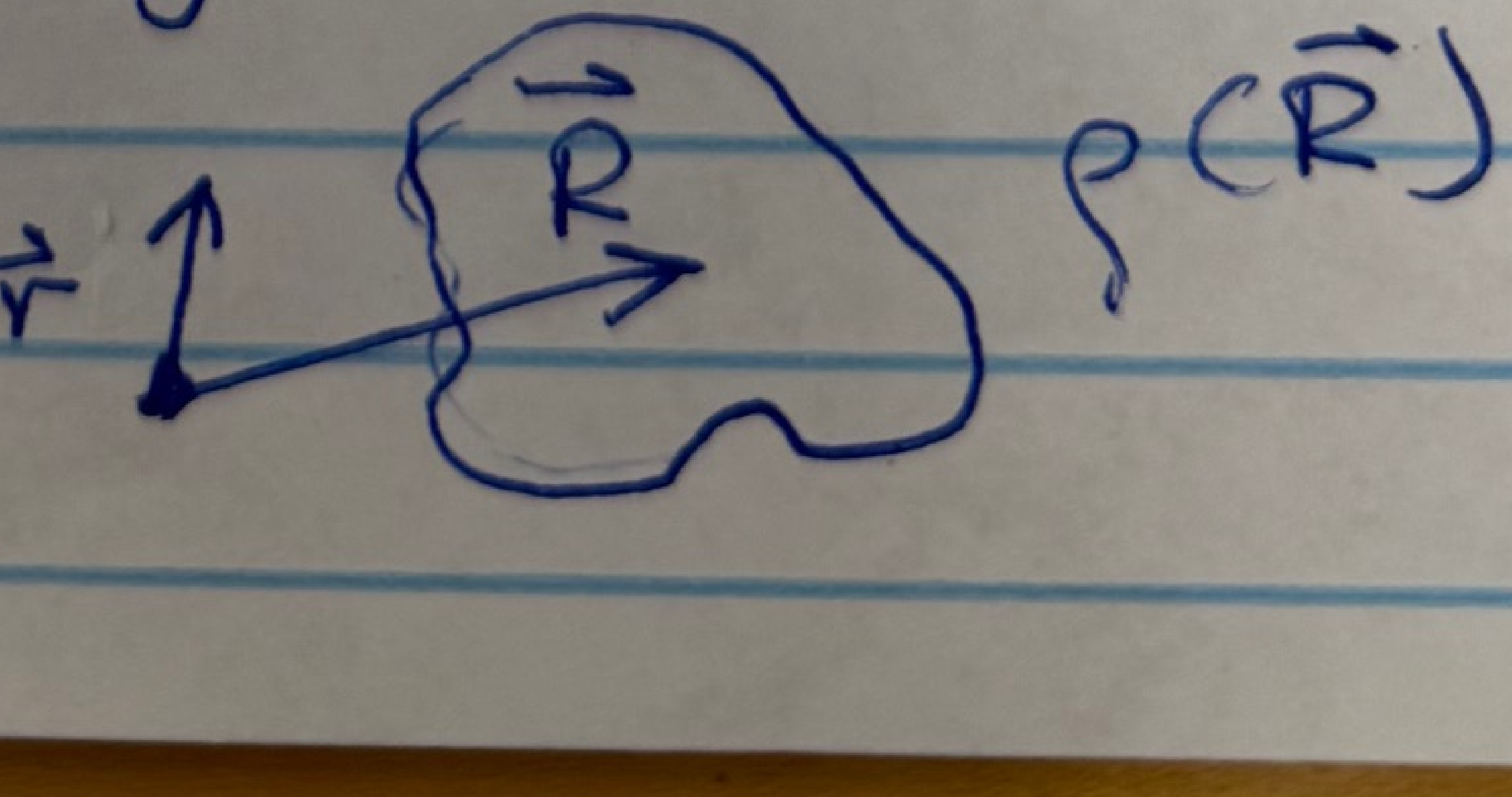
for point-like target $[V(r) \propto \delta(r)]$

$k = \frac{1}{4\pi\epsilon_0}$

$$\left.\frac{d\sigma}{d\Omega}\right)_{point} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} = |f(\theta)|^2 = \left[k\frac{Ze^2}{2T_{cm}\sin^2(\theta_{cm}/2)}\right]^2$$

↑ C.M. Kin. Energy

for Nucleus with charge distrib.



