Ph 203: L7

As shown in Lect. 4, hard to calc. heavy nuclei structure ab initio (1^{st} principles) for $A \gtrsim 20$. For $A > 20$ must rely on models that work for specific nuclei or general properties

e.g.

even-even nuclei

↳ Interacting Boson Approx.

↳ nucleons are paired

$\langle \rho \rangle$ inside nuclei

↳ Fermi Gas Model

Most Famous = Single-particle Shell Model ⇒ like AMO each shell is filled up to N, Z

Start w/ H.W. 1 spherical square Well

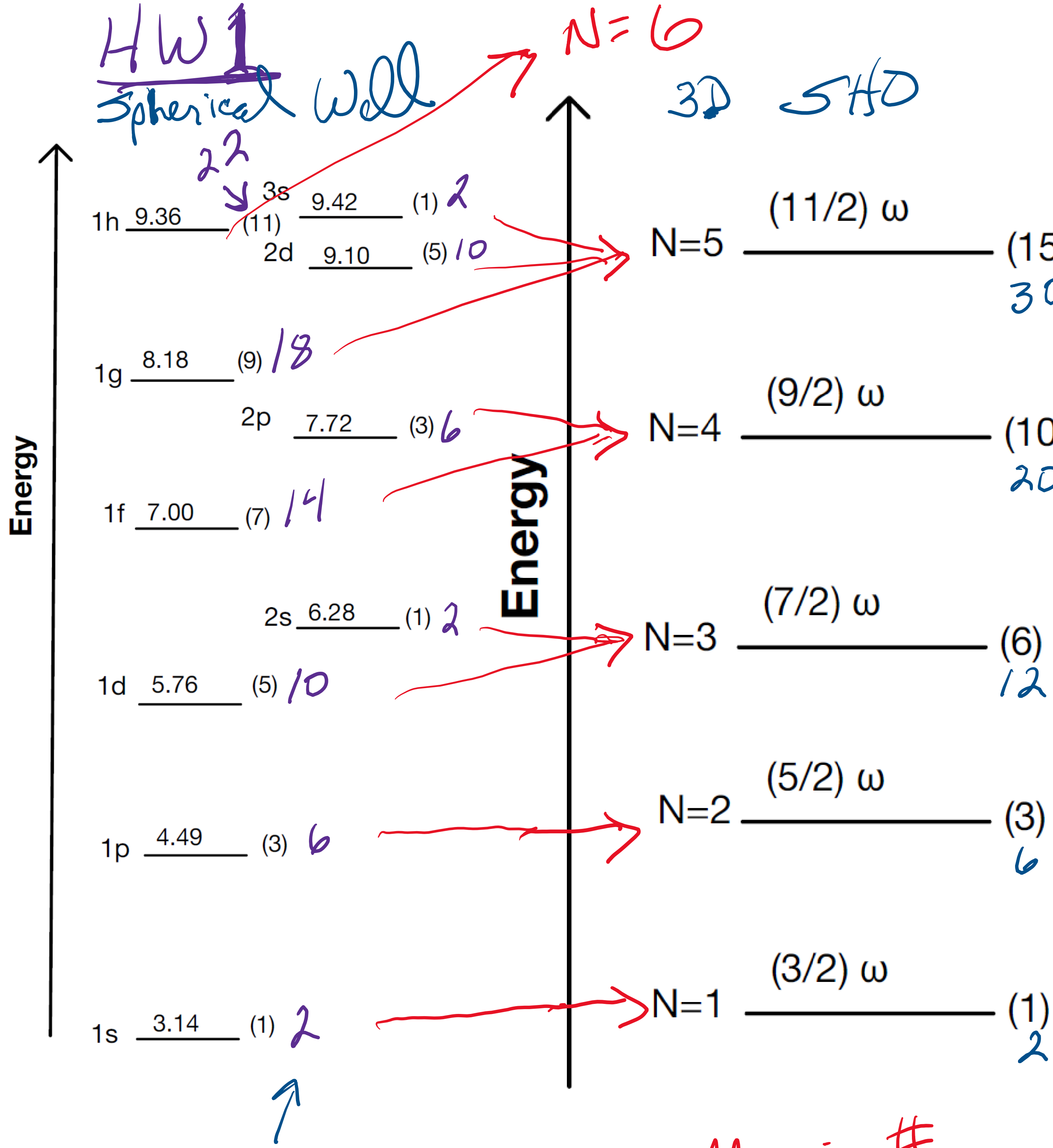
$V_0 \simeq -100\,MeV$

Harmonic oscillator & spherical well predict "separate" shells @ N or $Z = 2, 8, 20, 40, 70, 112, \ldots$

But! ⟶ (see Pic ...)

HW1 !

HW 1



$\frac{\text{\# nucleons}}{\text{level}}$

Magic = 2, 8, 18, 20, 34, 40, 58, ...

