Ph 203 L8 ①

Nucleon Models (constituent quarks, dynamical quarks, ...)

Nucleon Properties:

① Static: e.g. Mass, μ_N, J_N^π, t, t_3, decay, ...

② Semi-Dynamic ($\lambda \sim 1$–10 fm)

→ Electric & Magnetic Polarizability

→ Charge radii

→ Charge distrib.

③ Dynamic ($\lambda \lesssim 1$ fm)

→ Elastic Form Factors (G_E, G_M)

→ Deep Inelastic Structure Functions (W_1, W_2, g_1, ...)

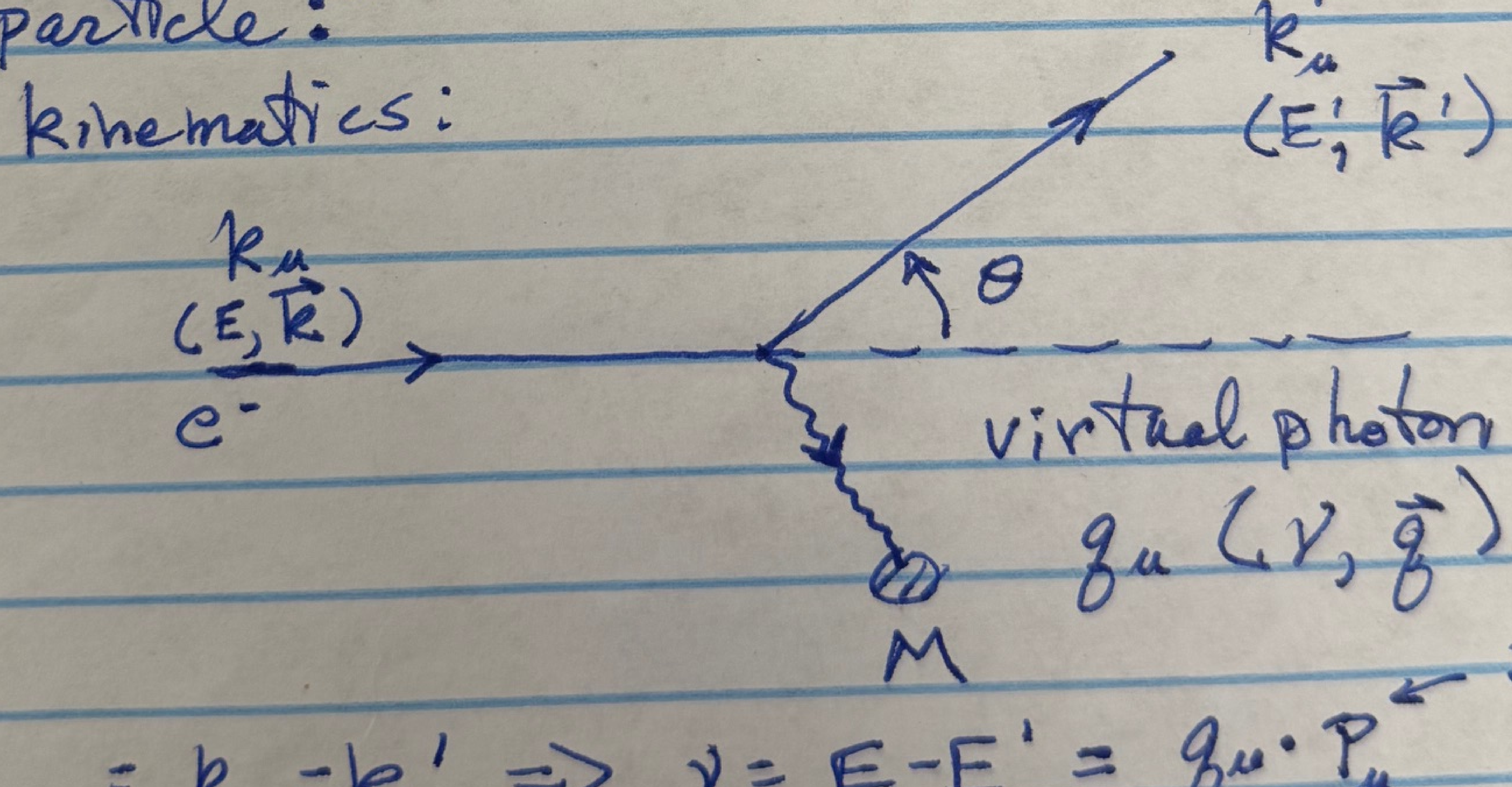
→ Transition Moments (e.g. $N \to \Delta$ = quark spin-flip)

⟺ ② & ③ access nucleon internal structure primarily via high energy ($E > 1$ GeV, $\lambda_{deB} \lesssim 1$ fm) e^-, μ scattering

Elastic, unpolarized EM scattering ($E < M_{Z^0} = 91.2$ GeV)

↳ e^- scattering from structureless, spinless ($M >> m_e$) particle:

"Lab" kinematics:



proton $(M, 0)$

$$q_\mu = k_\mu - k'_\mu \Rightarrow \nu = E - E' = \frac{q_\mu \cdot P_\mu}{M} \quad \text{invariant}$$

$$\text{w } E' = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)} \qquad E' \to E \text{ for } E >> M$$

↳ due to proton recoil

another invariant : $q_\mu \cdot q_\mu = q^2 \equiv -Q^2$
$= 2m_e^2 - 2EE' + 2\vec{k}\cdot\vec{k}'$

then assuming $E \gg m_e \Rightarrow |\vec{k}| \simeq E,\ |\vec{k}'| \simeq E'$

& $Q^2 = 4EE'\sin^2\left(\frac{\theta}{2}\right)$

<u>Target</u>

structureless
spinless

then cross section is :

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{4(\alpha Z)^2 E'^2 \cos^2\left(\frac{\theta}{2}\right)}{Q^4}\left(\frac{E'}{E}\right)}_{\text{Mott Cross Section} \equiv \sigma_{Mott}}$$

<u>Note:</u>

① $1/Q^2$ is photon propagator
② $\cos^2\left(\frac{\theta}{2}\right)$ forbids $180°$ scattering $\Rightarrow$ due to helicity conservation for $E_e \gg m_e$

structureless
spin $\frac{1}{2}$
(e.g. μ target)
[not e^+, e^- target!]

