Nuclear Physics: Spin, Strange Quarks, and Parity-Violation

M.J. Ramsey-Musolf

- *UMass Amherst*
- *T.D. Lee Institute/Shanghai Jiao Tong Univ.*
- *Caltech*

- *mjrm@umass.edu*
- *mjrm@sjtu.edu.cn*
- *微信 : mjrm-china*
- *https://michaelramseymusolf.com/*

About MJRM:

Science

Family

Friends

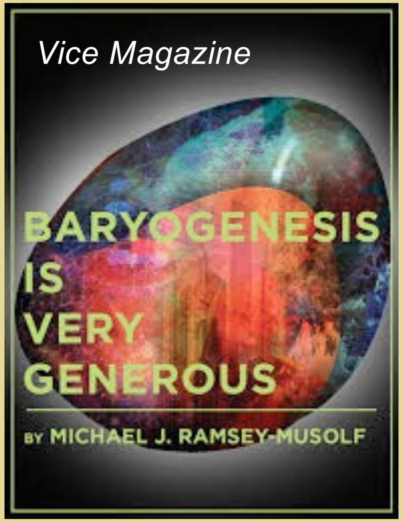
My pronouns: he/him/his
 # MeToo

Caltech Nuclear Physics
April 28, 2026

Michael Ramsey-Musolf

T.D. Lee Chair Professor, TDLI & Chair Professor, SJTU

Theoretical Physics



Vice Magazine

BARYOGENESIS IS VERY GENEROUS

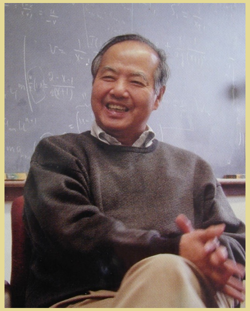
BY MICHAEL J. RAMSEY-MUSOLF

- *Why does the Universe contain more matter than antimatter ?*
- *What are the laws of nature beyond those of the Standard Model & General Relativity ?*
- *How do quantum field theories work ? How do they apply to processes in the early Universe ?*
- *How can experiments test our theoretical ideas?*

- *Ph.D. Princeton*
- *Post-doc MIT*
- *美国 → 中国 2019*

- *mjrm@sjtu.edu.cn*
- *微信：mjrm-china*

T. D. Lee Institute / Shanghai Jiao Tong U.



A point of convergence of the world's top scientists

A launch pad for the early-career scientists

Director

A world famous source of original innovation

Founded 2016

100+

faculty members from 17 countries and regions, with over 40% of them foreign (non-Chinese) citizens

Theory & Experiment

Particle & Nuclear Physics	Astronomy & Astrophysics	Quantum Science
Dark Matter & Neutrino	Laboratory Astrophysics	Topological Quantum Computation

https://tdli.sjtu.edu.cn/EN/

Key Ideas for This Talk

- *Deep inelastic scattering & partons*
    - *Valence vs. sea quarks & gluons*
- *Views of the nucleon: partons vs. constituent quarks*
- *The "Spin Crisis"*
    - *Ellis-Jaffe sum rule & strange quarks*
- *Weak neutral currents*
- *PV Electron scattering & nucleon structure*
    - *The Caltech history*
- *The future: PVES & BSM physics*

Outline

I. Spin Crisis p1

II. Deep inelastic leptoproduction

III. Spin Crisis p2

IV. Weak neutral currents

V. PV Electron scattering & strange quarks

VI. PVES & BSM Physics

I. The Spin Crisis p1

The Spin Crisis

Where does the Nucleon Spin come from?

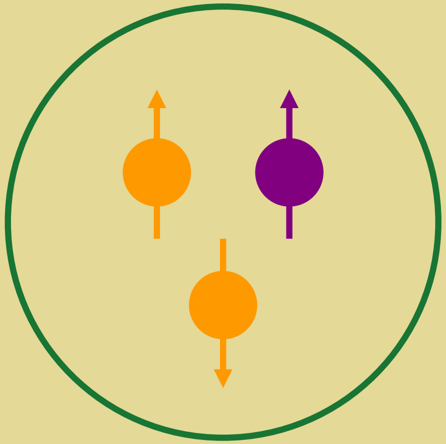
$$\frac{1}{2} = \frac{1}{2}\,\Delta\Sigma + \Delta G + L_q + L_G$$