Magic #
2, 8, 20, 40, 50

But <u>observed</u> Magic #'s are 2, 8, 20, 28, 50, 82, 126

→ see evidence for these Magic #'s

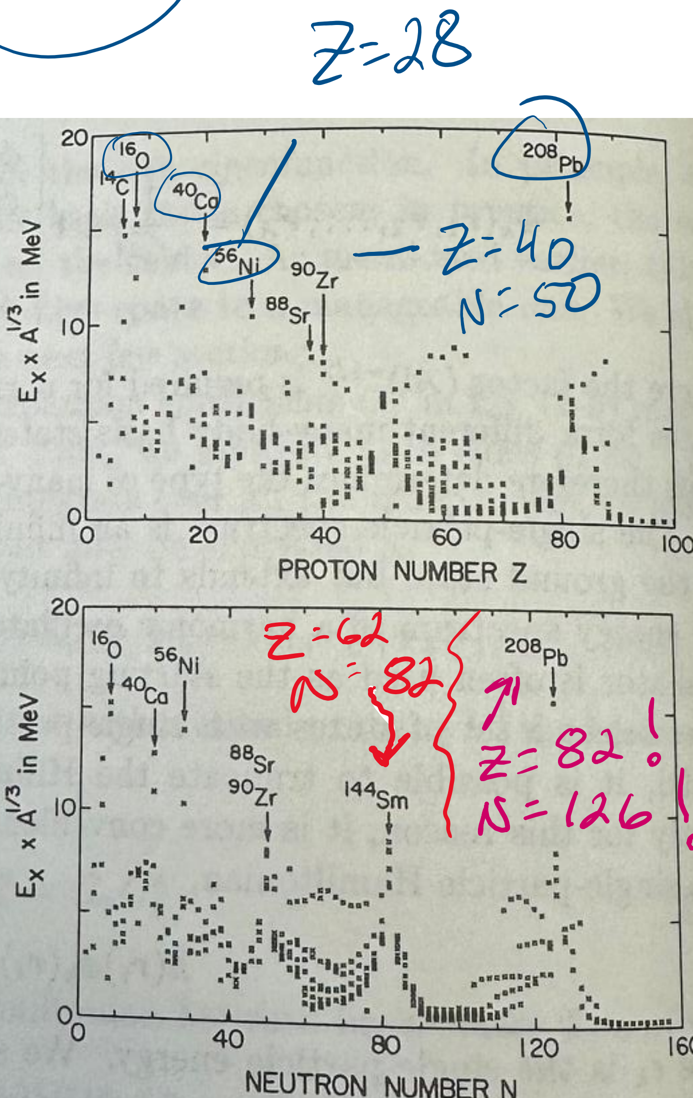
→ clearly, past N, Z = 20 something is "shifting" energy levels

Magic Numbers of Nucleons

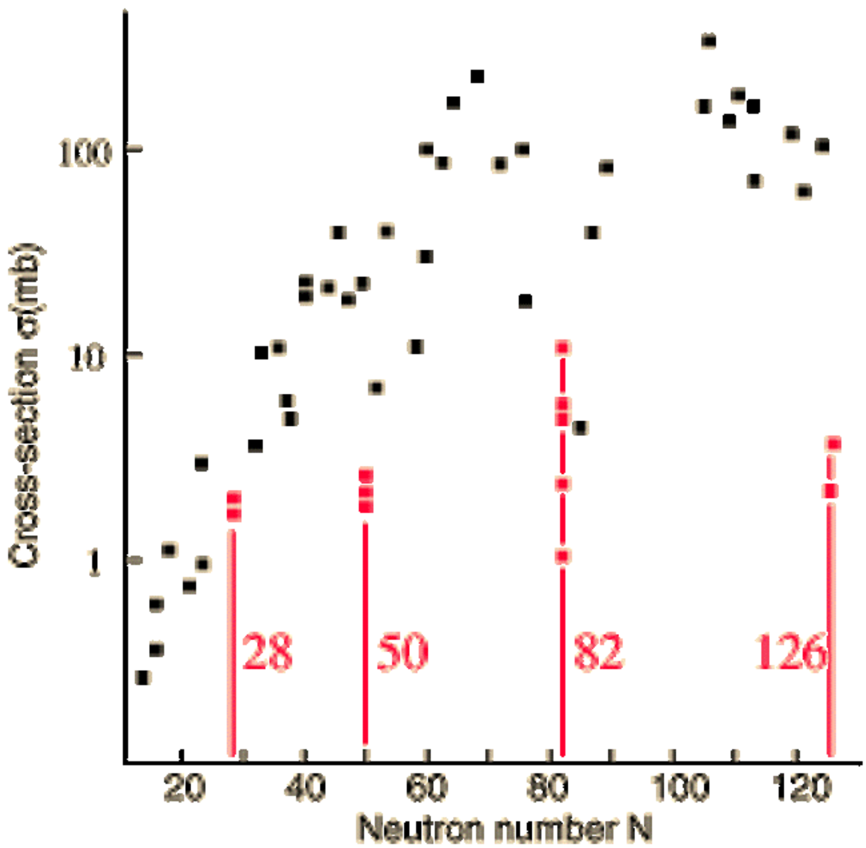
1st exited state energy x $A^{1/3}$

^{4}He

Figure 7-1: Energy of the first excited state of even-even nuclei as a function of proton number (upper) and neutron number (lower). All energies are multiplied by a factor $A^{1/3}$ to account for the general decrease in excitation energy with increasing nucleon number A. Among a total of 389 nuclei, form ^{4}He to ^{256}Fm, there are 372 with spin-parity 2^+ for the first excited state, only 7 with 0^+, 3 with 3^-, 2 with 1^-, and 5 with unknown spin. The highest excitation energy is found in ^{4}He (not shown) with $E_x = 20.1$ MeV or $E_x \times A^{1/3} = 31$ MeV. The data are taken from Ref. [95].



