# Ph 203. Solutions HW #1

**1.)** Energy levels.

a) Infinite potential well

$$V = \begin{cases} \infty & \text{for } r > a \\ 0 & \text{for } r < a \end{cases}$$

Spherically symmetric potential, so can write the total wavefunction as

$$\Psi_{klm}(r, \theta, \phi) = R_{kl}(r) Y_m^l(\theta, \phi),$$

with the radial part $R_{kl}(r) = \frac{u(r)}{r}$.
Schrödinger equation for $r < a$:

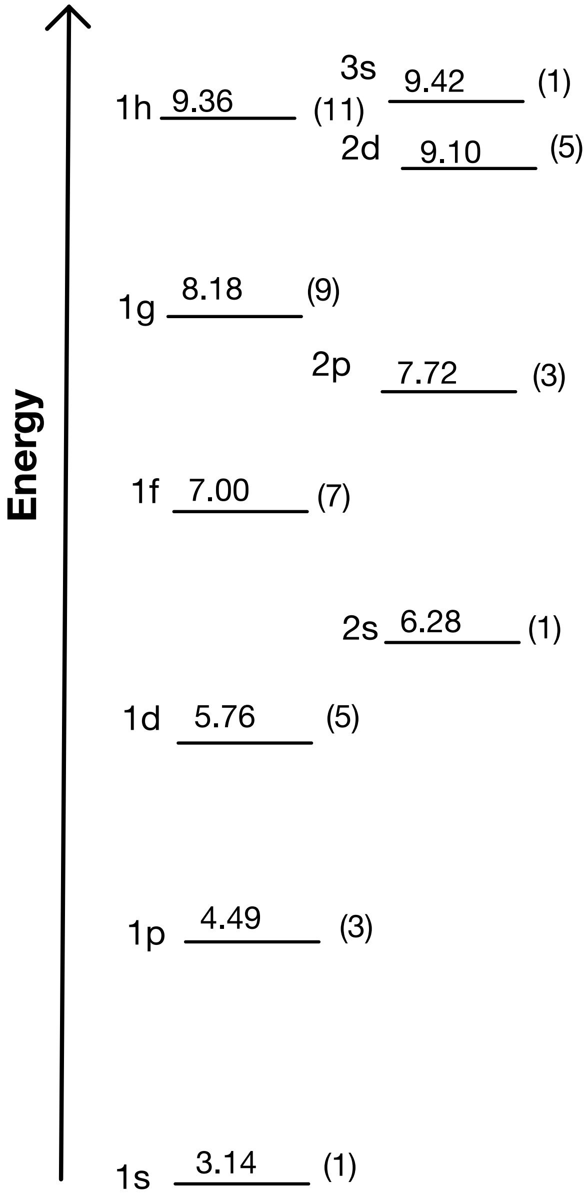
$$-\frac{1}{2\mu}\frac{\partial^2 u(r)}{\partial r^2} + \frac{l(l+1)}{2\mu r^2} u(r) = \frac{k^2}{2\mu} u(r),$$

with energy $E = \frac{k^2}{2\mu}$.
The solutions are spherical Bessel functions $j_l(kr)$ and energy levels are determined by $j_l(ka) = 0$.
Roots of $j_l(x) = 0$ are:

| | n=1 | n=2 | n=3 |
|---|---|---|---|
| $l = 0$ | 3.14 | 6.28 | 9.42 |
| $l = 1$ | 4.49 | 7.72 | |
| $l = 2$ | 5.76 | 9.10 | |
| $l = 3$ | 7.00 | | |
| $l = 4$ | 8.18 | | |
| $l = 5$ | 9.36 | | |

#fermions$(s = 1/2) = 2\times$ degeneracy
Energy levels, labeled by $nl$(degeneracy), (from lowest to highest):