quark spin

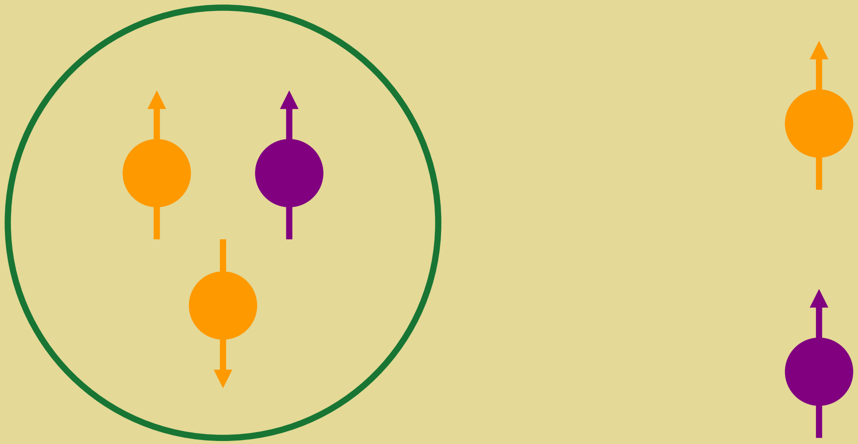
gluon spin

orbital motion

VS



The Constituent Quark Model gives a successful description



Proton

Up quark

Down quark

$Q^P = 2\ Q^U + Q^D$

$\kappa^P = Q^U\, \kappa^U{}_P + Q^D\, \kappa^D{}_P$

$Q^U = \ \ 2/3$

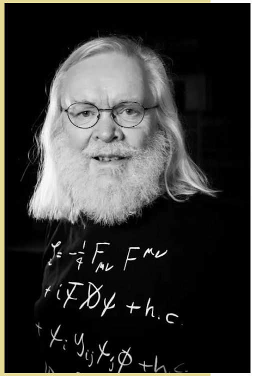
$Q^D = -1/3$

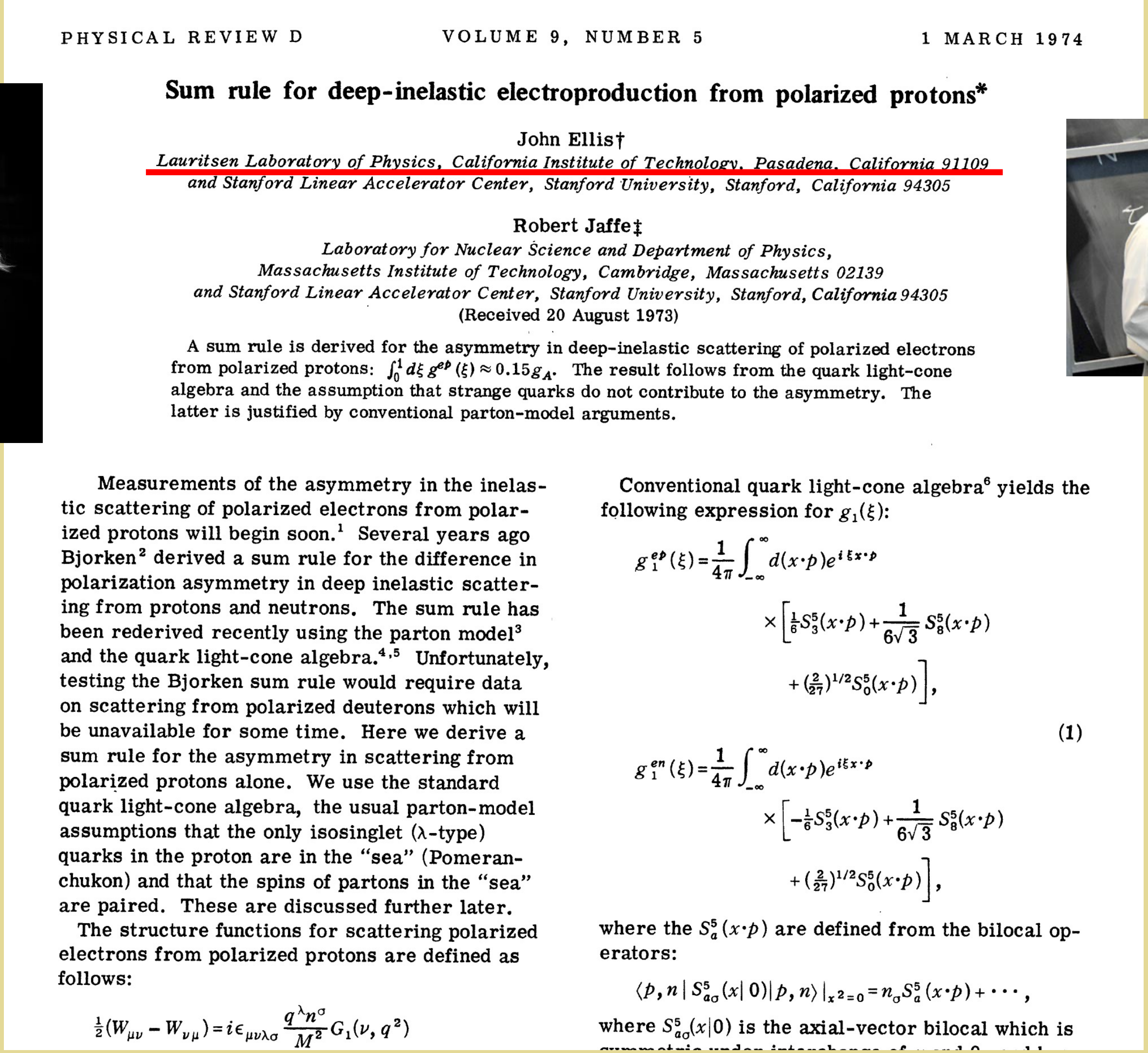
II. Deep Inelastic Leptoproduction

Blackboard: see scanned notes

III. The Spin Crisis p2

Ellis-Jaffe Sum Rule





PHYSICAL REVIEW D VOLUME 9, NUMBER 5 1 MARCH 1974

Sum rule for deep-inelastic electroproduction from polarized protons*

John Ellis†
Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

Robert Jaffe‡
Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 20 August 1973)

A sum rule is derived for the asymmetry in deep-inelastic scattering of polarized electrons from polarized protons: $\int_0^1 d\xi\, g^{ep}(\xi) \approx 0.15 g_A$. The result follows from the quark light-cone algebra and the assumption that strange quarks do not contribute to the asymmetry. The latter is justified by conventional parton-model arguments.

Measurements of the asymmetry in the inelastic scattering of polarized electrons from polarized protons will begin soon.[1] Several years ago Bjorken[2] derived a sum rule for the difference in polarization asymmetry in deep inelastic scattering from protons and neutrons. The sum rule has been rederived recently using the parton model[3] and the quark light-cone algebra.[4,5] Unfortunately, testing the Bjorken sum rule would require data on scattering from polarized deuterons which will be unavailable for some time. Here we derive a sum rule for the asymmetry in scattering from polarized protons alone. We use the standard quark light-cone algebra, the usual parton-model assumptions that the only isosinglet (λ-type) quarks in the proton are in the "sea" (Pomeranchukon) and that the spins of partons in the "sea" are paired. These are discussed further later.

The structure functions for scattering polarized electrons from polarized protons are defined as follows:

$$\tfrac{1}{2}(W_{\mu\nu} - W_{\nu\mu}) = i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda n^\sigma}{M^2} G_1(\nu, q^2)$$

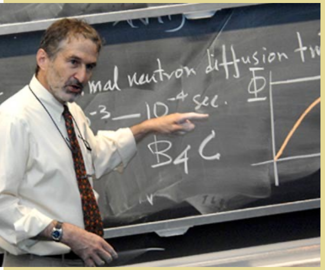
Conventional quark light-cone algebra[6] yields the following expression for $g_1(\xi)$:

$$\begin{aligned} g_1^{ep}(\xi) &= \frac{1}{4\pi} \int_{-\infty}^{\infty} d(x\cdot p) e^{i\xi x\cdot p} \\ &\quad \times \left[\tfrac{1}{6} S_3^5(x\cdot p) + \frac{1}{6\sqrt{3}} S_8^5(x\cdot p) \right. \\ &\quad \left. + \left(\tfrac{2}{27}\right)^{1/2} S_0^5(x\cdot p) \right], \\ g_1^{en}(\xi) &= \frac{1}{4\pi} \int_{-\infty}^{\infty} d(x\cdot p) e^{i\xi x\cdot p} \\ &\quad \times \left[-\tfrac{1}{6} S_3^5(x\cdot p) + \frac{1}{6\sqrt{3}} S_8^5(x\cdot p) \right. \\ &\quad \left. + \left(\tfrac{2}{27}\right)^{1/2} S_0^5(x\cdot p) \right], \end{aligned} \tag{1}$$

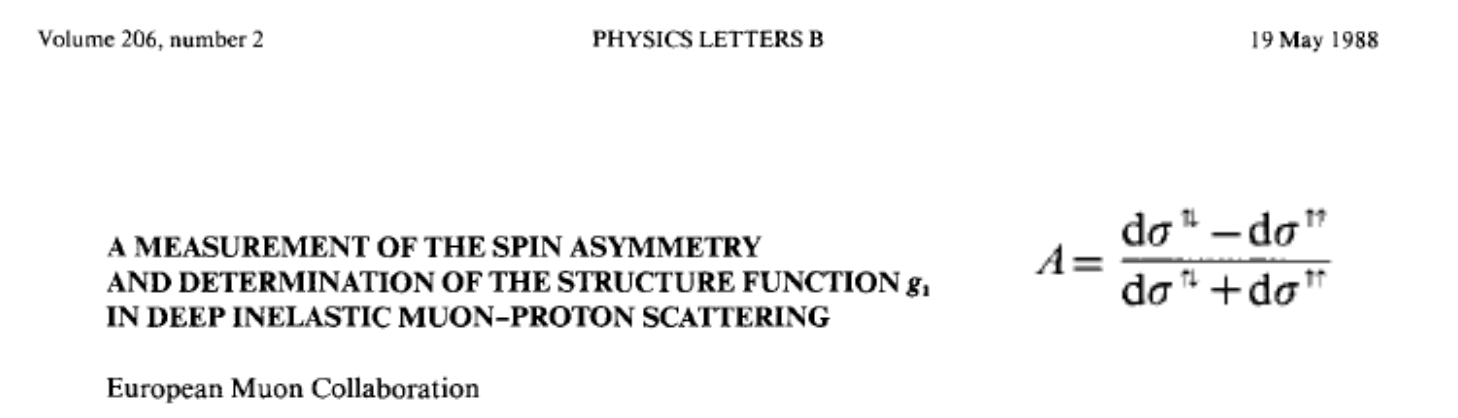
where the $S_a^5(x\cdot p)$ are defined from the bilocal operators:

$$\langle p, n \,|\, S_{a\sigma}^5(x|\,0) \,|\, p, n\rangle \,|_{x^2=0} = n_\sigma S_a^5(x\cdot p) + \cdots ,$$

where $S_{a\sigma}^5(x|0)$ is the axial-vector bilocal which is



The Spin Crisis

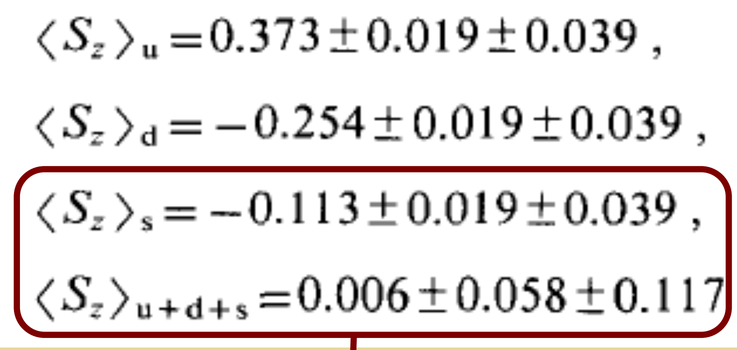


Volume 206, number 2 PHYSICS LETTERS B 19 May 1988

A MEASUREMENT OF THE SPIN ASYMMETRY AND DETERMINATION OF THE STRUCTURE FUNCTION g_1 IN DEEP INELASTIC MUON–PROTON SCATTERING