Magic Numbers of Nucleons

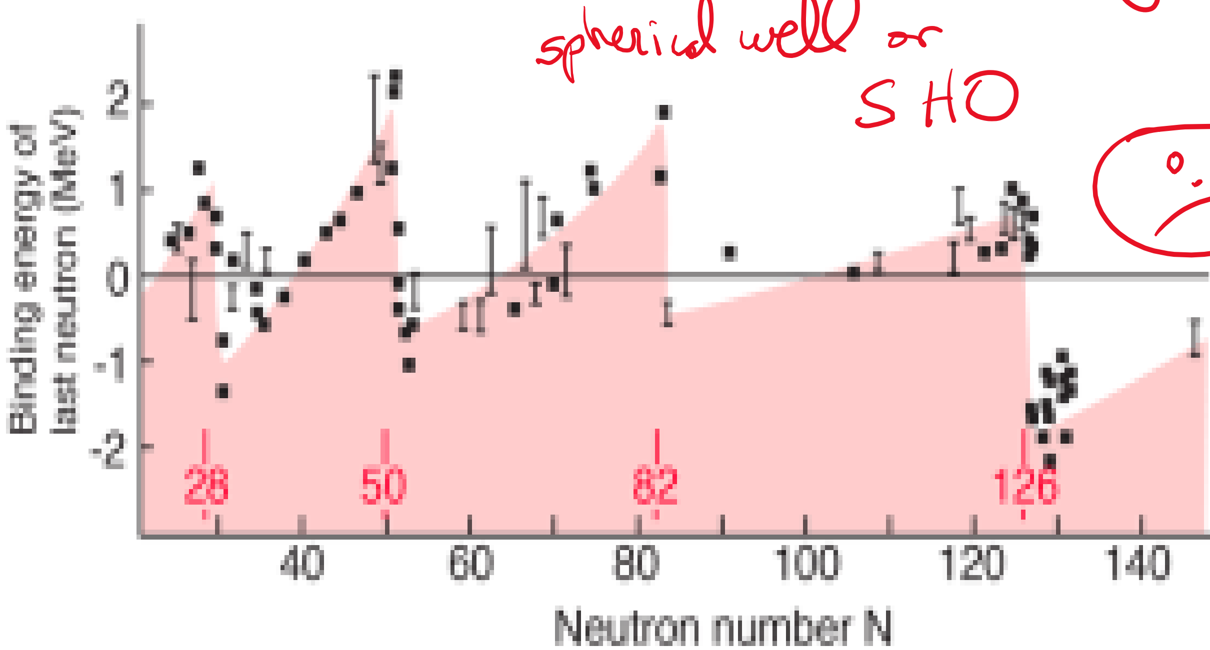


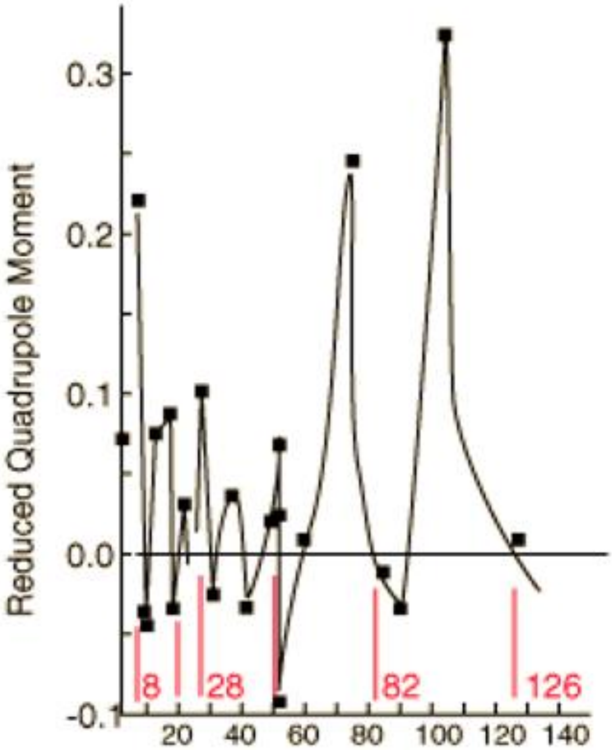
Nuclei with magic numbers of neutrons have neutron absorption cross-sections up to two orders of magnitude less than other nuclei with similar masses