②

$$V(\vec{r}) = \int \rho(\vec{R})\, V_{point}(\vec{r}-\vec{R})\, d^3R$$

$$V_{point} = \frac{Ze^2}{4\pi\epsilon_0 r}$$

then

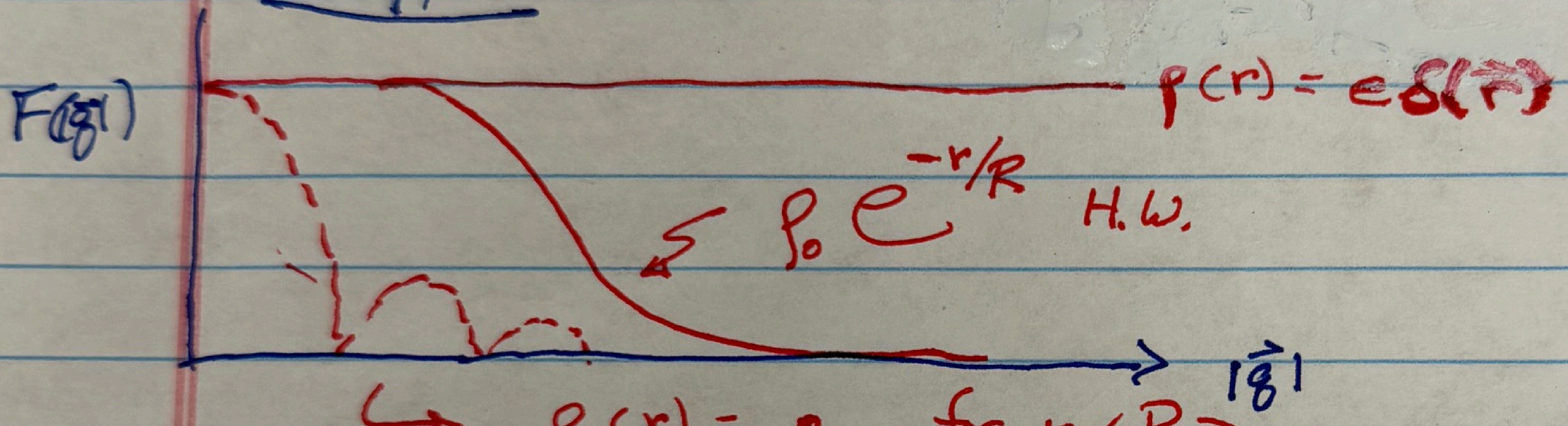
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \times |F(|\vec{q}|)|^2$$

↳ $F(|\vec{q}|) = \int \rho(\vec{R})\, e^{i\vec{q}\cdot\vec{R}}\, d^3R$

w/ $\int \rho(\vec{R})\, d^3R = 1$

& $\rho(\vec{r})$ is FT of $F(|\vec{q}|)$

Examples:



↳ $\rho(r) = \rho_0$ for $r \leq R$
$= 0$ f $r > R$ "Uniform sphere"

⇒

Can measure charge radius for nuclei via low $|\vec{q}|$ measurements of $F(|\vec{q}|)$ ⇒ e.g. $|\vec{q}| << \frac{1}{R_{ch}}$ using

$$(R_{ch}^{rms})^2 = \lim_{q\to 0}\left[6 \times \left|\frac{dF(|\vec{q}|)}{d|\vec{q}|^2}\right|\right]$$ ⇒ see H.W.