European Muon Collaboration

$$A=\frac{d\sigma^{\uparrow\downarrow}-d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow}+d\sigma^{\uparrow\uparrow}}$$



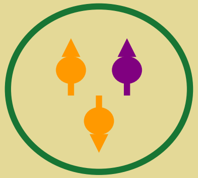
$$\langle S_z\rangle_u=0.373\pm0.019\pm0.039\,,$$

$$\langle S_z\rangle_d=-0.254\pm0.019\pm0.039\,,$$

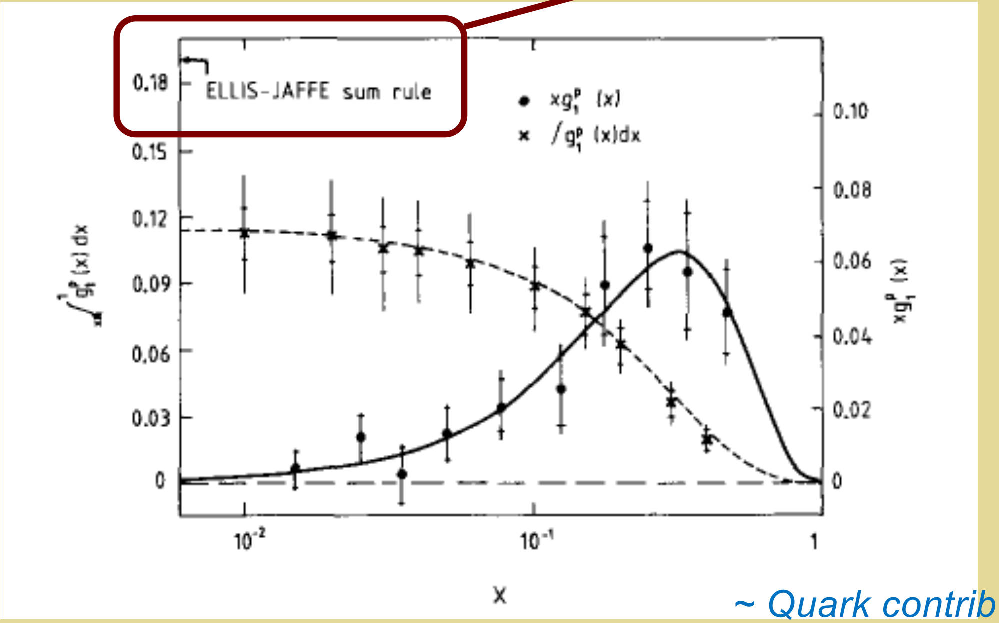
$$\boxed{\begin{aligned}\langle S_z\rangle_s&=-0.113\pm0.019\pm0.039\,,\\ \langle S_z\rangle_{u+d+s}&=0.006\pm0.058\pm0.117\end{aligned}}$$

Large negative strange sea polarization & small ΔΣ

What about s-quarks & nucleon magnetic moment and charge distribution ?



Δs = 0

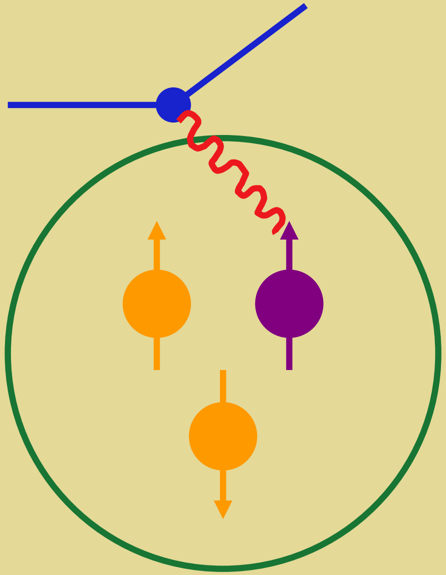


~ Quark contribution to nucleon spin

$$\boxed{g_1(x)}=\tfrac{1}{2}\sum e_i^2\boxed{\left[q_i^+(x)-q_i^-(x)\right]}\longrightarrow \Delta q$$

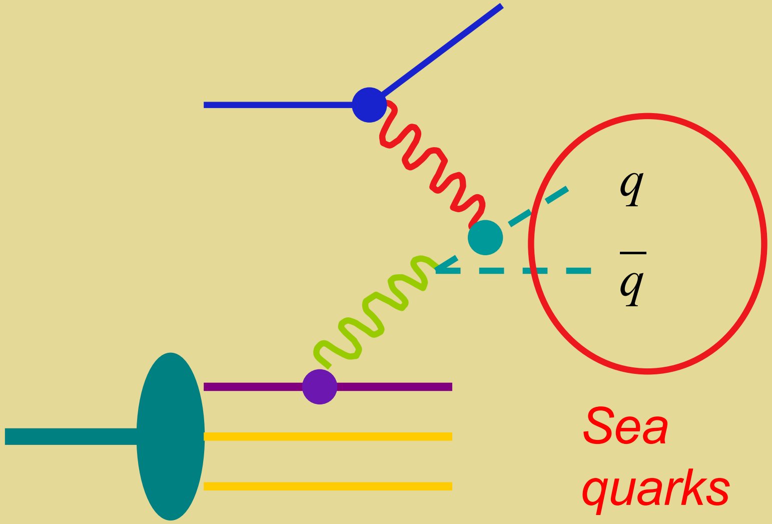
The Quark Model vs. QCD

Quantum
Chromodynamics



Constituent quarks (QM)

Q^P , μ^P



Sea quarks

Current quarks (QCD)

$F^P{}_2(x)$

IV. Weak Neutral Currents

We can uncover the sea with the Z^0

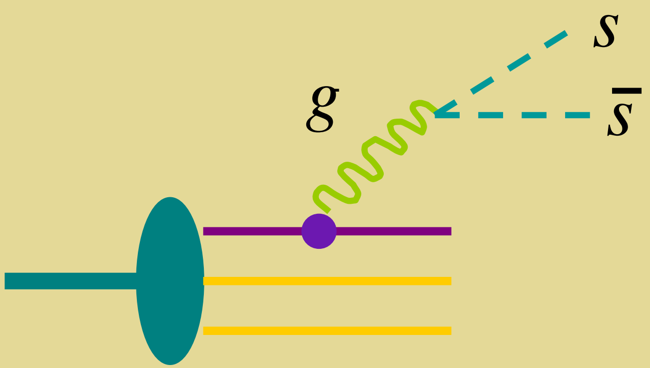
Light QCD quarks:

u	$m_u \sim 5$ MeV
d	$m_d \sim 10$ MeV
s	$m_s \sim 150$ MeV

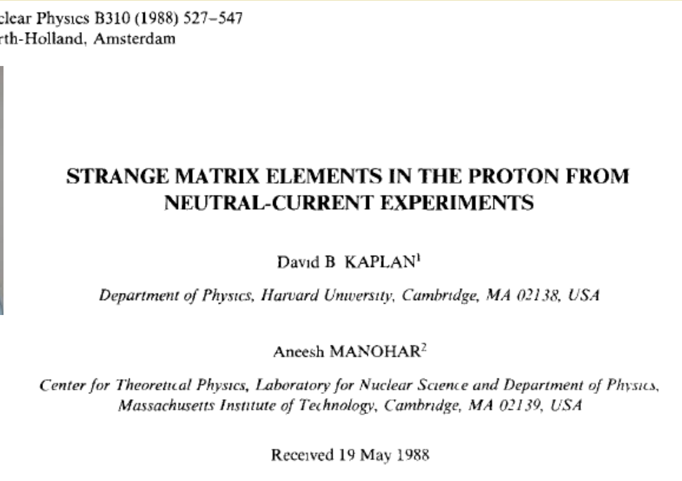
Lives only in the sea

Heavy QCD quarks:

c	$m_c \sim 1500$ MeV
b	$m_b \sim 4500$ MeV
t	$m_t \sim 175{,}000$ MeV



Weak Neutral Current is a Probe



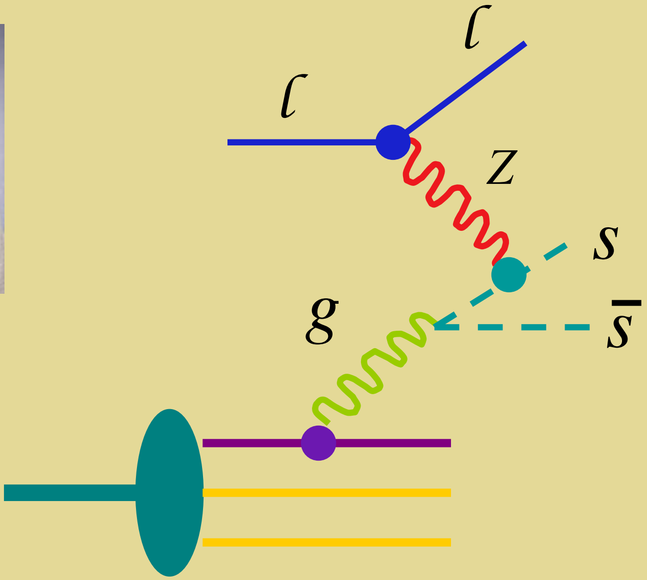
Nuclear Physics B310 (1988) 527–547
North-Holland, Amsterdam