Doesn't agree

spherical well or

SHO

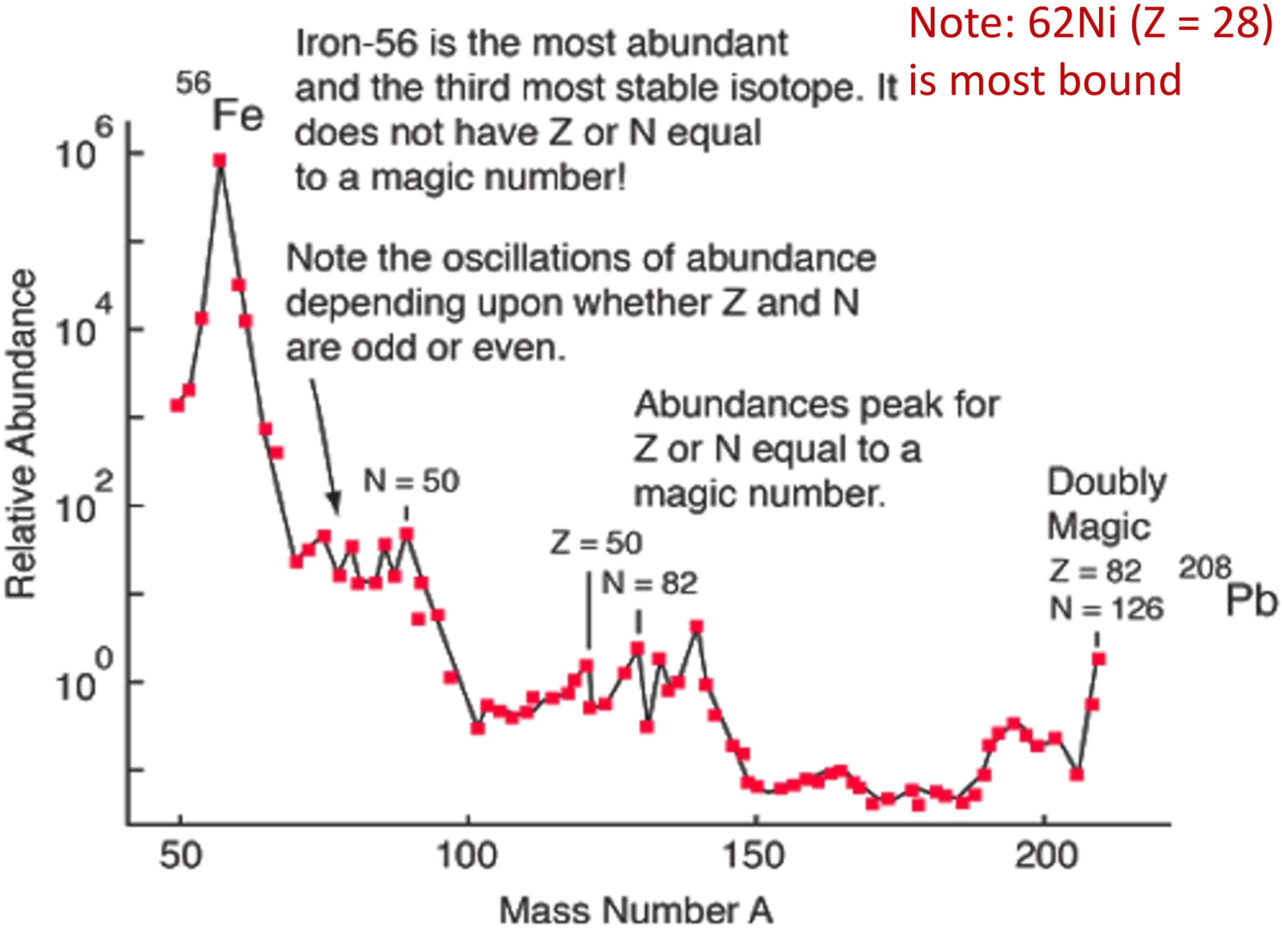




Quadrupole measurements for odd-A nuclei. The horizontal axis is either neutron number or proton number, whichever is odd.

For a shell model, those nuclei with closed shells should be spherically symmetric and have no quadrupole moment.

Note: 62Ni (Z = 28) is most bound



1948: Maria Goeppert Mayer published data (from Manhattan Project) that suggested $N, Z = 50, 82, 126$ were "magic"

Talking w Fermi they wondered about spin-orbit forces being important.

She wrote up her calcs that night & Fermi taught it in his NP class the following week!

She said: what if

$$V_{so} = -V_{so}(r)\,\hat{\vec{L}}\cdot\hat{\vec{S}} \quad \text{was "big"}$$

then since $\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$

each nucleon in shell with given l has either $j = l+\frac{1}{2}$ or $l-\frac{1}{2}$

then since $\hat{\vec{L}}\cdot\hat{\vec{S}} = \dfrac{\hat{\vec{J}}^2 - \hat{L}^2 - \hat{S}^2}{2}$, a nucleon w $j = l-\frac{1}{2}$ has higher energy than $j = l+\frac{1}{2}$ shell w energy splitting

$$\begin{aligned}
\Delta V_{so}^{l} = V_{so}^{l-\frac{1}{2}} - V_{so}^{l+\frac{1}{2}} &= -\left[j_-(j_-+1) - j_+(j_+ +1)\right] \\
&= -\left[(l-\tfrac{1}{2})(l+\tfrac{1}{2}) - (l+\tfrac{1}{2})(l+\tfrac{3}{2})\right] \\
&= -(-l-l-1) \\
&= \underline{2l+1}
\end{aligned}$$

splitting grows w l

This gives <u>correct</u> Magic #'s

(see Slide →)

Simple Shell Model (3D harmonic oscillator or 3D infinite square well predict shell closures at N or Z = 2, 8, 20, 40, 70)