R. Hofstadter '61 Nobel:

$\underline{R_A = 1.2\text{ fm} \times A^{1/3}}$, $A = N + Z$

↳ for $A > 12$

Note: $A^{1/3}$ implies "close-packing"

⟺ $\frac{\#\text{ nucleons}}{\text{Volume}} = \text{const.} \equiv \rho_N$ nucleon density same for all nuclei

③

this suggests "saturation" of Nucl. Force (due to repulsive core)

↳ $\rho_N^{central}$ does not increase w mass

↳ different from gravity

(II) Nuclear Binding Energy

Defined via $B(N,Z) = Z m_p + N m_n - M(N,Z)$

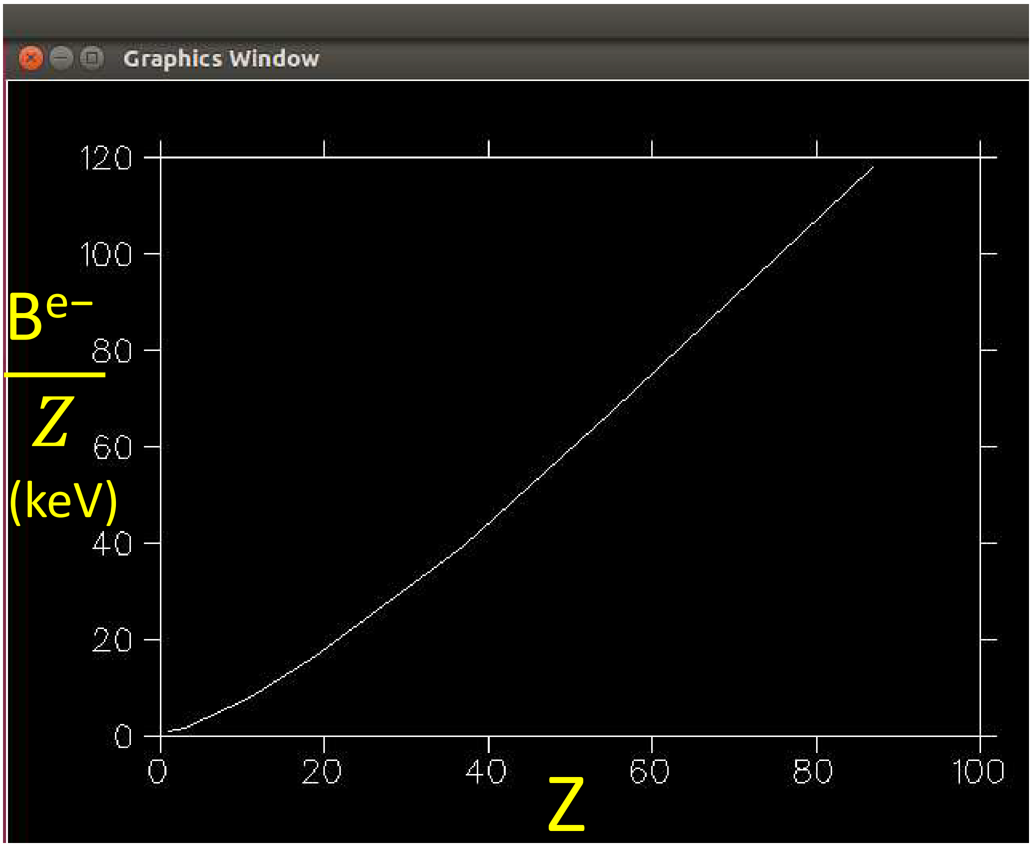
e⁻ binding removed

$B > 0$

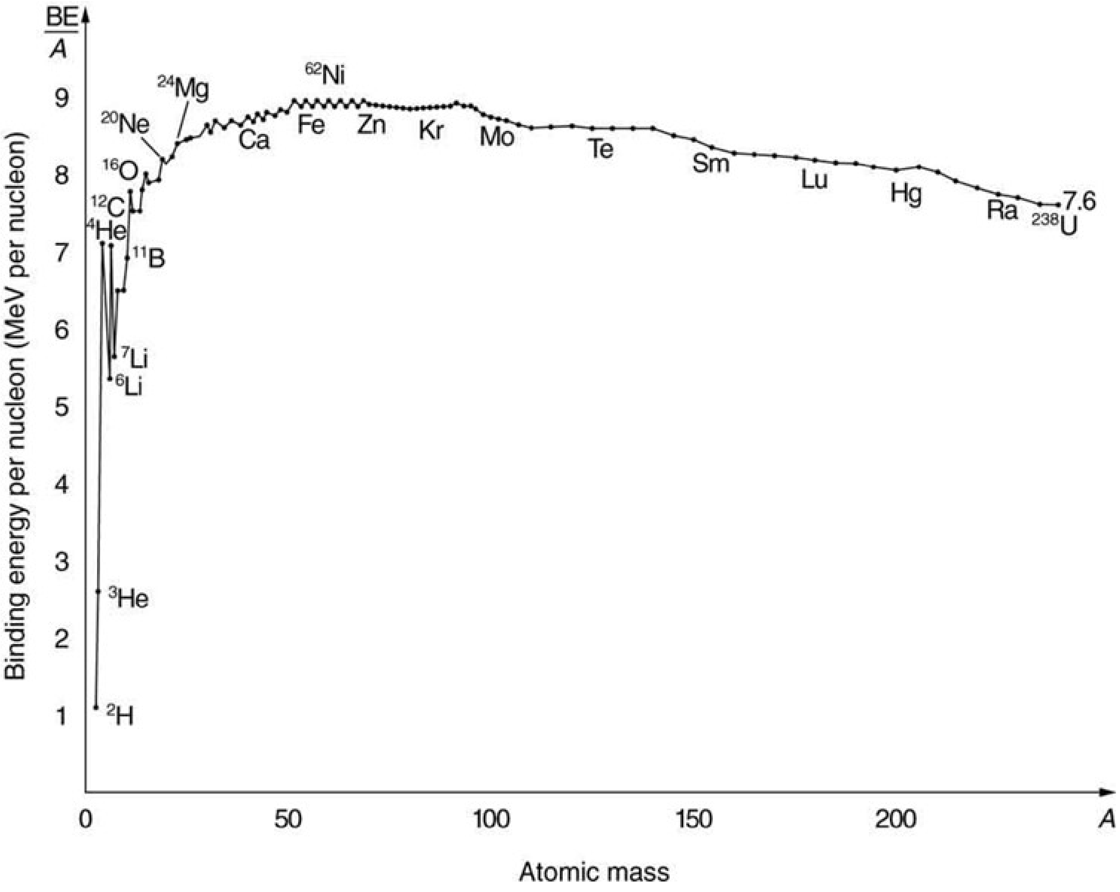
Pics of $\frac{B^{e^-}}{Z}$ for e⁻ binding (for comparison)

& $\frac{B^{nuc}}{A}$ vs A ⇒ signature of strong Int.

Binding Energy per Z or A



Binding energy per electron in Atoms



Binding energy per nucleon in Nuclei

BE shape allows energy "production" via

① Fusion for $A < 50$

e.g. $^{3}He + {}^{3}He \to {}^{4}He + 2p + 12.9\,\text{MeV}$

② Fission for $A > 50$

e.g. $^{238}U \to {}^{95}Sr + {}^{140}Xe + 3n + 171\,\text{MeV}$

III Nuclear Stability

251 Nuclides (fixed Z, N) are stable

(or have $\tau \gtrsim 10^{20}$ yrs $\Rightarrow \tau_{univ.} \simeq 1.4 \cdot 10^{10}$ yrs