$1s(1),\ 1p(3),\ 1d(5),\ 2s(1),\ 1f(7),\ 2p(3),\ 1g(9),\ 2d(5),\ 1h(11),\ 3s(1)$




b) Radial SHO: use separation of variables in cartesian coordinates

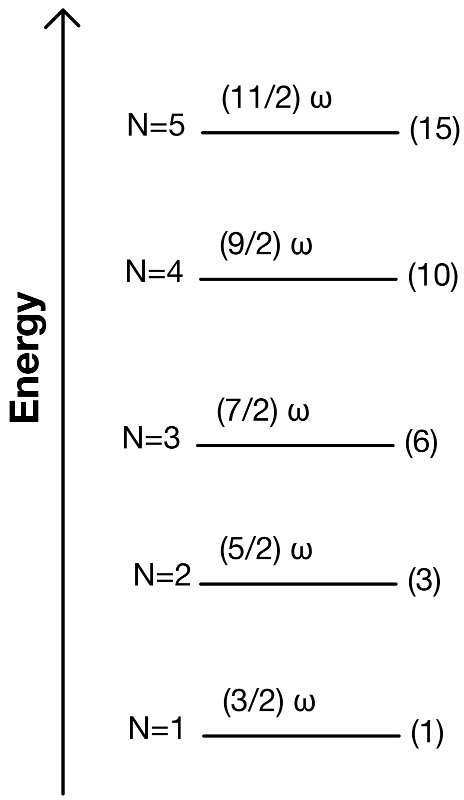
$$H = \omega\left(n_x + \frac{1}{2}\right) + \omega\left(n_y + \frac{1}{2}\right) + \omega\left(n_z + \frac{1}{2}\right)$$
$$= \omega\left(n_x + n_y + n_z + \frac{3}{2}\right) = \omega\left(n + \frac{3}{2}\right),$$

where $n = n_x + n_y + n_z$.

Energy levels (from lowest) labeled by $N = n + 1$:

| N(degeneracy) | $l$ |
|---|---|
| 1(1) | 0 |
| 2(3) | 1 |
| 3(6) | 0, 2 |
| 4(10) | 1, 3 |
| 5(15) | 0, 2, 4 |

Degeneracy - count all equivalent ways one can choose $n_x, n_y, n_z$ to get $n$
Angular momentum: $l = N - 1,\ N - 3,\ \ldots$




**2.)** Spin projection

$$\hat{S}_{\hat{n}} = \frac{1}{2}\vec{\sigma} \cdot \hat{n} = \frac{1}{2} \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix},$$

where $\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Eigenvalues: $\lambda = \pm\frac{1}{2}$. For $\lambda = \frac{1}{2}$, eigenvector is $\chi = (\cos\frac{\theta}{2}, \sin\frac{\theta}{2}e^{i\phi})$.

$$\Rightarrow a = \cos\frac{\theta}{2}, b = \sin\frac{\theta}{2}e^{i\phi}$$

*Answers may differ depending on phase rotations*

**3.)** Adding three spin 1/2 particles

Symmetric under exchange

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$
$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$
$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$
$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

Symmetric under $1 \leftrightarrow 2$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\uparrow\uparrow\downarrow - \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = -\sqrt{\frac{2}{3}}\downarrow\downarrow\uparrow + \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow)$$

Antisymmetric under $1 \leftrightarrow 2$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$$

**4.)** Bertulani 1.6

Incoming proton $p_1 = (E, 0, 0, p)$
proton at rest $p_2 = (m_p, 0, 0, 0)$

$$\Rightarrow s = (p_1 + p_2)^2 = 2m_p(m_p + E)$$

Final state particles: min $E$ when final particles at rest in C.M. frame

$$\sum_{final} p'_i = (4m_p, 0, 0, 0)$$

$$\Rightarrow s = \Big(\sum_{final} p'_i\Big)^2 = 16m_p^2$$

Equating $s$, we get $E = 7m_p$, so kinetic energy is $KE = 6m_p \approx 5.6$ GeV

**5.)** Parity of the pion

$$\pi^- + d \rightarrow n + n$$

**Initial state**

Pion is spin 0 and $\pi^-$ captured at rest $\Rightarrow l = 0$.
Parity of the intial state $= P_\pi(-1)^l P_d = P_\pi$ for $l = 0, P_d = +1$.
Deuteron is spin 1.
Total angular momentum of the initial state: $J = 1$

**Final state**

n-n has overall antisymmetric wavefunction and total angular momentum $J = 1$.
Possibilities ($s$ is total spin, $l$ is orbital angular momentum):

[1] $s = 0,\ l = 0 \rightarrow J = 0$, not allowed

[2] $s = 0,\ l = 1 \rightarrow$ symmetric wavefunction, not allowed

[3] $s = 1,\ l = 0 \rightarrow$ symmetric wavefunction, not allowed

[4] $s = 1,\ l = 1$, can have:

- $J = 0 \rightarrow$ symmetric wavefunction, not allowed
- $J = 1 \rightarrow$ antisymmetric wavefunction
- $J = 2 \rightarrow$ symmetric wavefunction, not allowed

Only the $J = 1$ $s = 1$ $l = 1$ option gives a totally antisymmetric state with $J = 1$. Since $l = 1$, the parity of the final state is $P_{2n} = -1$. Since parity is conserved in this process, we deduce $P_\pi = -1$.