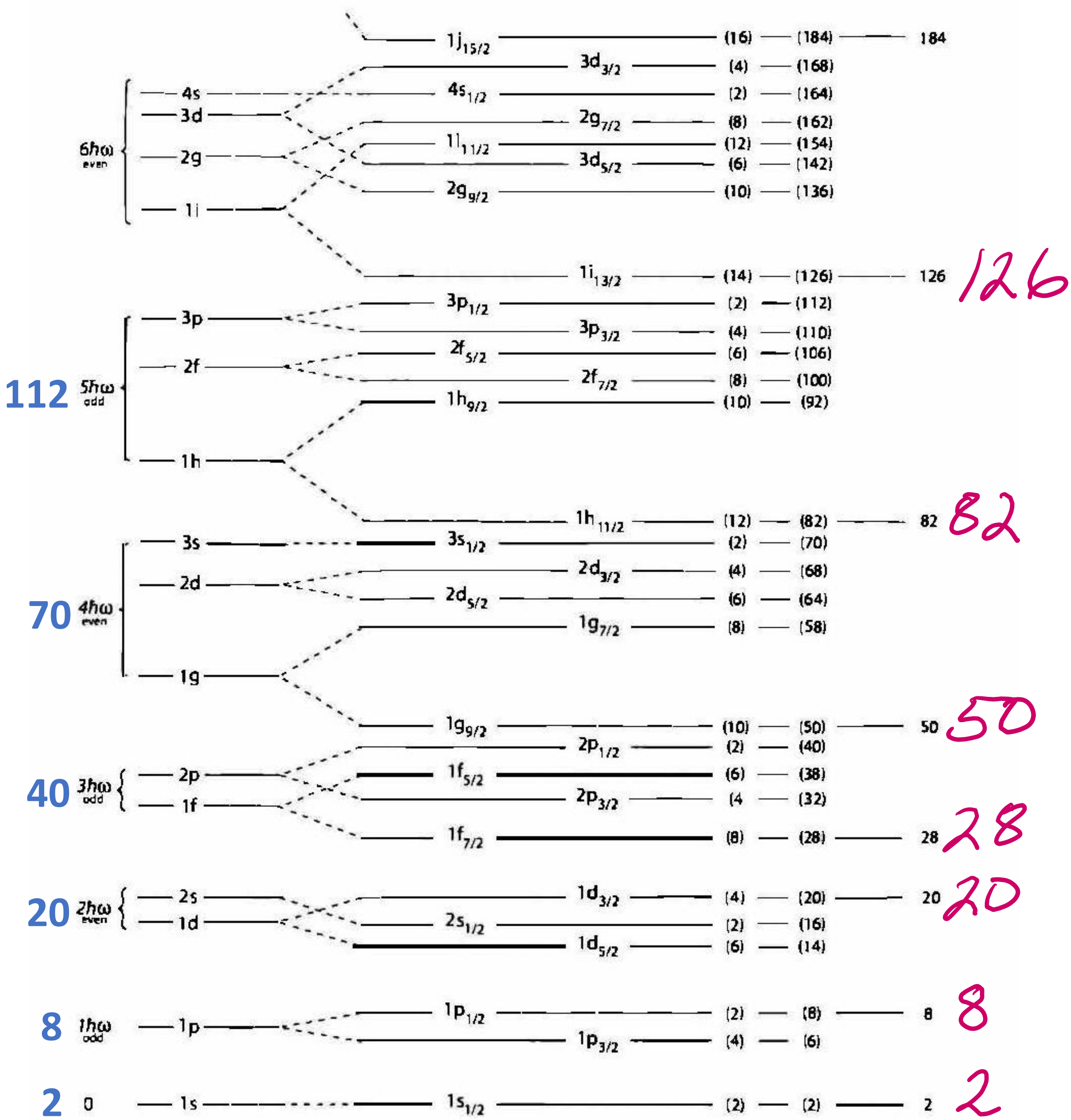


Figure 5.9 Level scheme of the shell model showing the break of the degeneracy in j caused by the spin-orbit interaction term and the emergence of the magic numbers in the shell closing. The values in the first set of parentheses indicate the number of nucleons of each type that the level admits and the values in the second set of parentheses provide the total number of nucleons of each type up to that level. Finally, the numbers outside parentheses indicate the total number of nucleons at shell-closure, reobtaining the magic numbers in their entirety. The ordering of the levels is not rigid, and there could be level inversions when changes occur in the form of the potential [MJ55].

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Shell Model also predicts many gnd state spin/Paritys:
↳ assume "pairing" interaction such that
all p & n pairs cancel ∴

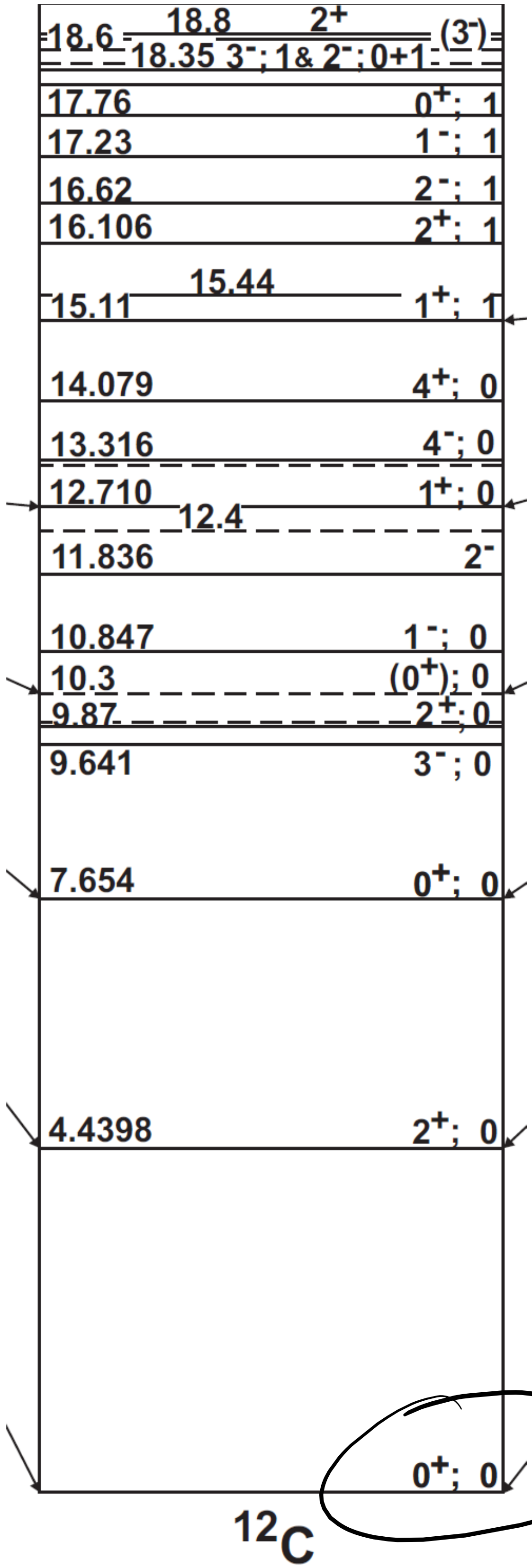
① Even-Even nuclei $\underline{\underline{\text{all}}}$ have $J^{\pi} = 0^{+}$ for gnd state
Z N