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{Mott}\left(\frac{E'}{E}\right)\Big[1 + \underbrace{\frac{Q^2}{2M^2}\tan^2\frac{\theta}{2}}_{\text{allows } 180° \text{ scattering (dipole-dipole)}}\Big]$$

structured
spin $\frac{1}{2}$
(e.g. proton/neutron)

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott}\left(\frac{E'}{E}\right)\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right]$$

↳ $\tau = \frac{Q^2}{4M^2}$,

↳ $G_E(Q^2)$ & $G_M(Q^2)$ are Electric & Magnetic Form Factors w/

	p	n
$G_E(0)$	1	0
$G_M(0)$	χ_p	χ_n

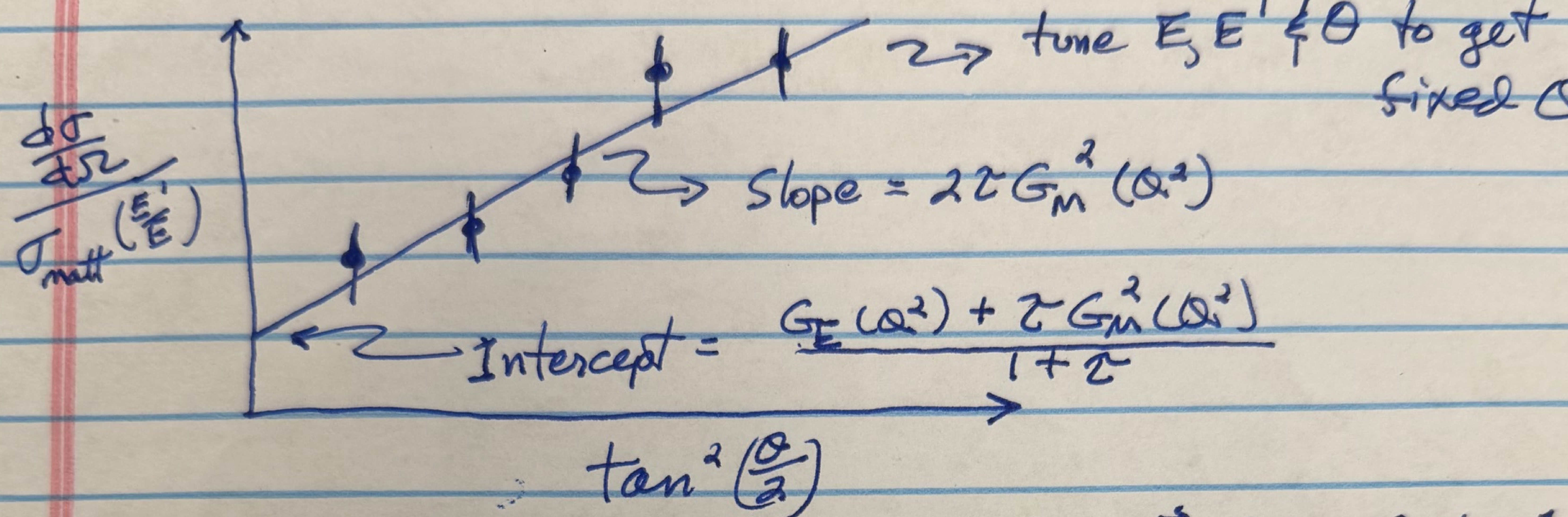
$+1.79$; -1.91

$$\kappa = \frac{\mu - \mu_{Dirac}}{\mu}$$

= anomalous moment

& $G_E(Q^2 \to 0)$ = Fourier Transform of charge density
$G_M(Q^2 \to 0)$ = " " " magnetization "

Use $\frac{d\sigma}{d\Omega}(\theta)$ @ fixed Q^2 to measure $G_E(Q^2)$ $G_M(Q^2)$