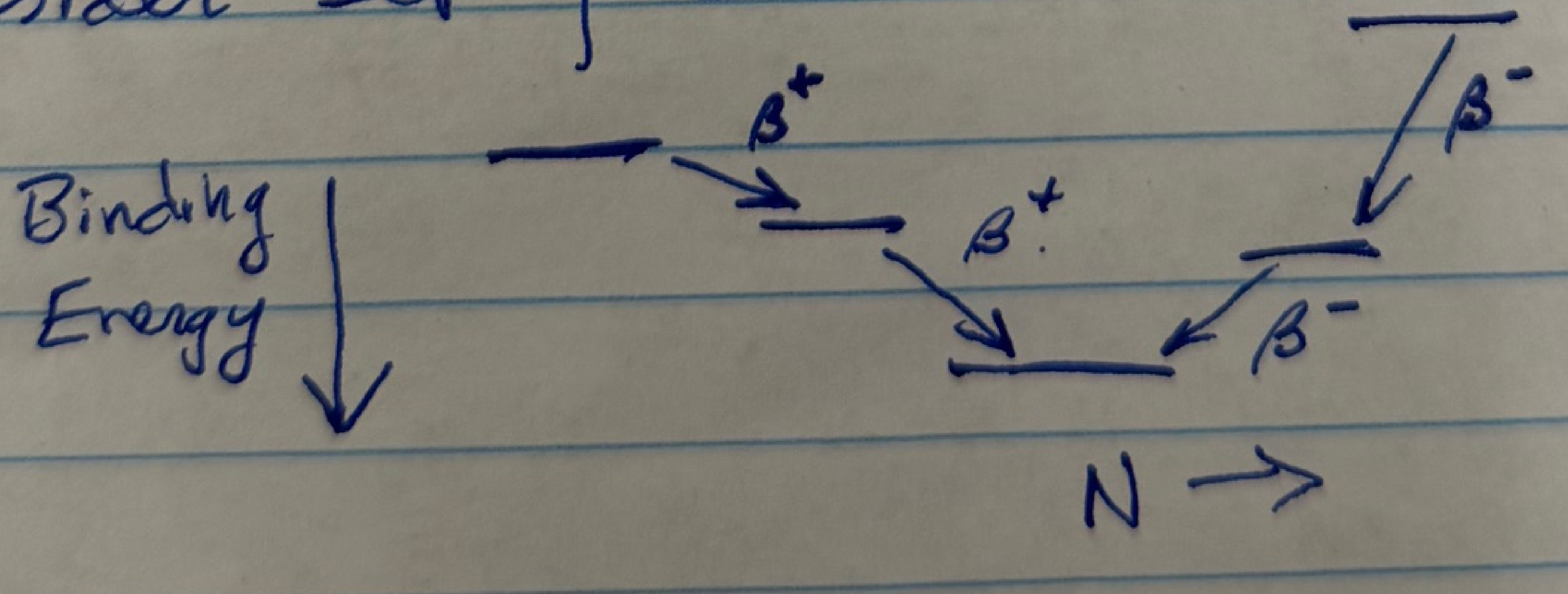
~3000 observed Nuclides have $\tau < 10^{20}$ yrs