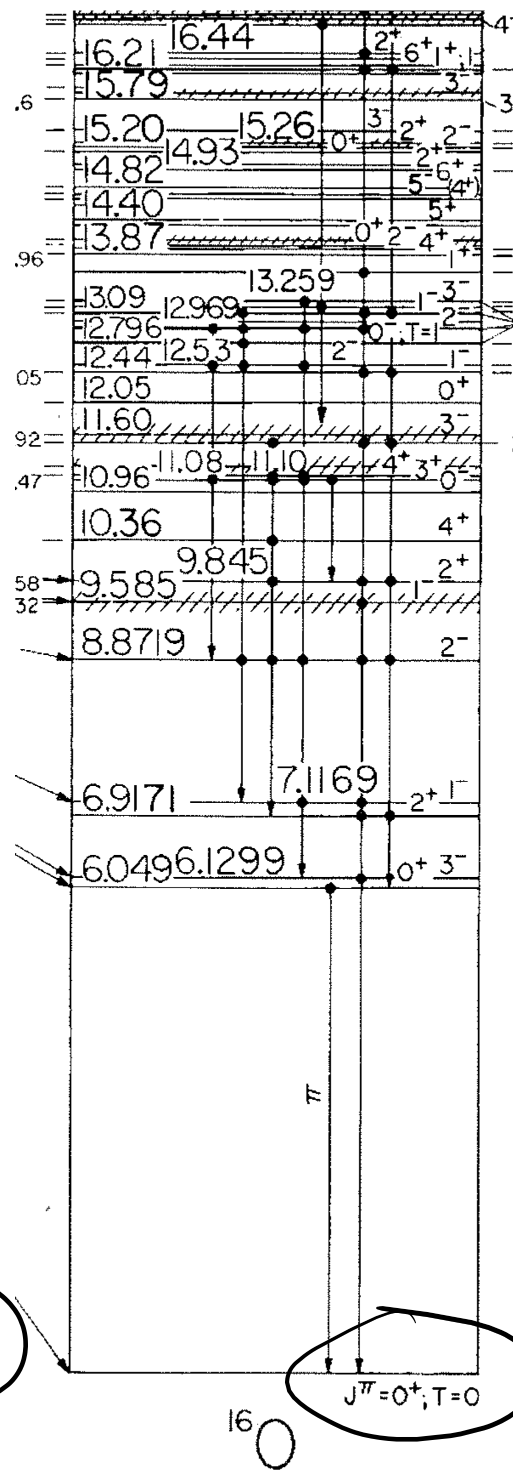
↳ Perfect!

② Odd-Even nuclei gnd state J^{π} from last, "valence" nucleon (see ^{17}O, ^{17}F pics)
$Z=8$, $N=9$ — $Z=9$, $N=8$

↳ works well (not perfect)

↳ called "Mirror" Nuclei


18.6
18.8
2+
18.35 3-; 1& 2-; 0+1
(3-)
17.76
0+; 1
17.23
1-; 1
16.62
2-; 1
16.106
2+; 1
15.44
15.11
1+; 1
14.079
4+; 0
13.316
4-; 0
12.710
1+; 0
12.4
11.836
2-
10.847
1-; 0
10.3
(0+); 0
9.87
2+; 0
9.641
3-; 0
7.654
0+; 0
4.4398
2+; 0
0+; 0
12C


16.44
16.21
15.79
15.20
15.26
14.93
14.82
14.40
13.87
13.259
13.09
12.969
12.796
12.53
12.44
12.05
11.60
11.08
11.10
10.96
10.36
9.845
9.585
8.8719
7.1169
6.9171
6.049
6.1299
Jπ=0+; T=0
16O

Even - Even

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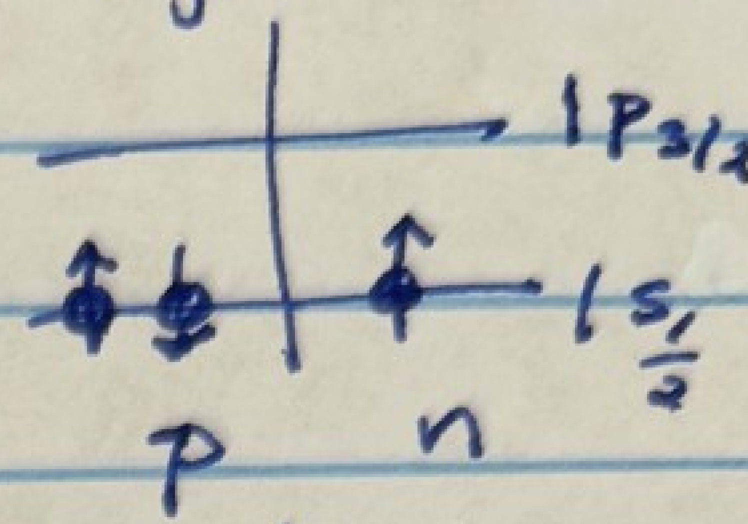
This provides trial W.F. to calc. mag. moments, Quad moments (sometimes), nucleon momenta, ..., other Matrix elements

Sometimes show W.F. via Shell diags (HW)

e.g., ^3He in $J_z = +\frac{1}{2}$



However, for full Anti sym. W.F. in spin/isospin space use Slater Determ. eg.