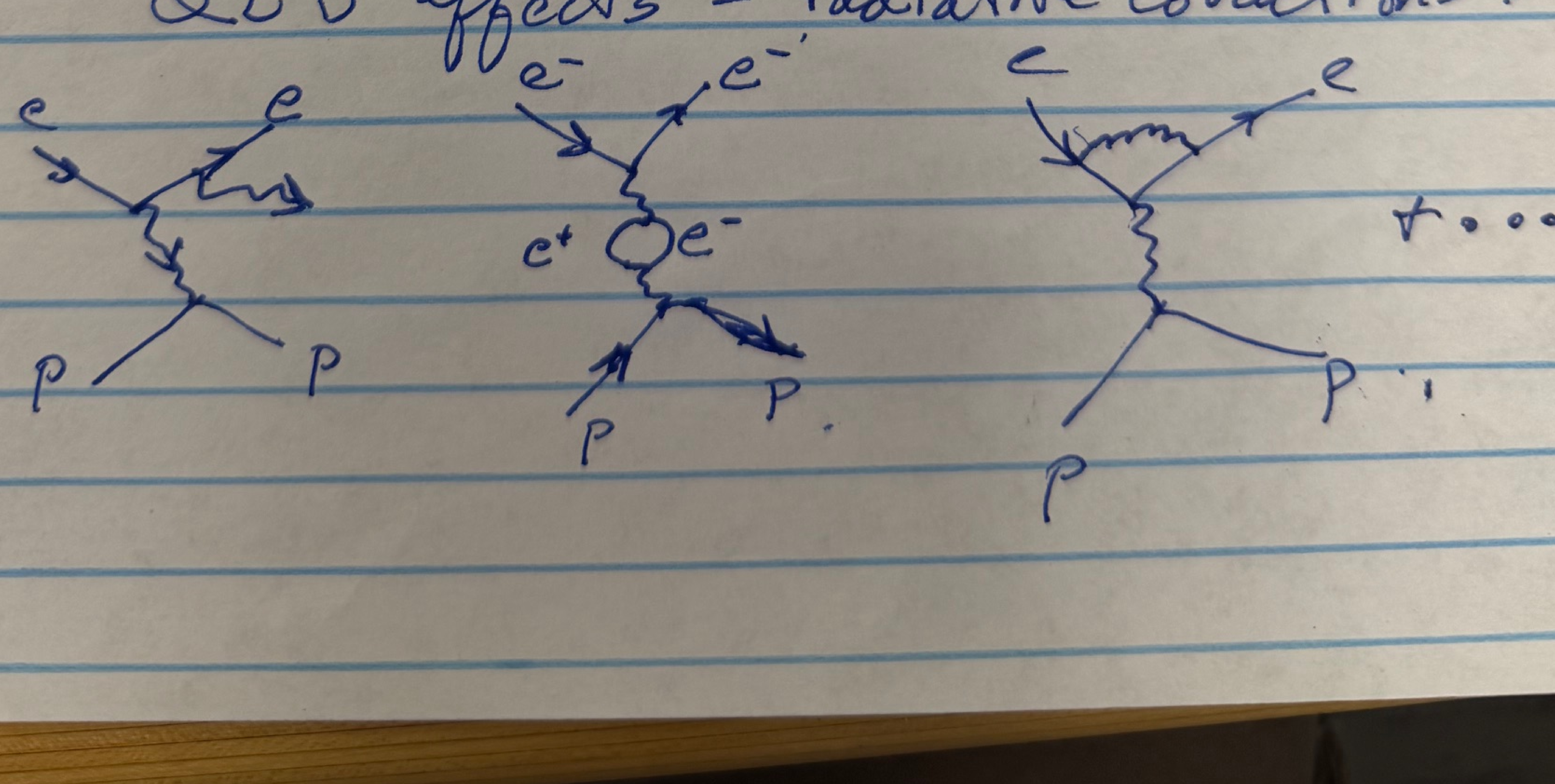


$^3\vec{He}\uparrow \simeq p\uparrow p\downarrow n\uparrow$

$e^- + p$ for proton
$e^- + d$, $\vec{e}^- + \vec{d}$, $\vec{e} + {}^3\vec{He}$ for neutron
also $n + e^-$ works @ very low Q^2

Summary of Data ⇒ see Pics

Note: data must be corrected for higher order QED effects = radiative corrections:



Recall low energy scattering to get R_{ch}.
↳ Can recast (see HW5):

$$R_{ch}^2 = -6 \lim_{Q^2 \to 0} \frac{dG_E(Q^2)}{dQ^2}$$

For $n + \overset{\text{Atomic}}{e^-}$: $(R^{rms}_{ch-n})^2 = -0.116 \pm 0.002$ fm
↳ due to $q < 0$ @ larger r
← charge

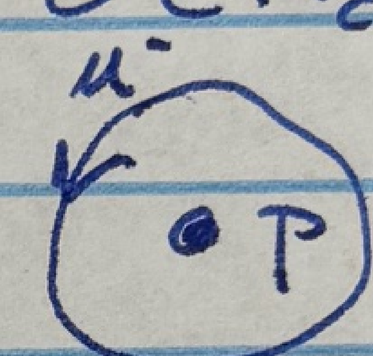
For $e^- + p$ scattering (pre-2017)

$$\underline{R^{rms}_{ch-p} = 0.877 \pm 0.007 \text{ fm}}$$

Note: H atom spectroscopy generally agreed w/ R_{ch-p} from $e^- + p$

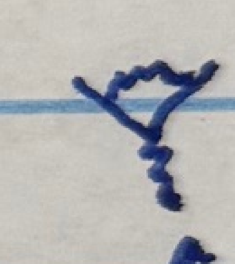
$\Leftrightarrow \Delta E = E_0 + O(R_{ch}^2)$

New muonic atom exp. for PSI lab in Swiss.



in '98 proposed

measure $2S_{1/2} \to 2P_{3/2}$ $\underbrace{\text{Lamb shift}}_{\text{Rad corr again}}$



here $\Delta E = E_0 + O(R_{ch}^2)$
↖ much bigger for $p + \mu^-$ vs $p + e^-$

2010
$\Leftrightarrow$ $\Leftrightarrow$ $\underline{R^{rms}_{ch-p} = 0.841 \pm 0.001 \text{ fm}}$

5σ !

More recently PRAD exp. @ Jefferson Lab ...
See pics

We are now in a position to calculate the contribution of each multipole order to the transition probability given in Eq. (5-13). On inserting the density of final states and using the multipole operator given in Eq. (5-25) for H', we can express the transition probability for multipole λ from an initial nuclear state $|J_iM_i\zeta\rangle$ to a final nuclear state $|J_fM_f\xi\rangle$ as

$$\mathcal{W}(\lambda; J_i\zeta \rightarrow J_f\xi) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{k^{2\lambda+1}}{\hbar}B(\lambda; J_i\zeta \rightarrow J_f\xi) \tag{5-27}$$

where the reduced transition probability $B(\lambda; J_i\zeta \rightarrow J_f\xi)$ may be written in terms of the reduced matrix element of the multipole operator for either electric or magnetic transition, in the same way as we did in Eq. (5-7),

$$B(\lambda; J_i\zeta \rightarrow J_f\xi) = \sum_{\mu M_f}|\langle J_fM_f\xi|\boldsymbol{O}_{\lambda\mu}|J_iM_i\zeta\rangle|^2 = \frac{1}{2J_i+1}|\langle J_f\xi\|\boldsymbol{O}_{\lambda}\|J_i\zeta\rangle|^2 \tag{5-28}$$

It is worth noting here that the reduced transition probabilities are quantities with dimensions. For electric transitions, the units are $e^2\text{fm}^{2\lambda}$, and for magnetic transitions, $\mu_N^2\text{fm}^{2\lambda-2}$. The transition rate $\mathcal{W}$ is the number of decays per unit time. In relating the numerical values of $\mathcal{W}$ and reduced transition probability, one must be careful with the factors e^2 in electric transitions and μ_N^2 for magnetic transitions. For example, the values in Table 5-1 are obtained using the following relations:

$$\mathcal{W}(\lambda) = \begin{cases} \alpha\hbar c\dfrac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\dfrac{1}{\hbar}\Big(\dfrac{1}{\hbar c}\Big)^{2\lambda+1}E_\gamma^{2\lambda+1}\,B(E\lambda \text{ in } e^2\text{fm}^{2\lambda}) \\ \alpha\hbar c\Big(\dfrac{\hbar c}{2M_pc^2}\Big)^2\dfrac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\dfrac{1}{\hbar}\Big(\dfrac{1}{\hbar c}\Big)^{2\lambda+1}E_\gamma^{2\lambda+1}\,B(M\lambda \text{ in } \mu_N^2\text{fm}^{2\lambda-2}) \end{cases}$$

where we have used short-hand notation $B(E\lambda)$ for reduced electric transition probability and $B(M\lambda)$ for reduced magnetic transition probability. The numerical values for e^2 and μ_N^2 may be obtained using the relation $e^2 = \alpha\hbar c$ in cgs units.