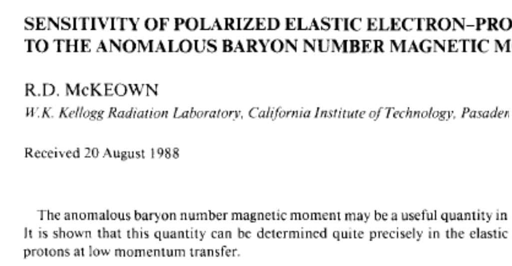
STRANGE MATRIX ELEMENTS IN THE PROTON FROM NEUTRAL-CURRENT EXPERIMENTS

David B KAPLAN[1]
Department of Physics, Harvard University, Cambridge, MA 02138, USA

Aneesh MANOHAR[2]
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received 19 May 1988





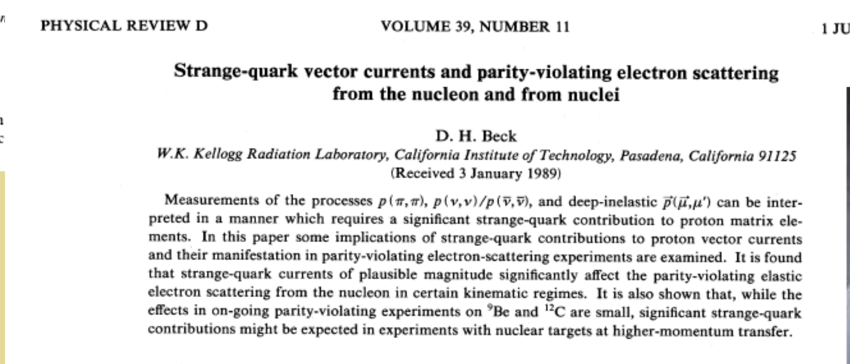
PHYSICS LETTERS B 16 March 1989

SENSITIVITY OF POLARIZED ELASTIC ELECTRON–PROTON SCATTERING TO THE ANOMALOUS BARYON NUMBER MAGNETIC MOMENT

R.D. McKEOWN
W.K. Kellogg Radiation Laboratory, California Institute of Technology, Pasaden

Received 20 August 1988

The anomalous baryon number magnetic moment may be a useful quantity in
It is shown that this quantity can be determined quite precisely in the elastic
protons at low momentum transfer.



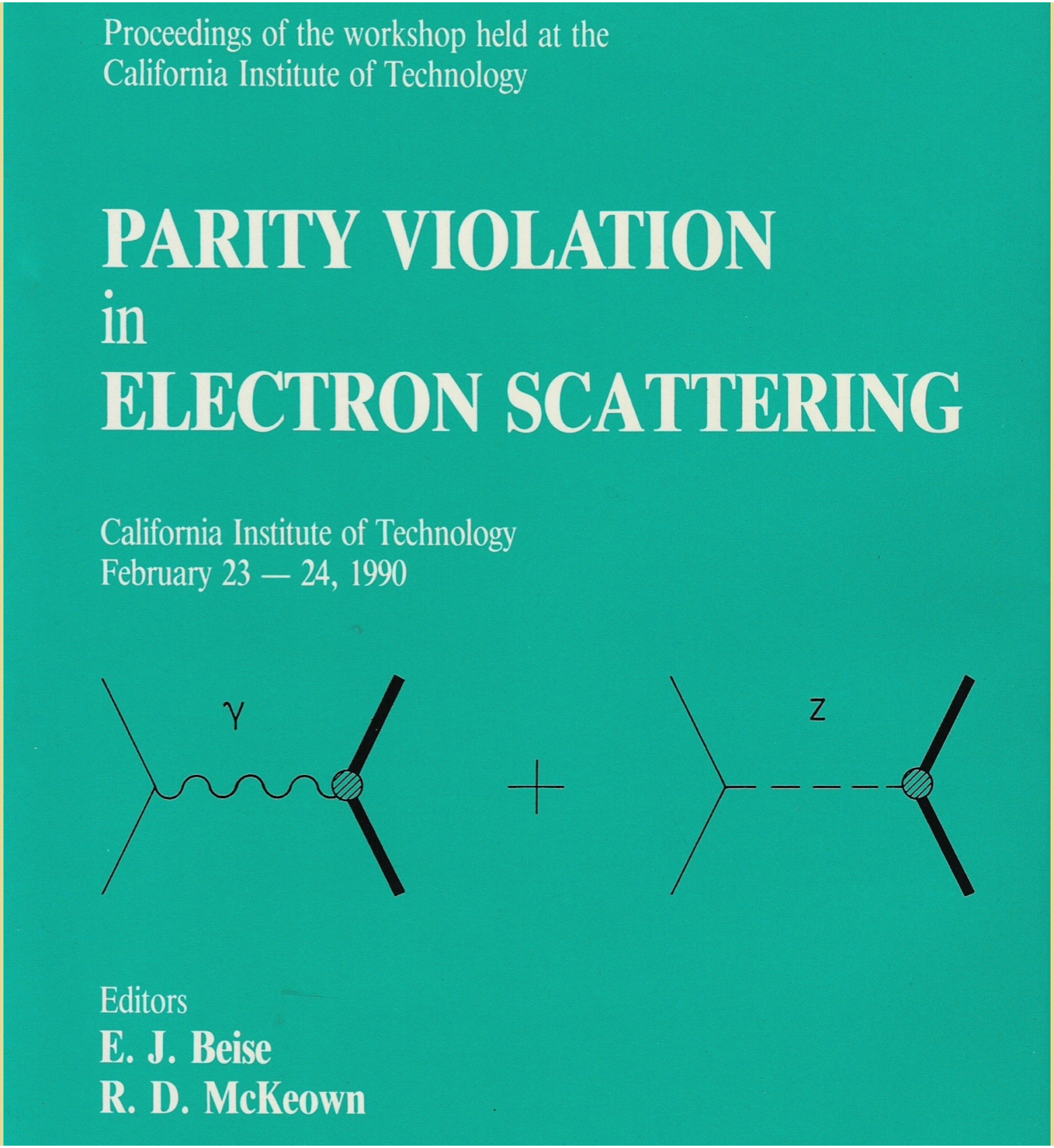
PHYSICAL REVIEW D VOLUME 39, NUMBER 11 1 JUNE 1989

Strange-quark vector currents and parity-violating electron scattering from the nucleon and from nuclei

D. H. Beck
W.K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

(Received 3 January 1989)

Measurements of the processes $p(\pi,\pi)$, $p(\nu,\nu)/p(\bar{\nu},\bar{\nu})$, and deep-inelastic $\vec{p}(\vec{\mu},\mu')$ can be interpreted in a manner which requires a significant strange-quark contribution to proton matrix elements. In this paper some implications of strange-quark contributions to proton vector currents and their manifestation in parity-violating electron-scattering experiments are examined. It is found that strange-quark currents of plausible magnitude significantly affect the parity-violating elastic electron scattering from the nucleon in certain kinematic regimes. It is also shown that, while the effects in on-going parity-violating experiments on ^{9}Be and ^{12}C are small, significant strange-quark contributions might be expected in experiments with nuclear targets at higher-momentum transfer.