$$|J\, J_z\, T\, T_3\rangle = |\tfrac{1}{2}\, {+\tfrac{1}{2}}\, \tfrac{1}{2}\, {+\tfrac{1}{2}}\rangle = |{}^3He,\ J_z = +\tfrac{1}{2}\rangle$$

$$|{}^3He\ {+\tfrac{1}{2}}\rangle = \frac{1}{\sqrt{6}} \begin{vmatrix} p_\downarrow & p_\uparrow & n_\uparrow \\ p_\downarrow & p_\uparrow & n_\uparrow \\ p_\downarrow & p_\uparrow & n_\uparrow \end{vmatrix} =$$

$$\frac{1}{\sqrt{6}} \Big[|p_\uparrow n_\uparrow p_\downarrow\rangle - |p_\downarrow n_\uparrow p_\uparrow\rangle + |p_\downarrow p_\uparrow n_\uparrow\rangle - |p_\uparrow p_\downarrow n_\uparrow\rangle + |n_\uparrow p_\downarrow p_\uparrow\rangle - |n_\uparrow p_\uparrow p_\downarrow\rangle \Big]$$

For bulk properties can use:

Fermi Gas Model

⟹ Assume nucleus is degenerate Fermi Gas (non-interacting)

① ⟹ clearly fermions

② ⟹ Mostly non-interacting

⟹ since $R_A \simeq 1.2 A^{1/3}$ fm then

$$\rho_N \simeq \frac{A}{\frac{4}{3}\pi R_A^3} = 0.14 \frac{\text{nucleons}}{\text{fm}^3}$$

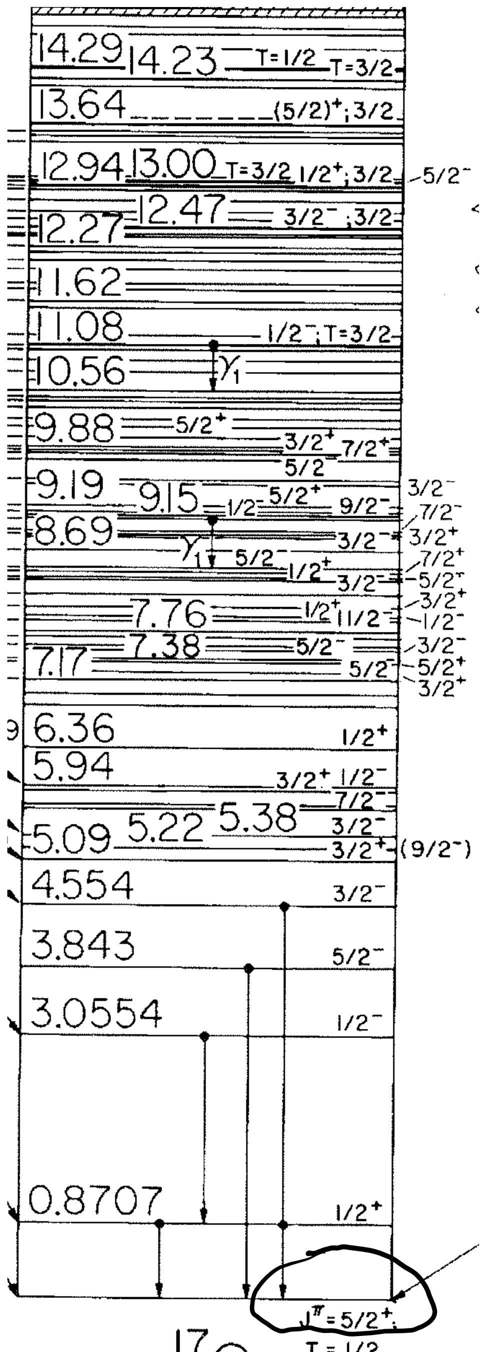
⟹ $\sim 7 \frac{\text{fm}^3}{\text{nucleon}}$