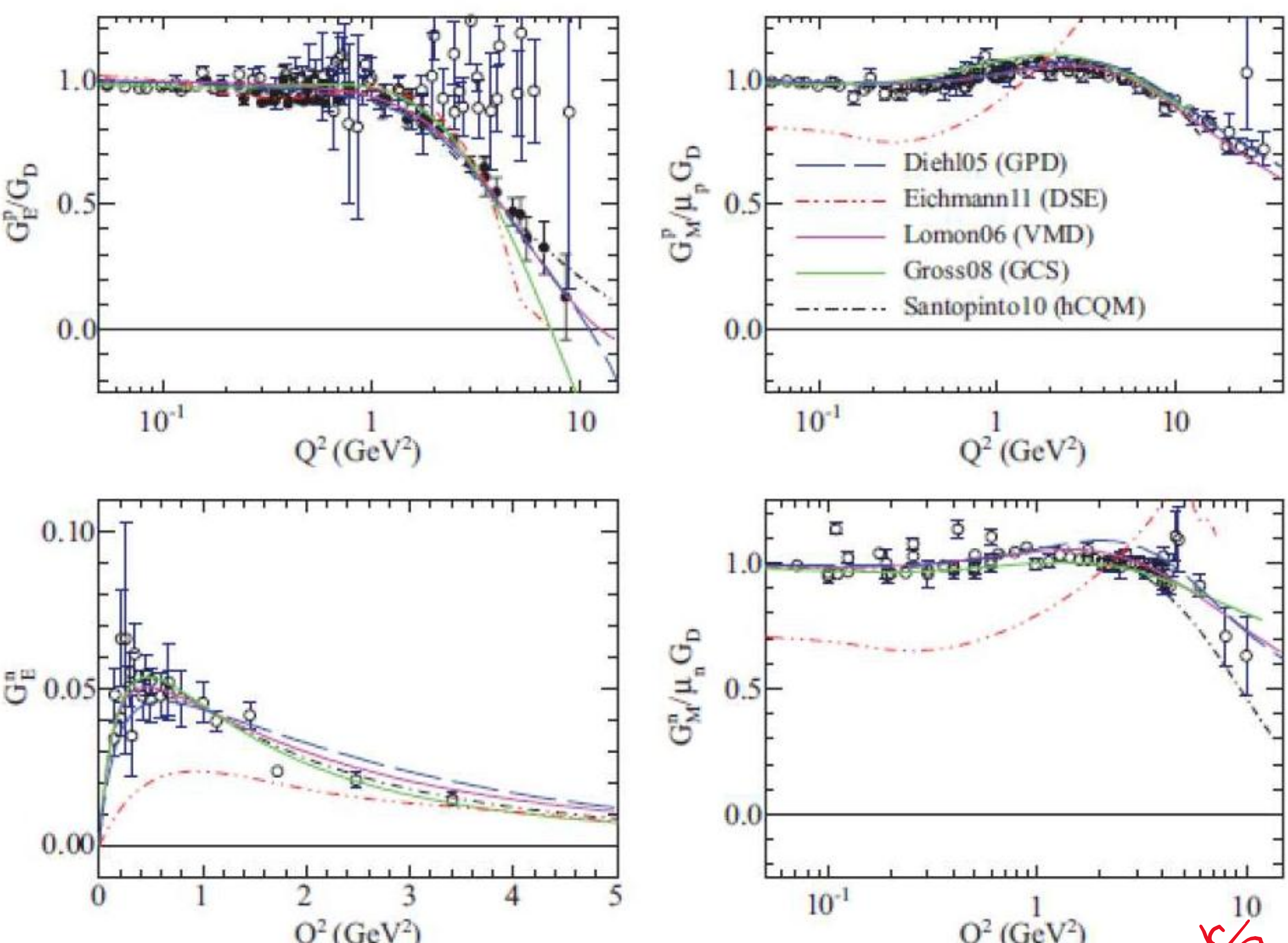
Table 5-1: Electromagnetic transition probabilities for the lowest four multipoles.

$\mathcal{W}(E1)=1.59\times10^{15}\ E_\gamma^3\times B(E1)$	$\mathcal{W}(M1)=1.76\times10^{13}\ E_\gamma^3\times B(M1)$
$\mathcal{W}(E2)=1.23\times10^{9}\ E_\gamma^5\times B(E2)$	$\mathcal{W}(M2)=1.35\times10^{7}\ E_\gamma^5\times B(M2)$
$\mathcal{W}(E3)=5.71\times10^{2}\ E_\gamma^7\times B(E3)$	$\mathcal{W}(M3)=6.31\times10^{0}\ E_\gamma^7\times B(M3)$
$\mathcal{W}(E4)=1.70\times10^{-4}E_\gamma^9\times B(E4)$	$\mathcal{W}(M4)=1.88\times10^{-6}E_\gamma^9\times B(M4)$

E_γ are in MeV, $B(E\lambda)$ in $e^2\text{fm}^{2\lambda}$, and $B(M\lambda)$ in $\mu_N^2\text{fm}^{(2\lambda-2)}$.

If we take that the electric charge in a nucleus consists of point charges carried by individual protons and the magnetization currents are due to the magnetic dipole moments of individual nucleons and the orbital motion of protons, the electric and

Proton and Neutron Form Factors [scaled by $G_D(Q^2)$]