Proceedings of the workshop held at the
California Institute of Technology

PARITY VIOLATION
in
ELECTRON SCATTERING

California Institute of Technology
February 23 — 24, 1990

Editors
E. J. Beise
R. D. McKeown

Theoretical interpretation

V. PV Electron Scattering & Strange Quarks

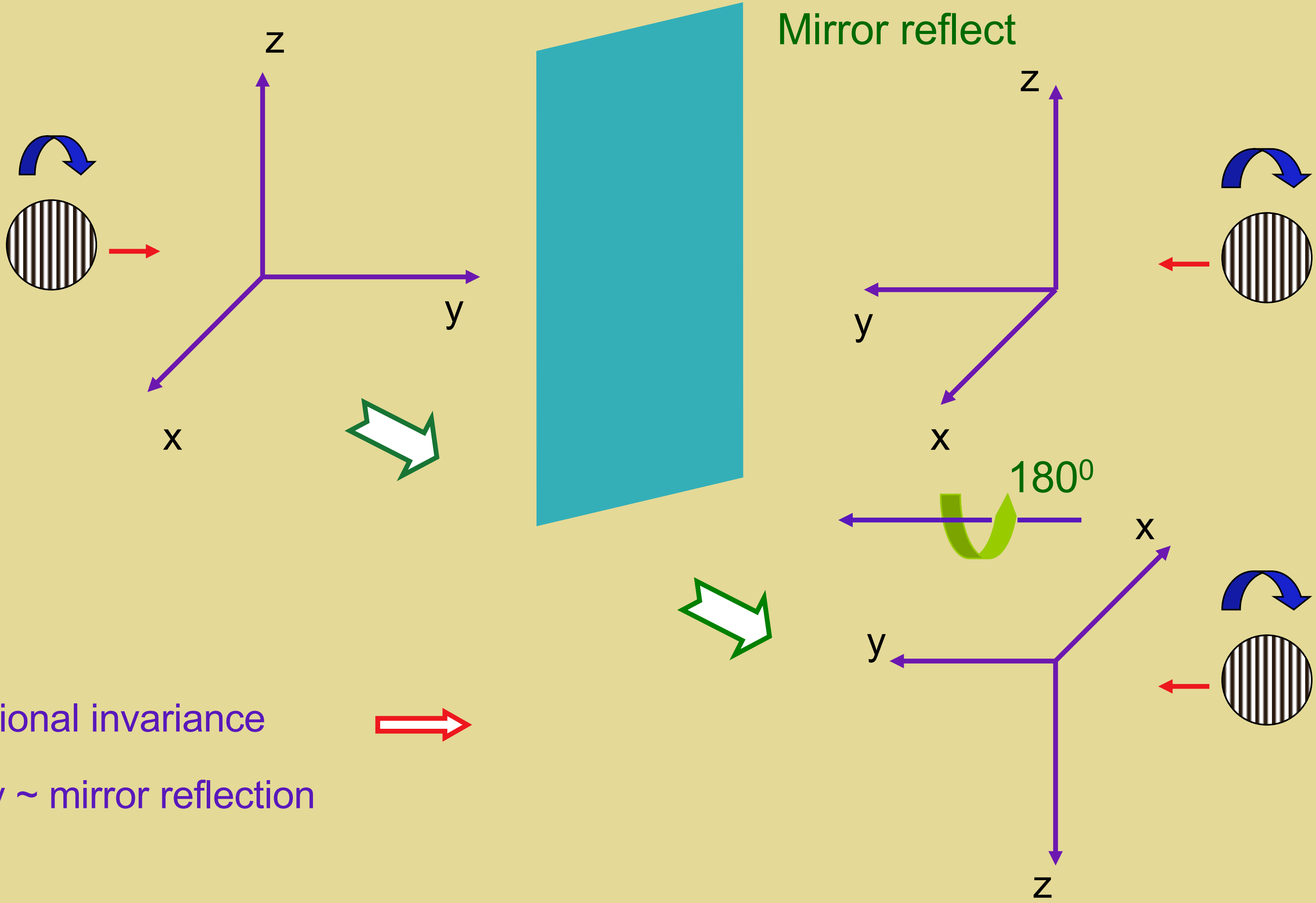
Symmetries Score Card

Force	P	C	T
Gravity	Yes	Yes	Yes
E.M.	Yes	Yes	Yes
Strong	Yes	Yes	Yes
Weak	No	No	No

C: $e^+ \longleftrightarrow e^-$

T: $t \longleftrightarrow -t$

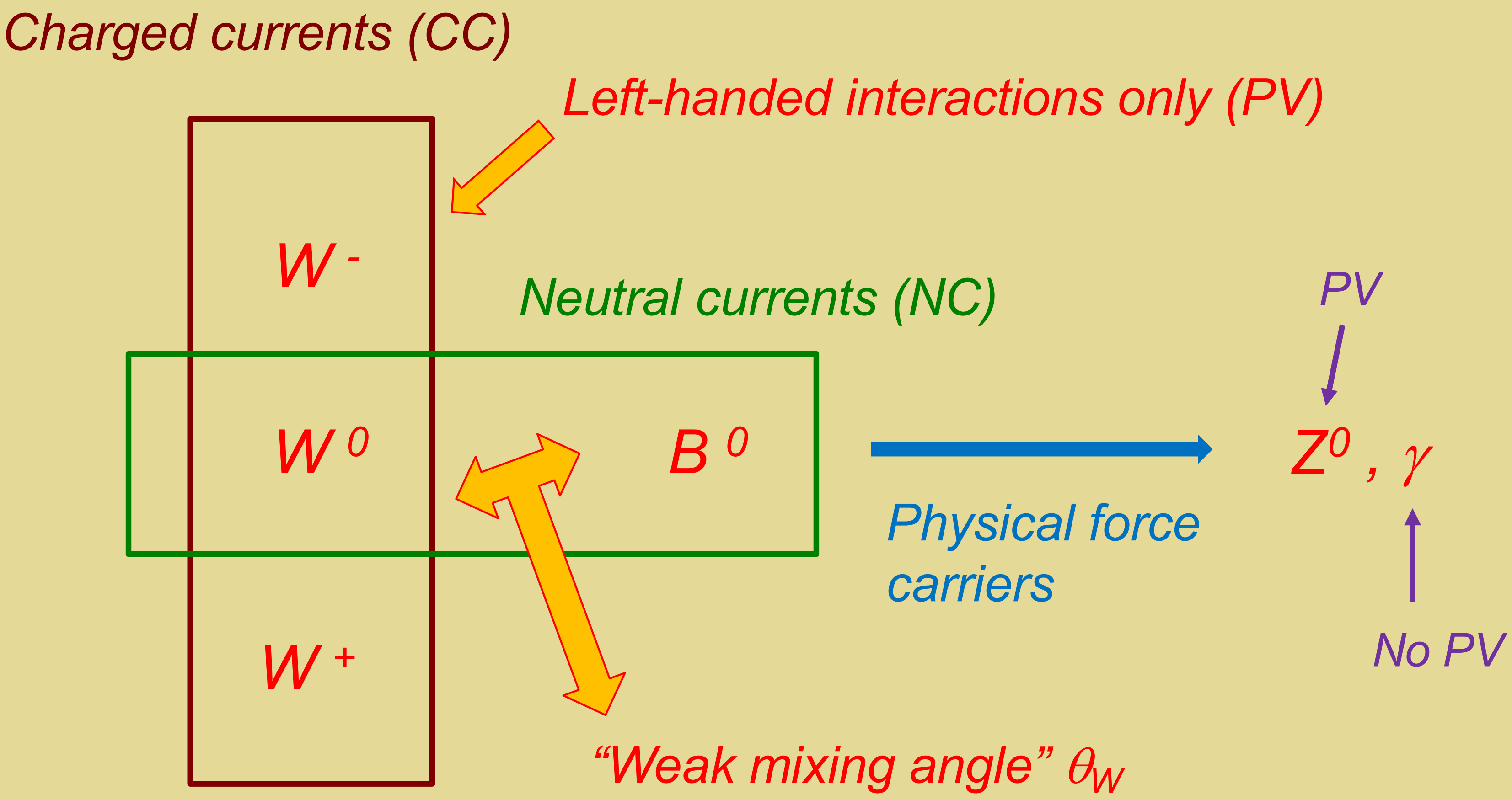
Exploit Parity Symmetry



Rotational invariance ⇒

Parity ~ mirror reflection

Fermion Electroweak Interactions & PV



Charged currents (CC)

Left-handed interactions only (PV)