~7000 total Nuclides predicted to exist

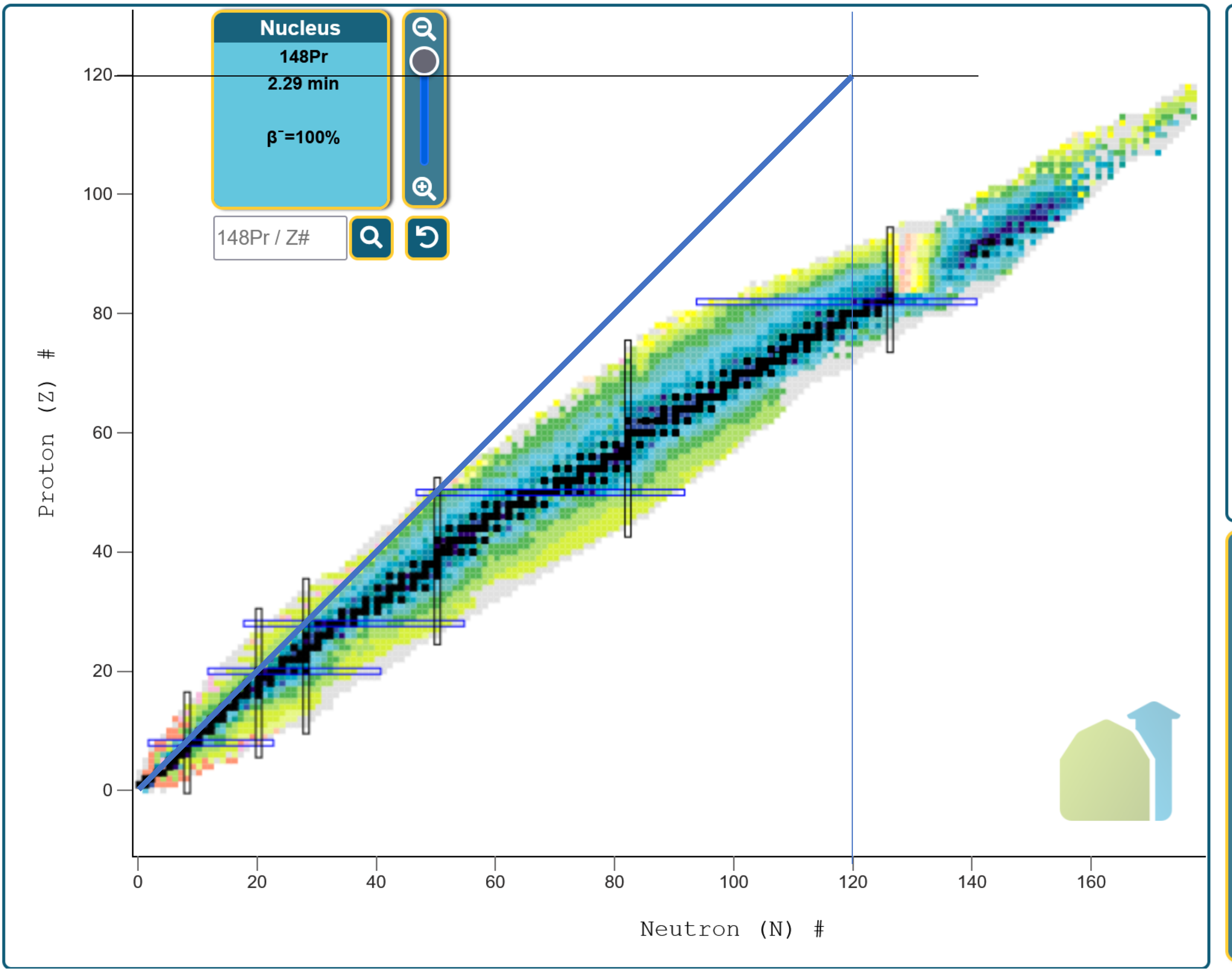
What are Decay Processes?

A. Weak decays

Consider set of Isobars ≡ same A, but diff. N, Z



- β^- decay: $A(N,Z) \to A(N-1, Z+1) + e^- + \bar{\nu}_e$ ($n \to p$)
- β^+ decay: $A(N,Z) \to A(N+1, Z-1) + e^+ + \nu_e$ ($p \to n$)