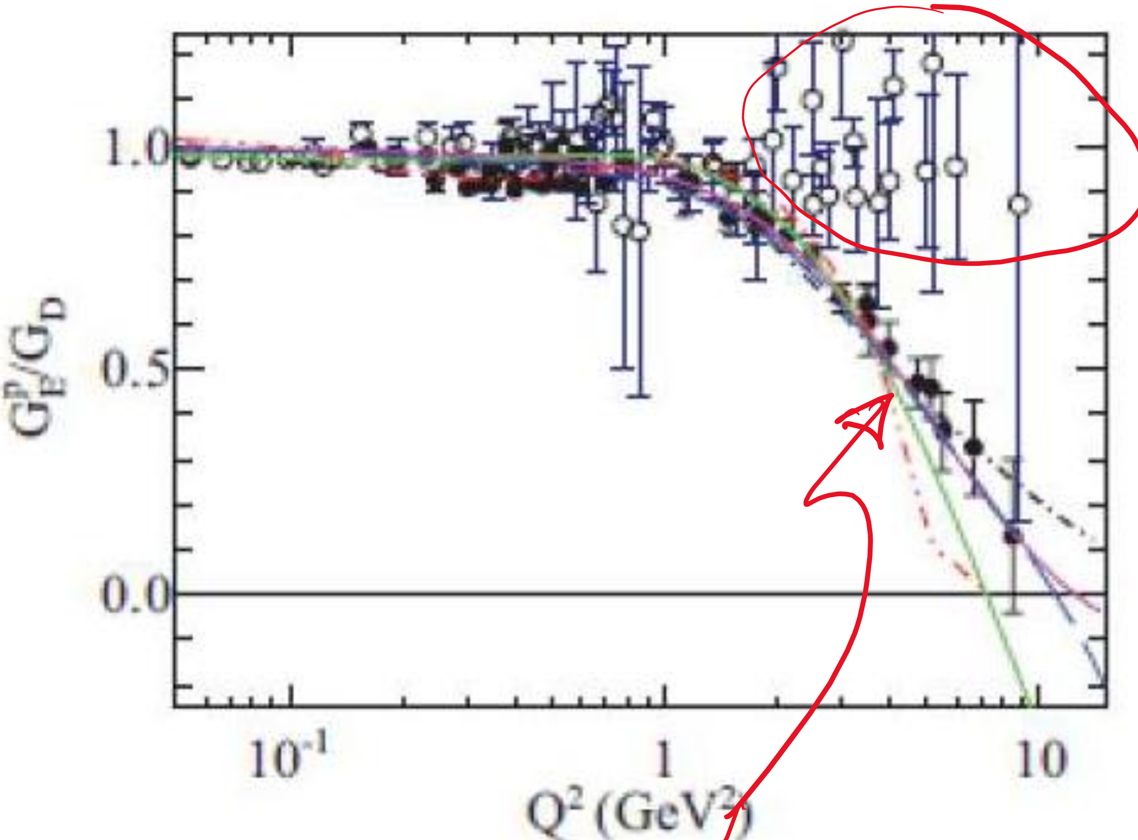
$$G_D(Q^2) = \frac{1}{\left(1+\frac{Q^2}{Q_0^2}\right)^2} \Longrightarrow \simeq \text{F.T. of } e^{-r/R_0}$$

2

$$\hookrightarrow Q_0^2 \simeq 0.71\ \text{GeV}^2$$

Interesting $G^P_E(Q^2)$ Puzzle

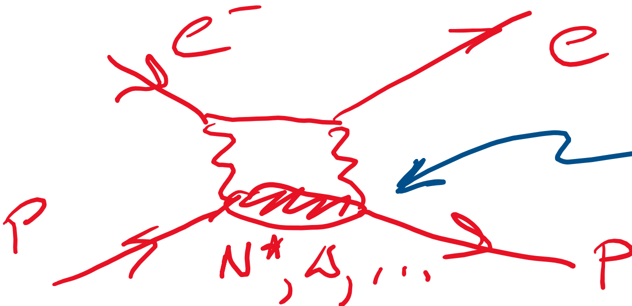
$e^- + p$



$\vec{e}^- + p \to e' + \vec{p}$

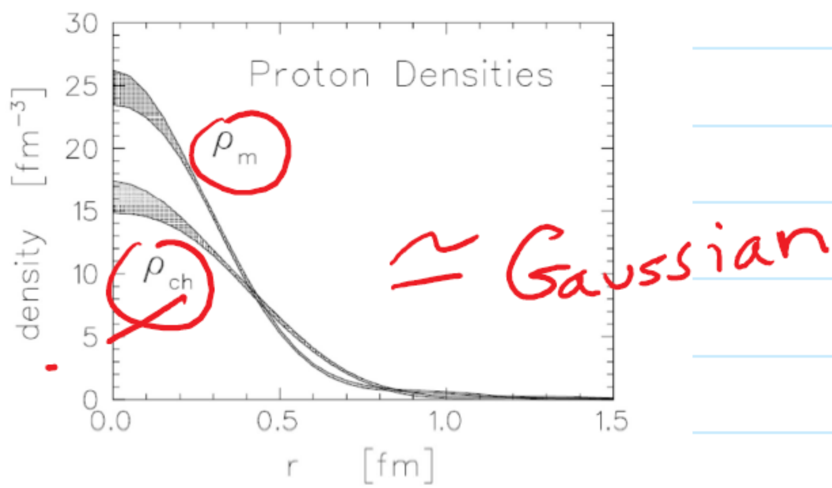
Which exp. is correct?

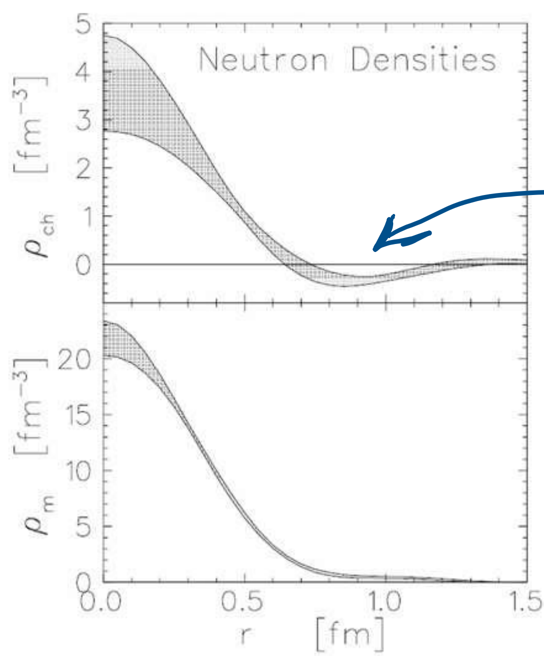
Rad. Corr. again!



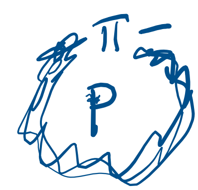
very hard to calc.
assumed small
but not for Hi Q $e^- + p$

Proton and Neutron Charge and Magnetization Densities





Pion cloud

n ≃ 

or

n ≃ 

Charge (ρ_{ch}) and magnetization (ρ_m) densities for the neutr
ation with $\lambda_E = \lambda_M = 2$.