W^-

Neutral currents (NC)

PV

W^0

B^0

Z^0 , γ

Physical force carriers

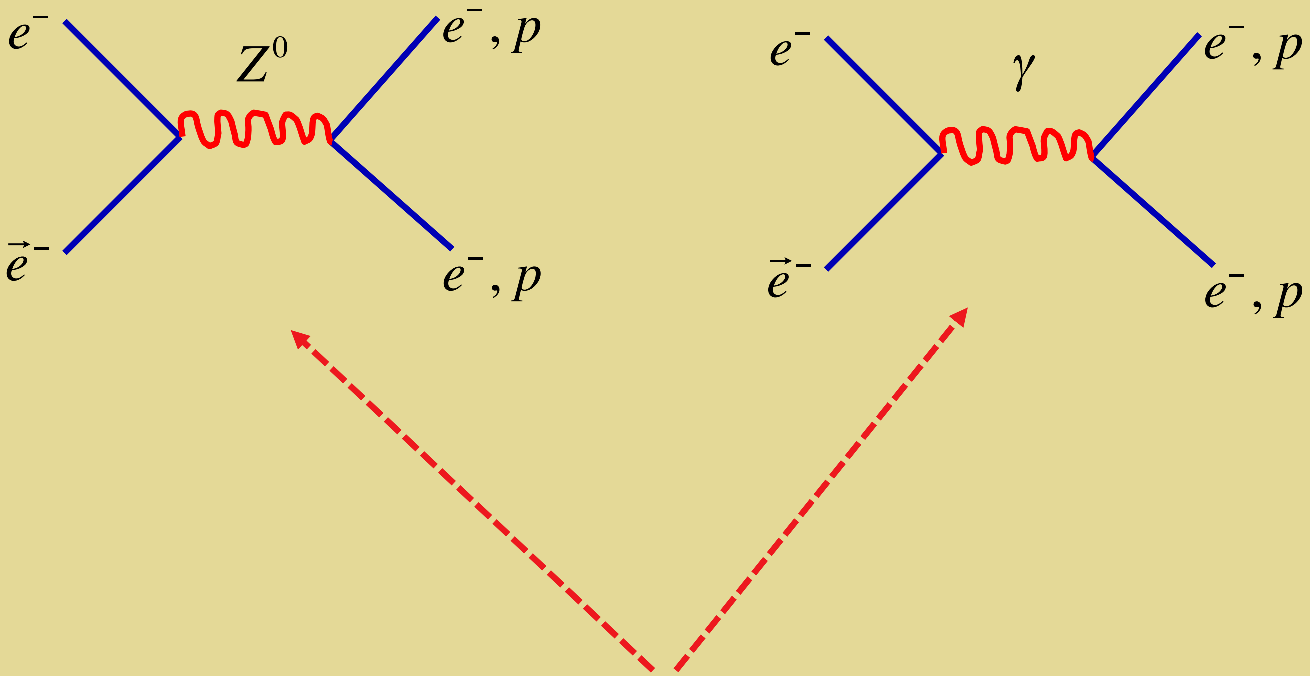
No PV

W^+

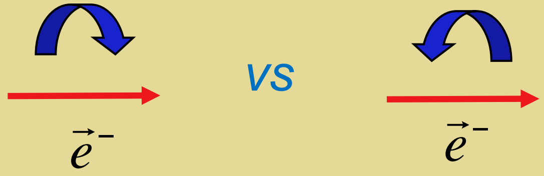
“Weak mixing angle” θ_W

Weak interaction flavor basis:
“primordial” force carriers

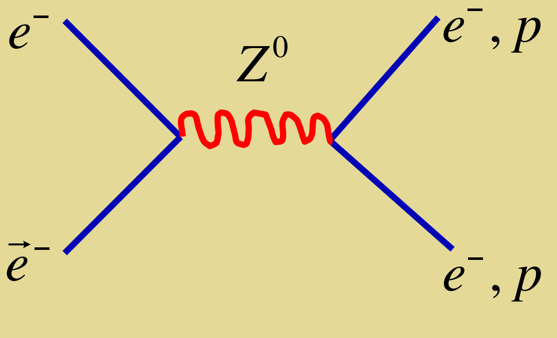
Parity-Violation: Scattering & Atoms

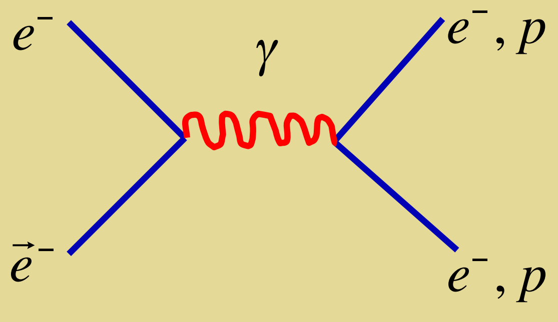


PV: quantum interference



Parity-Violation & Weak Charges





Parity-Violating electron scattering

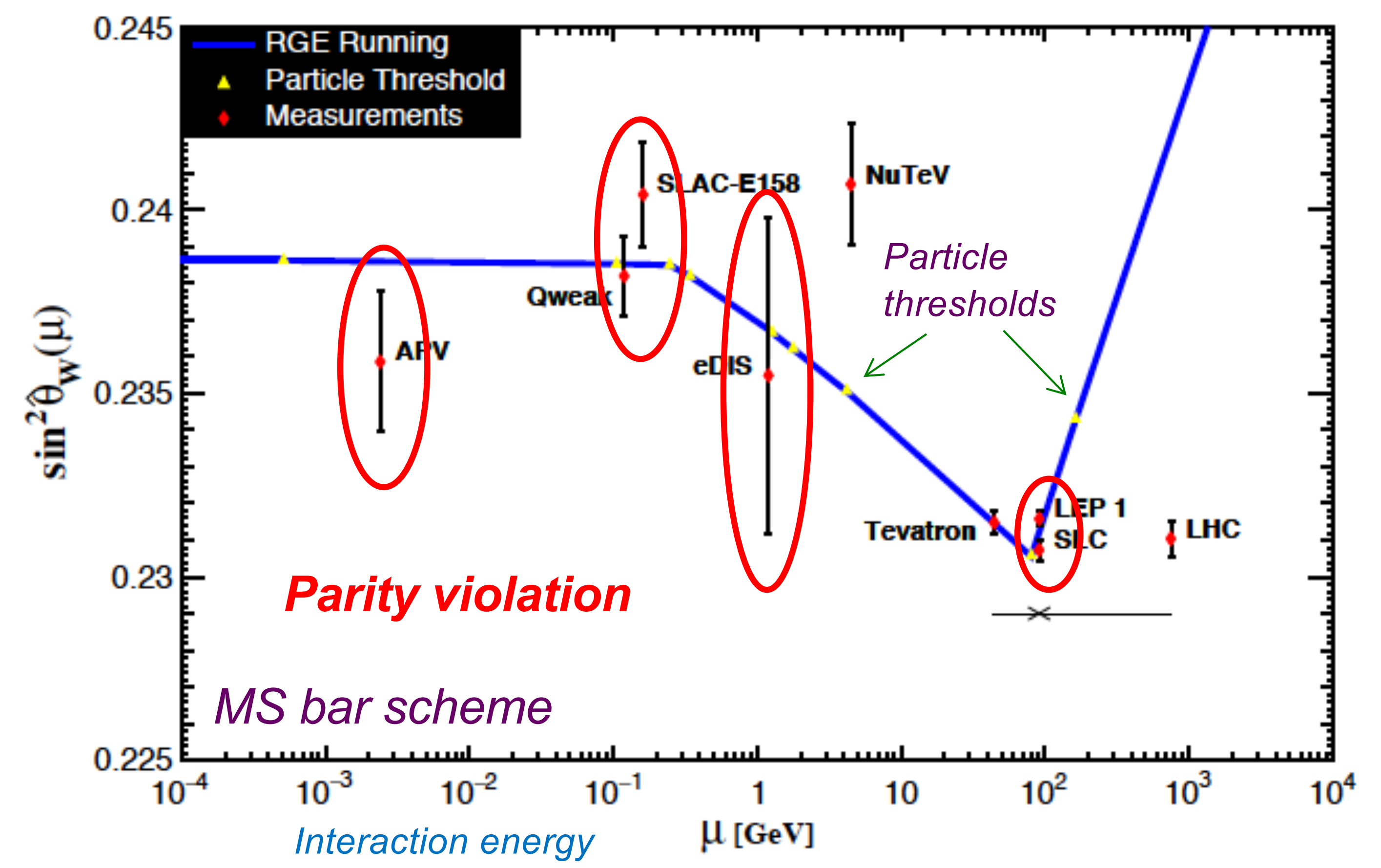
Sensitive to weak mixing

$$A_{PV} = \frac{N_{\uparrow\uparrow} - N}{N_{\uparrow\uparrow} + N} \sim 10^{-6} \left(\frac{Q}{M_p}\right)^2 \left[\boxed{Q_W} + F(Q^2, \theta)\right]$$