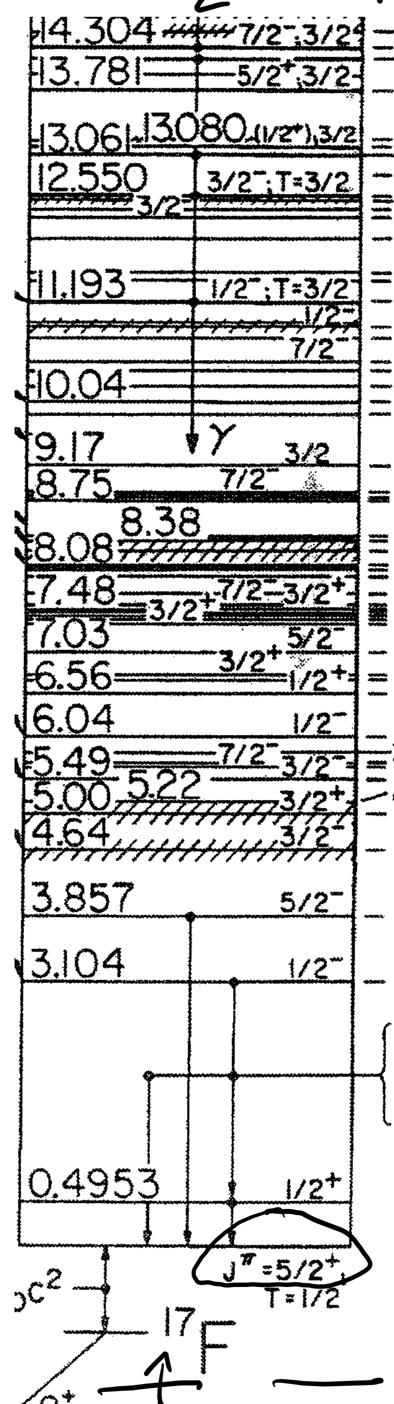
~ 3.5 fm between nucleons

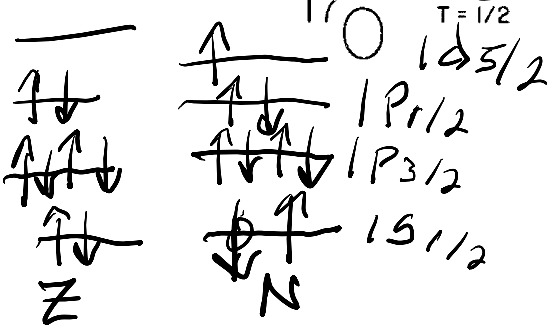
↳ "weak" interaction

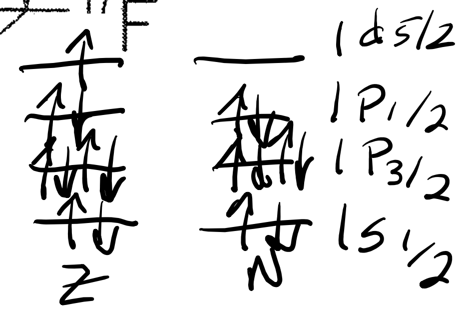
(Even-Odd) Mirror Nuclei

N → Z
Z → N









③ Degenerate (?)

typical 1^{st} excited state in nucleus

$\sim 0.5 - 5$ MeV
$T_{Room} \sim \frac{1}{40}$ eV
} Degenerate!

in stars $T_{star} \sim .03 - 0.1$ MeV

↳ still mostly degen.

∴ nucleons occupy lowest available states up to Fermi Energy E_F

& from Stat. Mech.,

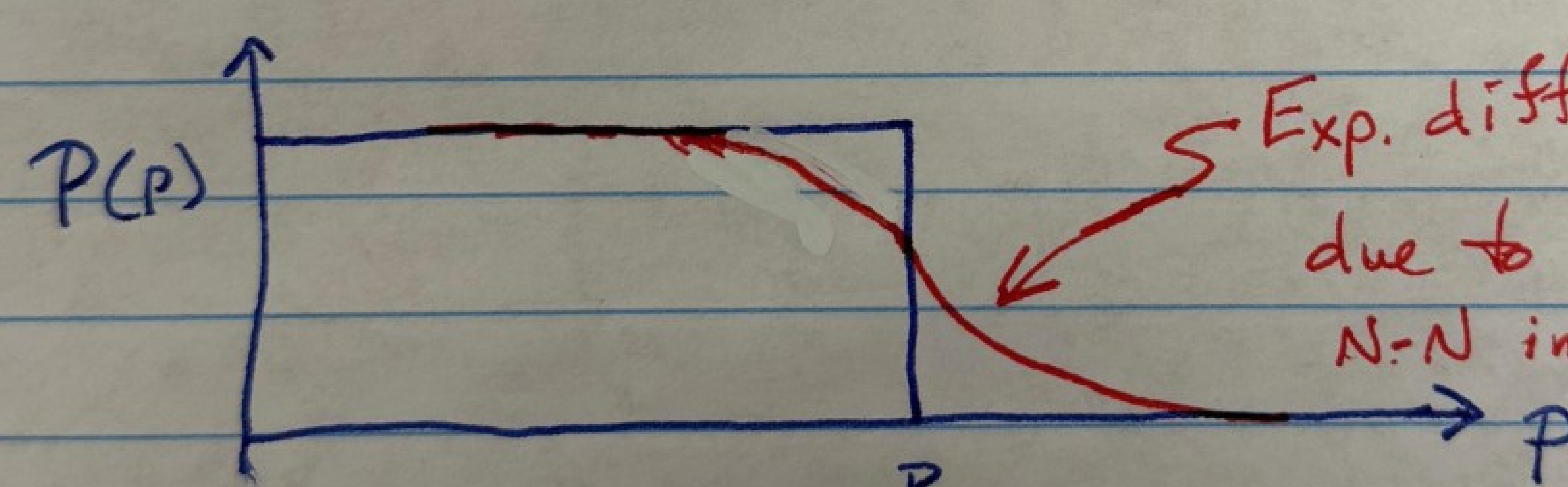
$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \rho_N\right)^{2/3} \simeq 30\,\text{MeV}$$

$$\therefore P_F = \sqrt{2 M_N E_F} \simeq 250\,\text{MeV}/c$$

also

$$E_{tot} = \frac{3}{5} A E_F \quad \& \quad E^n_{tot} = \frac{3}{5} N E^n_F$$
$$E^p_{tot} = \frac{3}{5} Z E^p_F$$

∴ Free Fermi Gas Model Predicts:



Exp. differs by ~15% due to short-range N-N interactions (aka correlations) due to hard core V(r)

Can use all of above to derive:

Semi-Empirical Mass Formula

Collect terms to calc. $E_B^{A(N,Z)}$ ∴

$$M_A(N,Z) = Zm_p + Nm_n - E_B \quad w/$$

$$E_B = a_1 A - a_2 A^{2/3} - a_3\left[\frac{Z(Z-1)}{A^{1/3}}\right] - a_4\left[\frac{(A-2Z)^2}{A}\right] + \frac{\delta}{A^{1/2}}$$

Each nucleon feels same ave. binding energy due to Nuclear Well

Correction for nucleons near surface (Area $\propto R_A^2$)

Coulomb energy to bring Z protons to $R_A \sim A^{1/3}$

Difference in Fermi Energy if $N \neq Z$ (see Bertulani)

"Pairing" Interaction with $\delta = +a_5$ if N, Z = even-even

$\delta = -a_5$ if N, Z = odd-odd