CREMA = Charge Radius Exp using Muonic Atoms

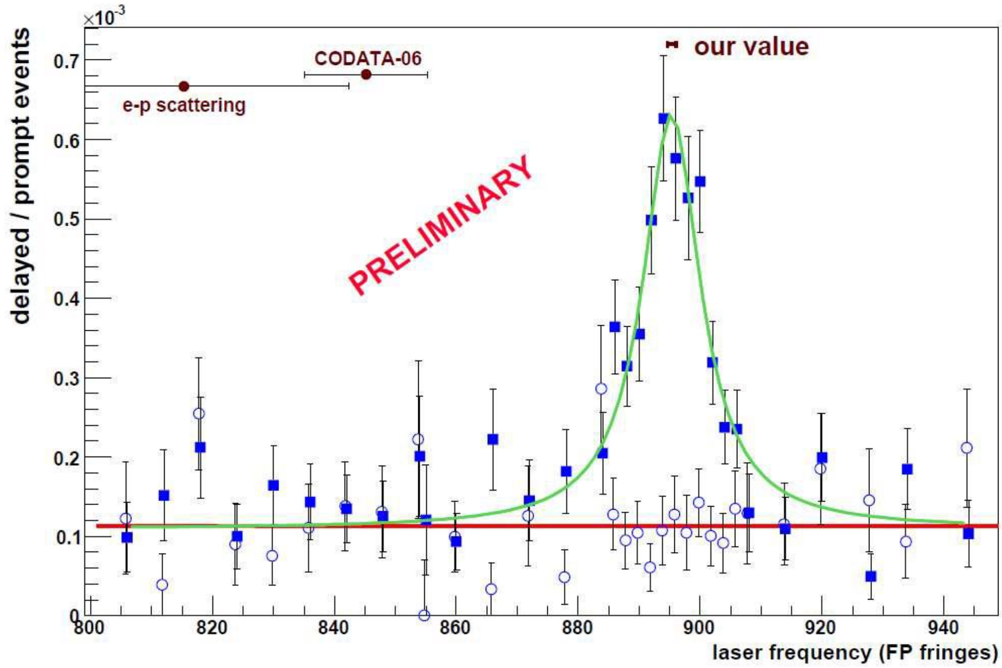
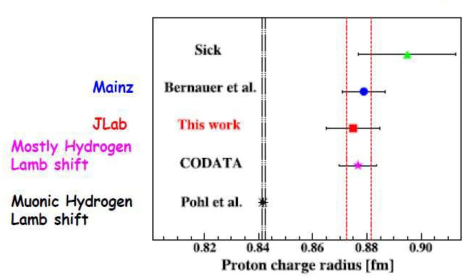


Figure 8: The first resonance in μp. Solid squares indicate data taken with laser, open circles are laser-OFF data taken at the same time (using muons arriving during the dead-time of the laser). The red horizontal line indicates the background determined from the laser-ON time spectra fitting the regions just before and after the laser peak (see Fig. 2). The green solid function is a fit of a simple Lorenztian to the laser data. Predictions of the resonance position (e-p scattering [6] and CODATA [5]) are given. Our precision is indicated, too. One FP fringe corresponds to 1.5 GHz.

Proton Charge Radius Puzzle



The figure is from X. Zhan et al., PLB 705, 59 (2011)

Connection to radius of the proton:

$$F(q^2) = 1 - \frac{1}{6}\frac{q^2 <r^2>}{\hbar^2} +$$

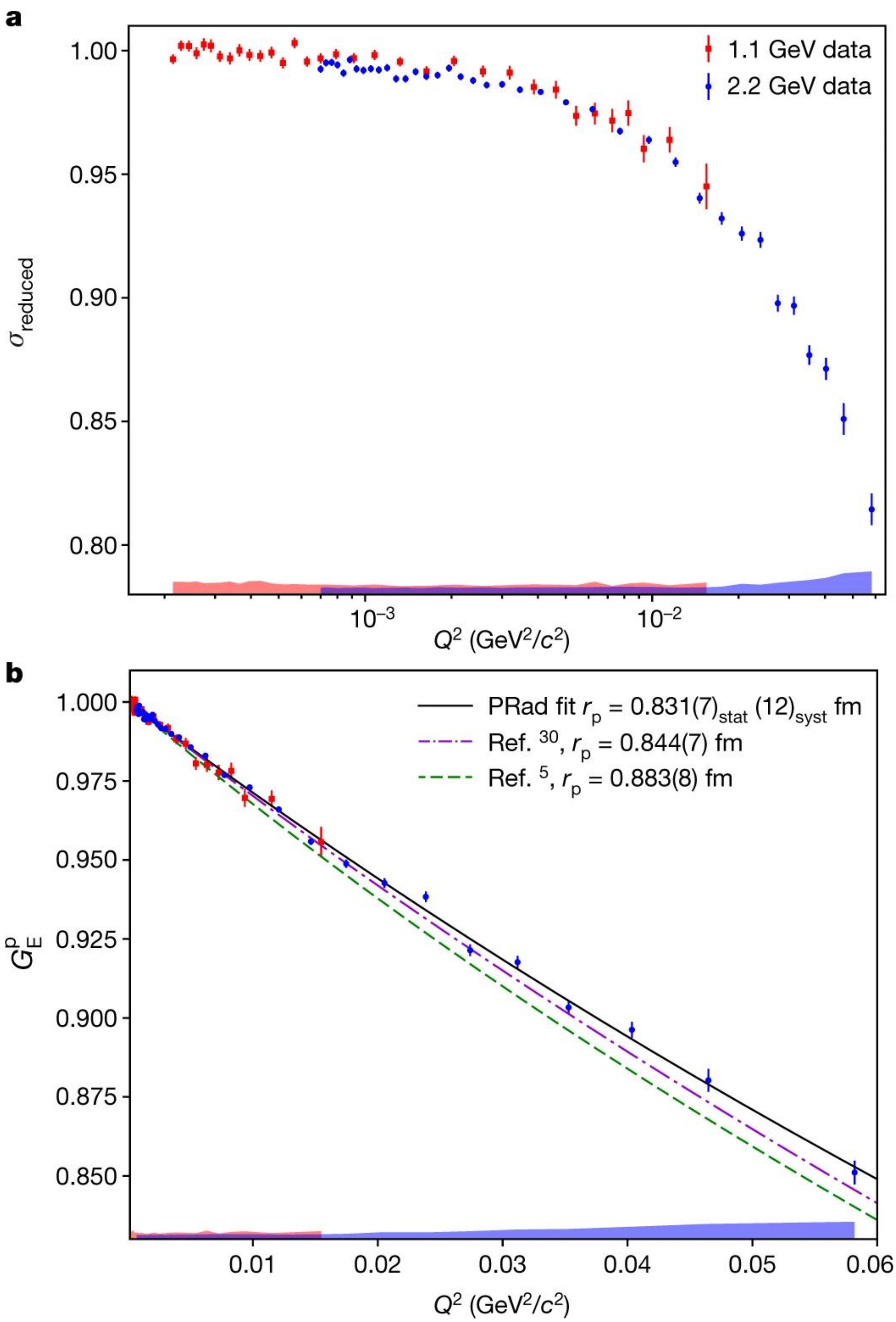
$$<r^2> = -6\hbar^2 \frac{dF(q^2)}{dq^2}|_{q^2}$$

The New York Times, July 13, 2010.