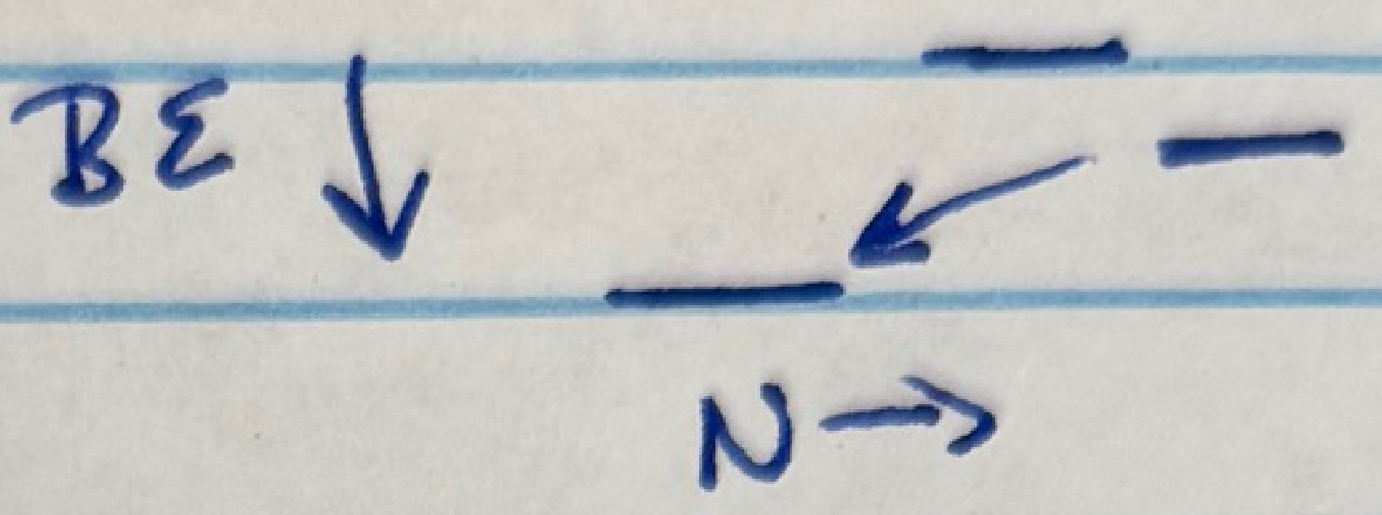

Nucleus
148Pr
2.29 min
β^-=100%
148Pr / Z#
120
100
80
60
40
20
0
Proton (Z) #
0
20
40
60
80
100
120
140
160
Neutron (N) #

- e^--capture: $A(N, Z) + e^- \to A(N+1, Z-1) + \nu_e$

(p → n)

Above account for ≳ 90% of known unstable nuclei but also:

- $\underbrace{\text{Double } \beta^-\text{-decay}}_{\text{DBD}}$ possible



$A(N, Z) \to A(N-2, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$

$\tau > 10^{18}$ yrs

0ν DBD violates Lepton # conservation

↪ $A(N, Z) \to A(N-2, Z+2) + 2e^-$

↪ Beyond Standard Model

$\nu_e = \bar{\nu}_e$ ⇒ Majorana vs

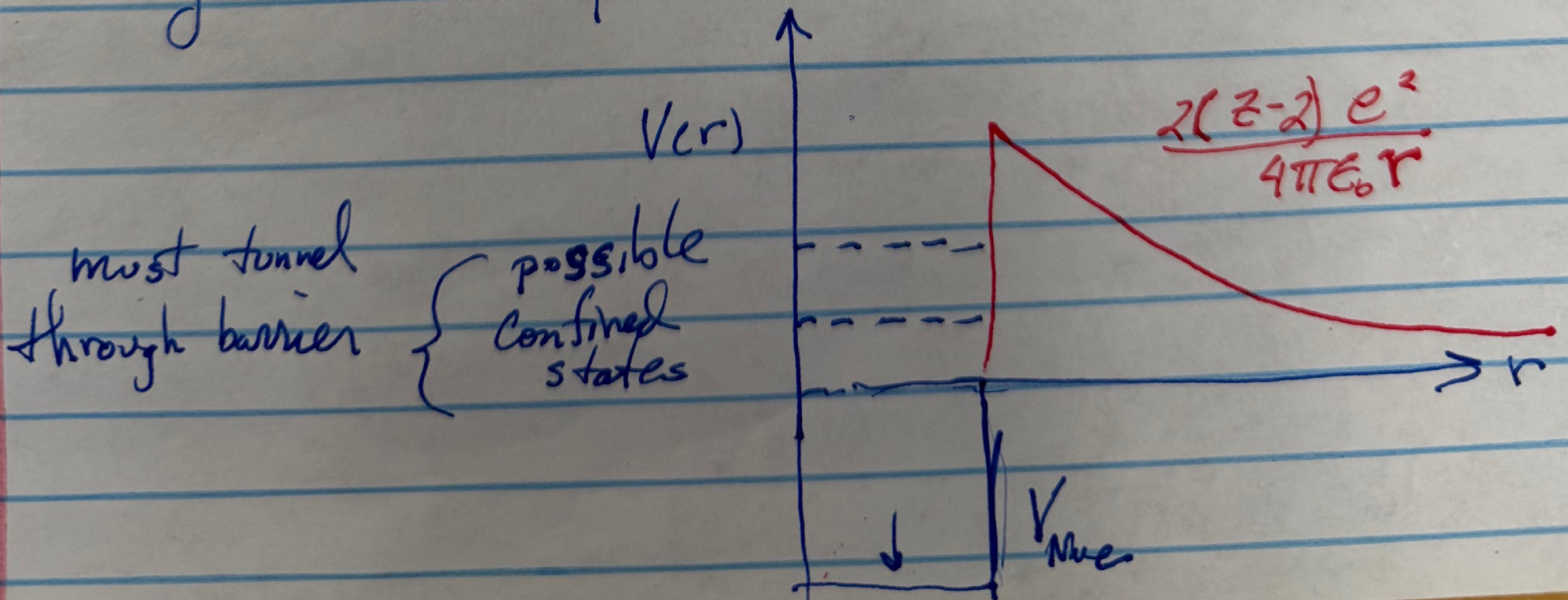
$\nu_e \neq \bar{\nu}_e$ ⇒ Dirac ν

B. Strong Decays

- α-decay

↪ often viewed as pre-existing α particle confined by Coulomb Potential

↗ ^4He



tunnelling prob. can be very small:

$$\tau_{^{238}U \rightarrow {}^{234}Th + \alpha} = 4.5 \cdot 10^{9} \text{ yrs}$$

Note:

1. α-decay primarily occurs for nuclei w $A > 56$
2. Source of 99% of terrestrial ^4He

- Spontaneous Fission

e.g. $^{252}_{98}Cf \rightarrow {}^{97}_{36}Kr + {}^{151}_{62}Sm + 4n$

Z ↗

other options possible

$\langle N_n \rangle = 3.7$

- Cluster decays: ^{12}C, ^{14}C, ^{34}Si observed

- 2 proton decay

↳ correlated pair of protons emitted

- n & p "dripping" n/p drip line...

↳ not considered decay since it occurs "instantly" e.g. time to cross nucleus $\sim 10^{-21}$ sec

non-Poissonian process

Look @ Chart of Nuclei

Google: NNDC, IAEA

IV Nuclear Excited States

Nuclei often possess many energy levels that are β-stable & particle-stable ($\alpha, n, p, \ldots$) but not γ/EM-stable, i.e. many bound states compared to deuteron

→ Often need 4-10 MeV excitation before p or n are unbound (aka "drip off")

→ These states can decay to gnd. state via EM transitions

- ↳ photon/γ-ray emission
- ↳ internal conversion (virtual γ kicks out inner shell e^-)
- ↳ e^+e^- decay

γ-decay Selection Rules

Multipolarity: $|J_f - J_i| \leq \lambda \leq J_f + J_i$

Parity Change: $\pi_i \pi_f = (-1)^\lambda$ $E\lambda$

$\pi_i \pi_f = (-1)^{\lambda+1}$ $M\lambda$

∴ e.g. $2^+ \to 0^+$ is E2

$1^+ \to 0^+$ is M1

$0^+ \to 0^+$ is impossible ($J_\gamma = 1$) but e^+e^- is OK

$2^+ \to 1^+$ is either M1 or E2

γ-decay Rate

$$\Gamma \propto E_\gamma^{2\lambda+1} \; ; \quad \text{e.g.} \quad \frac{\Gamma_{E3}(E_\gamma)}{\Gamma_{E1}(E_\gamma)} \simeq 4 \cdot 10^{-11} \frac{E_\gamma^7}{E_\gamma^3}$$

E_γ in MeV

Big Suppress. if $E_\gamma <$ MeV ↪ Int. Conv.