

Physics 203

Homework 4

- 1.) Determine the non-relativistic elastic scattering form factor

$$F(|\mathbf{q}|) = \int d^3r \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

for the following two densities:

- a) $\rho(r) = \rho_0 = \text{constant}$ for $r \leq R$ and $\rho = 0$ for $r > R$,
 b) $\rho(r) = \rho_0 e^{-r/R}$

- 2.) Beginning with the non-relativistic elastic scattering form factor

$$F(|\mathbf{q}|) = \int d^3r \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

show that the mean square radius of the charge distribution is related to the derivative of the form factor, i.e.:

$$\langle r^2 \rangle = \lim_{q \rightarrow 0} 6 \left| \frac{dF(|\mathbf{q}|)}{d|\mathbf{q}|^2} \right|$$

- 3.) The semi-empirical mass formula (with coefficients in MeV) is

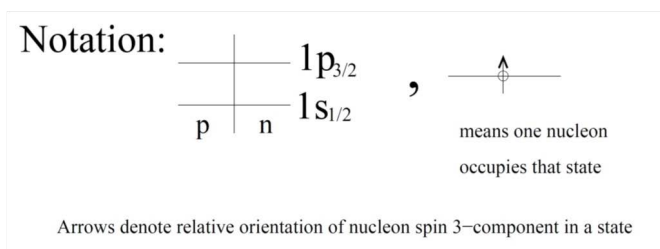
$$E_b(\text{MeV}) = 16A - 18A^{2/3} - .71 \frac{Z(Z-1)}{A^{1/3}} - 23 \frac{(A-2Z)^2}{A} + \frac{\Delta}{A^{1/2}}$$

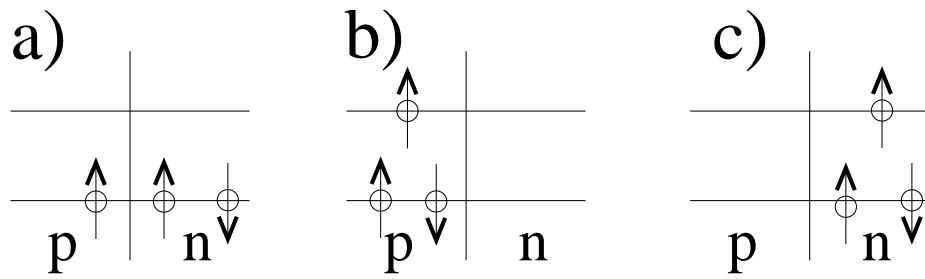
with $\Delta = +11$ MeV for even-even nuclei, $\Delta = 0$ MeV for odd-even nuclei and $\Delta = -11$ MeV for odd-odd nuclei. Use this formula to calculate the binding energy per nucleon vs. A for stable nuclei for $A = 1 - 200$. Sketch the resulting curve and determine THE most stable nucleus. [Hint: use the above formula to find stable nuclei by identifying for a given A what value of Z gives the largest binding energy].

- 4.) Use the Fermi Gas Model to determine the fourth term in the semi-empirical mass formula (Prob. 3). You should be able to determine both the dependence on A and Z as well as the coefficient.

- 5.) Use the semi-empirical mass formula (in Prob. 3) to investigate the stability of ^{235}U against emission of (a) a proton, (b) a neutron, (c) an alpha particle (this is a ^4He nucleus; Note - don't use the semi-empirical formula to calculate the mass of the alpha!). For any of these cases where the decay is possible calculate the kinetic energy of the emitted particle.

- 6.) What are the nuclei, spins, parities, isospins, and "3"-component of isospin for the diagrams [a) - h)], shown on the next page, representing states of the three nucleon system. Note that these diagrams are shorthand for the full states but are sufficient to identify the nuclei. Use the shell model as a guide, and assume the state of lowest excitation energy in each case. If some of the total spins are not unique, list the range of possible values. Group isospin multiplets together and indicate any that are missing (Hint: Find the state of maximum "3"-component of isospin first, and then identify other members of the multiplet). The notation of the diagrams is:





f)
$$\left[\sqrt{\frac{1}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \downarrow \\ \hline | \uparrow \downarrow \\ \hline | \uparrow \\ \hline \end{array} + \sqrt{\frac{2}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \downarrow \\ \hline | \downarrow \\ \hline | \uparrow \\ \hline \end{array} \right]$$

g)
$$\left[\sqrt{\frac{2}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \downarrow \\ \hline | \downarrow \\ \hline | \uparrow \\ \hline \end{array} + \sqrt{\frac{1}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \\ \hline | \downarrow \uparrow \\ \hline | \uparrow \\ \hline \end{array} \right]$$

h)
$$\left[\sqrt{\frac{2}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \downarrow \\ \hline | \downarrow \\ \hline | \uparrow \\ \hline \end{array} - \sqrt{\frac{1}{3}} \begin{array}{c} | \uparrow \\ \hline | \uparrow \\ \hline | \downarrow \\ \hline | \uparrow \\ \hline \end{array} \right]$$