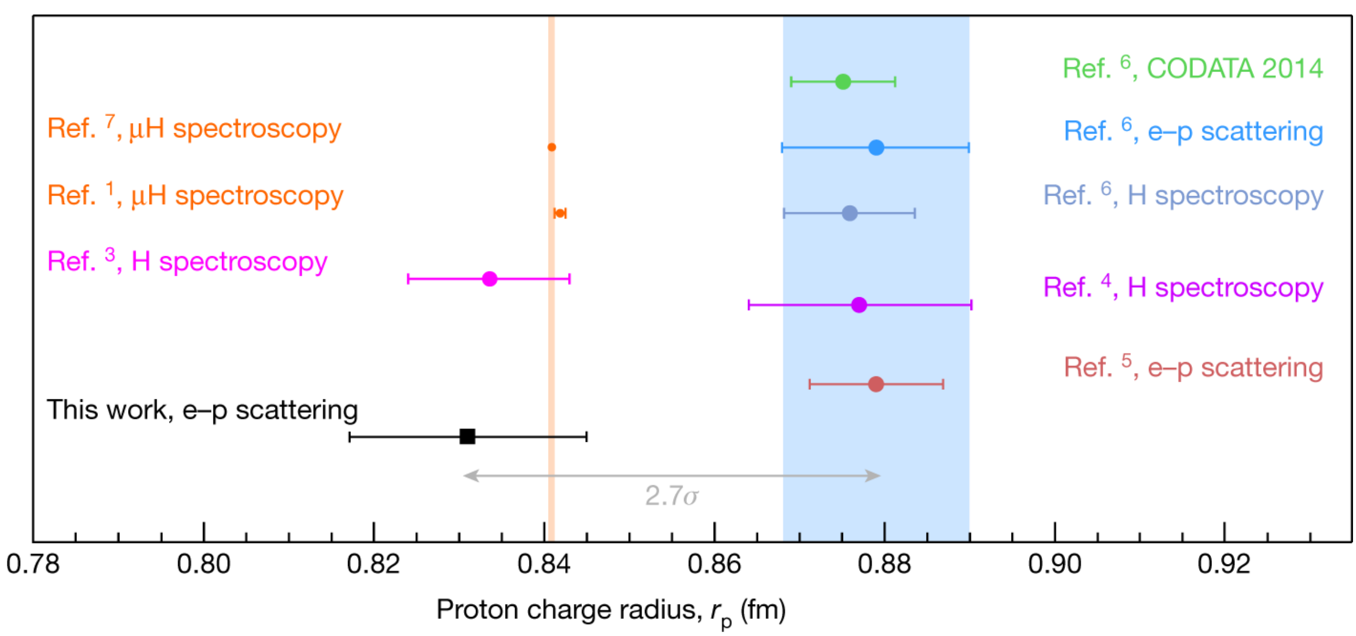
It went from 0.8768±0.0069 fm to 0.8418±0.0007 fm

"For a Proton, a Little Off the Top (or Side) Could Be Big Trouble"

PRAD (Proton Radius) Exp

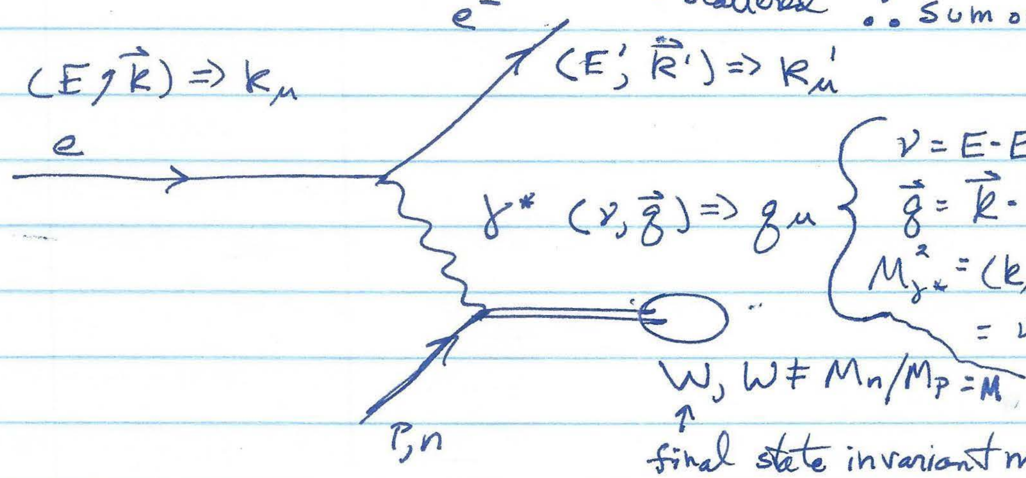


PRAD = Proton RADius



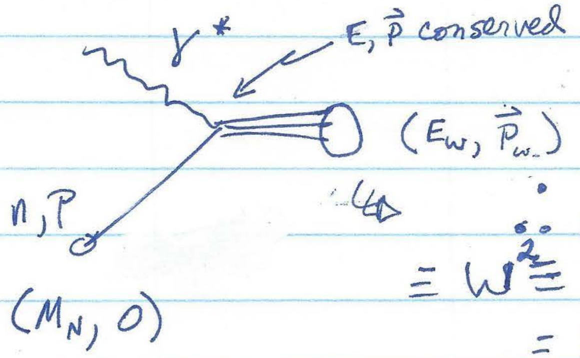
To probe quark structure of nucleon, consider unpolarized $e^- - N$ "inclusive" inelastic scattering

↳ only scattered e^- is detected ∴ sum over final states



$$\begin{cases} \nu = E - E' \\ \vec{q} = \vec{k} - \vec{k}' \\ M_{\gamma^*}^2 = (k_\mu - k'_\mu)^2 \equiv -Q^2 \\ \quad = \nu^2 - \vec{q}^{\,2} \end{cases}$$

Can characterize the inelasticity via final state invariant mass $= W^2$



∴ invariant mass of final state is

$$\equiv W^2 \equiv E_W^2 - \vec{P}_W^{\,2} = (M + \nu)^2 - \vec{q}^{\,2}$$

$$= M^2 + \nu^2 + 2M\nu - \vec{q}^{\,2}$$

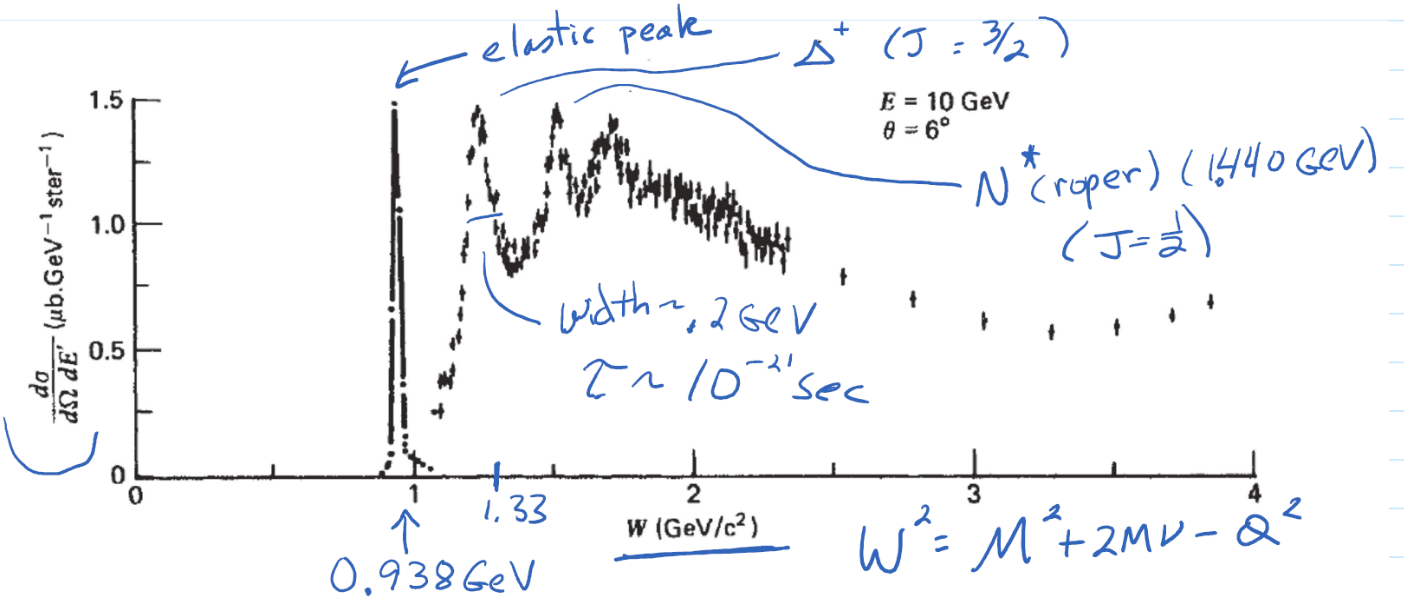
$$= M^2 + 2M\nu - Q^2$$

then $W^2 = M^2$ if $Q^2 = 2M\nu$

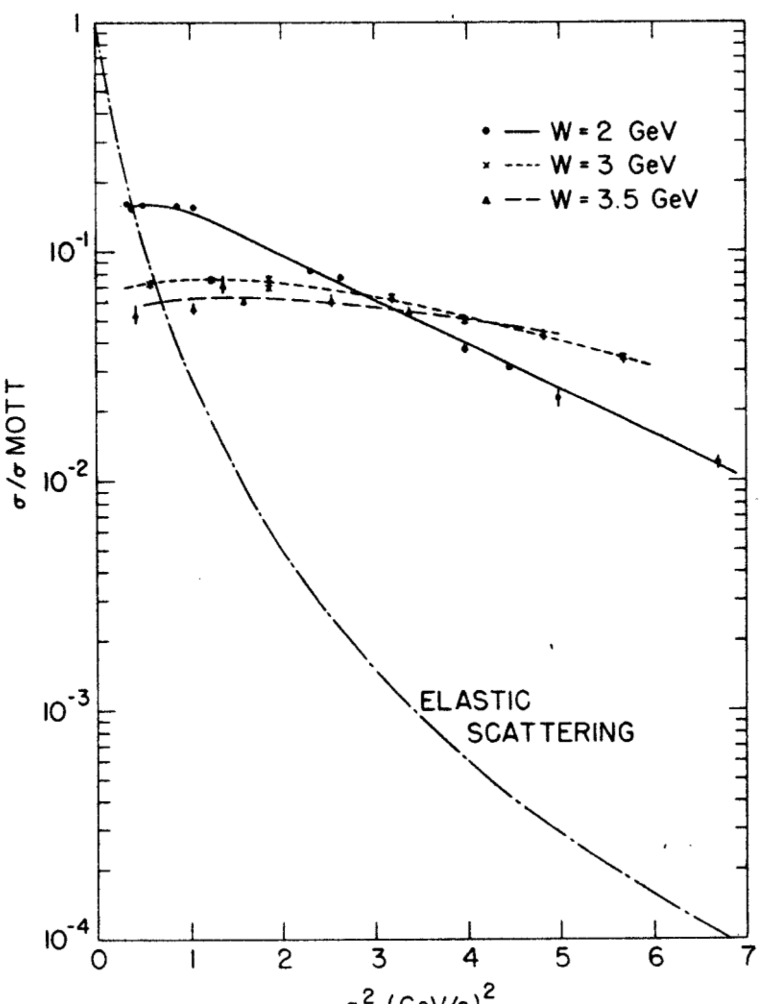
&

$W^2 > M^2$ if $Q^2 < 2M\nu$

e⁻ + proton Inelastic Scattering



"Just a bunch of messy resonances"



Or is it?

"For larger invariant mass, it looks like scattering from point-like spin ½ particle"