Atomic parity-violation

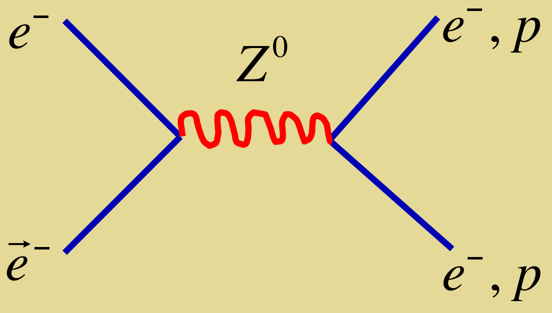
$$E_1^{PV} / \beta = i\, e\, \mathcal{M} \times 10^{-11}\, a_0\, (\boxed{Q_W /} N) / \beta$$

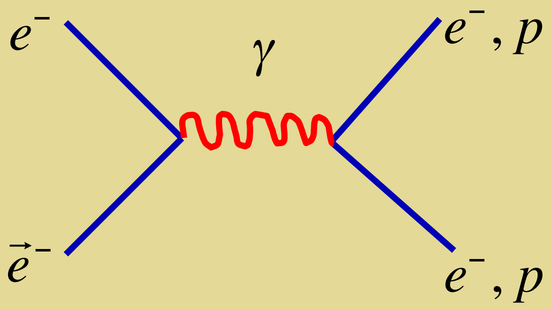
Weak Mixing Depends on Energy Scale



Marciano & Czarnecki ’00
Erler & MJRM ‘05
Erler & Ferro-Hernandez ‘18

Parity-Violation & Weak Charges





Parity-Violating electron scattering

$$A_{PV} = \frac{N_{\uparrow\uparrow} - N}{N_{\uparrow\uparrow} + N} \sim 10^{-6} \left(\frac{Q}{M_p}\right)^2 \left[Q_W + F(Q^2, \theta)\right]$$

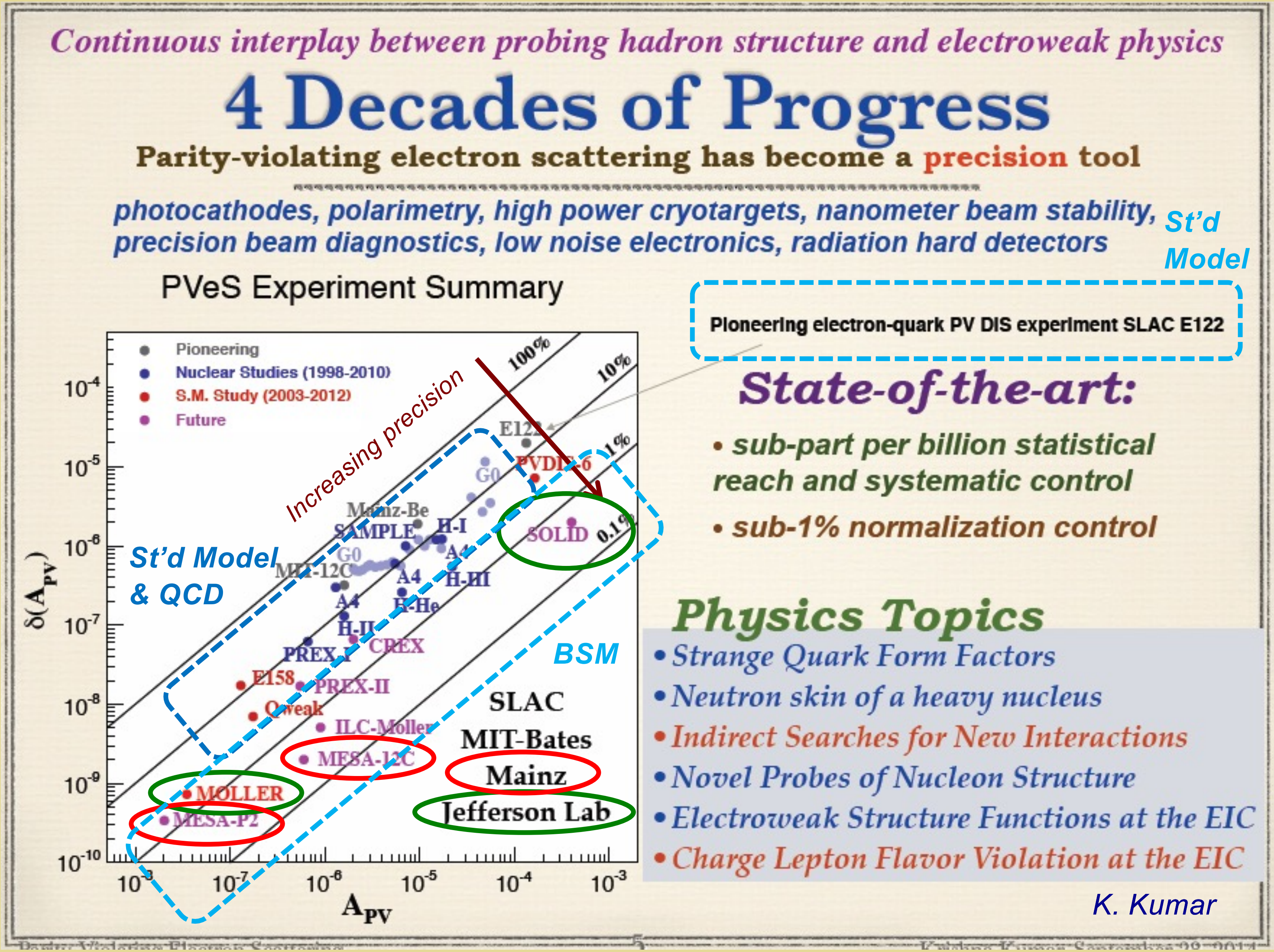
"Weak Charge" ~ 0.1 in SM

Enhanced transparency to beyond Standard Model physics

Small QCD uncertainties (Marciano & Sirlin; Erler & R-M)

Nucleon internal structure: strong interaction (QCD) dynamics at low energy

PV Electron Scattering



Continuous interplay between probing hadron structure and electroweak physics

4 Decades of Progress

Parity-violating electron scattering has become a precision tool

photocathodes, polarimetry, high power cryotargets, nanometer beam stability, precision beam diagnostics, low noise electronics, radiation hard detectors

PVeS Experiment Summary

Pioneering electron-quark PV DIS experiment SLAC E122

State-of-the-art:

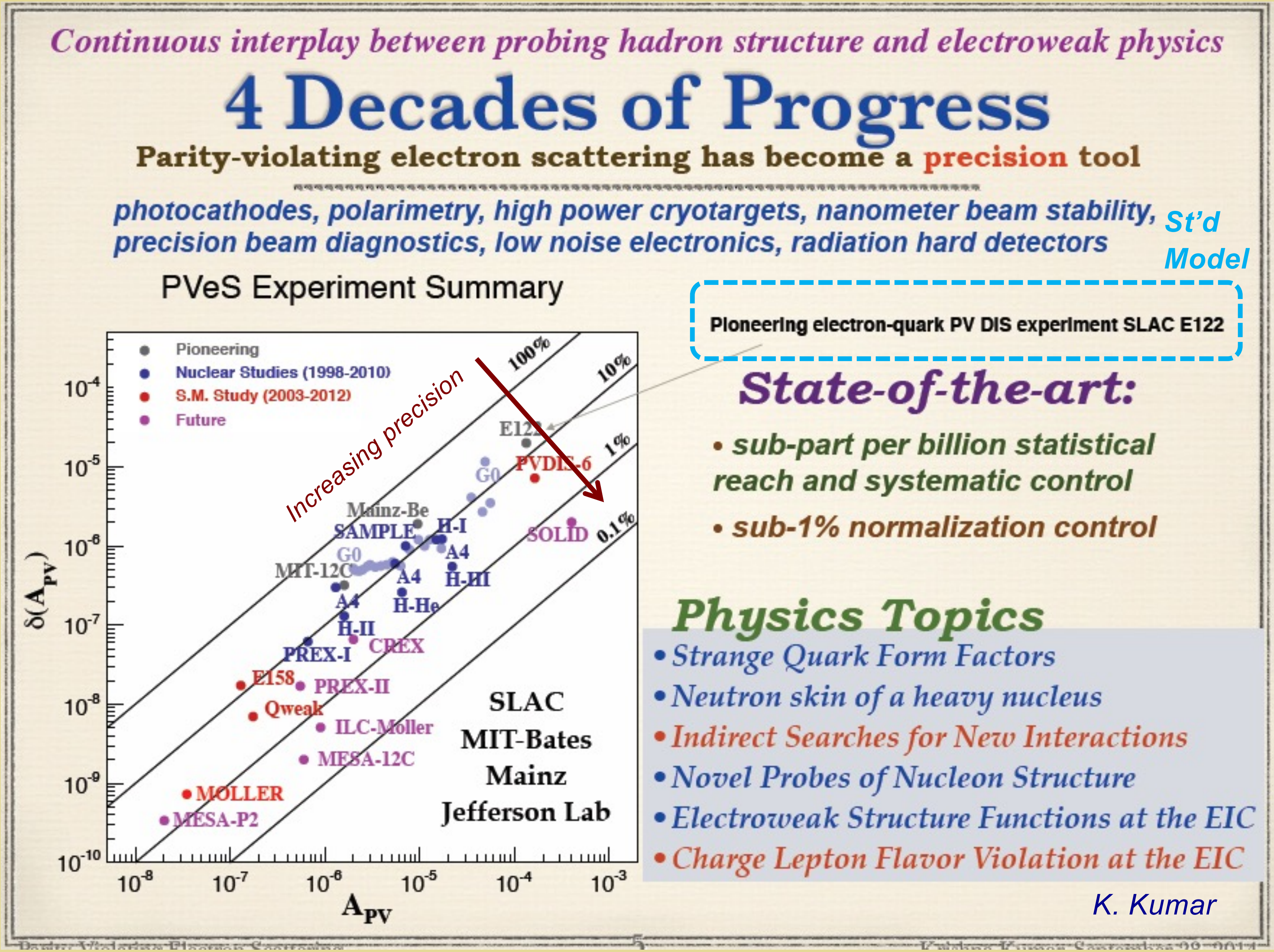
- *sub-part per billion statistical reach and systematic control*
- *sub-1% normalization control*

Physics Topics

- *Strange Quark Form Factors*
- *Neutron skin of a heavy nucleus*
- *Indirect Searches for New Interactions*
- *Novel Probes of Nucleon Structure*
- *Electroweak Structure Functions at the EIC*
- *Charge Lepton Flavor Violation at the EIC*

K. Kumar

PV Electron Scattering



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St'd Model

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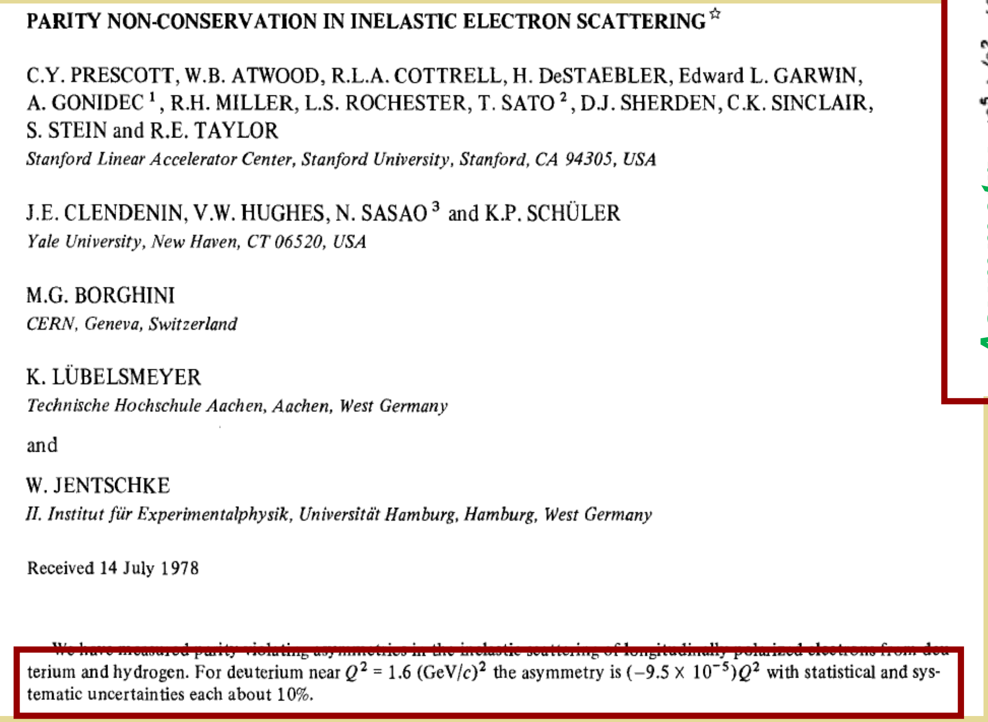
K. Kumar

SLAC '77: PV Deep Inelastic Scattering

Glashow-Weingerg-Salam:
Standard Model



PARITY NON-CONSERVATION IN INELASTIC ELECTRON SCATTERING☆

C.Y. PRESCOTT, W.B. ATWOOD, R.L.A. COTTRELL, H. DeSTAEBLER, Edward L. GARWIN,
A. GONIDEC [1], R.H. MILLER, L.S. ROCHESTER, T. SATO [2], D.J. SHERDEN, C.K. SINCLAIR,
S. STEIN and R.E. TAYLOR
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA

J.E. CLENDENIN, V.W. HUGHES, N. SASAO [3] and K.P. SCHÜLER
Yale University, New Haven, CT 06520, USA

M.G. BORGHINI
CERN, Geneva, Switzerland

K. LÜBELSMEYER
Technische Hochschule Aachen, Aachen, West Germany

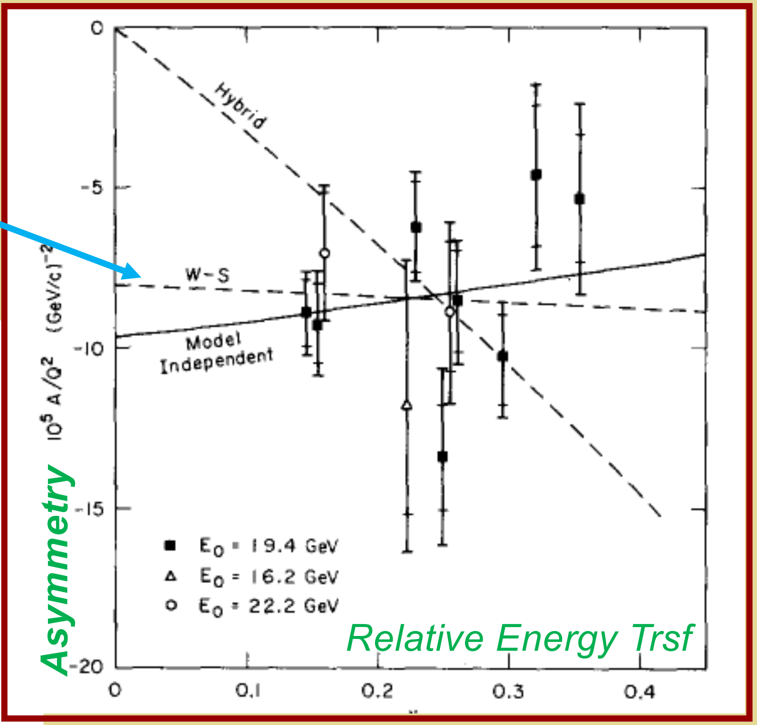
and

W. JENTSCHKE
II. Institut für Experimentalphysik, Universität Hamburg, Hamburg, West Germany

Received 14 July 1978

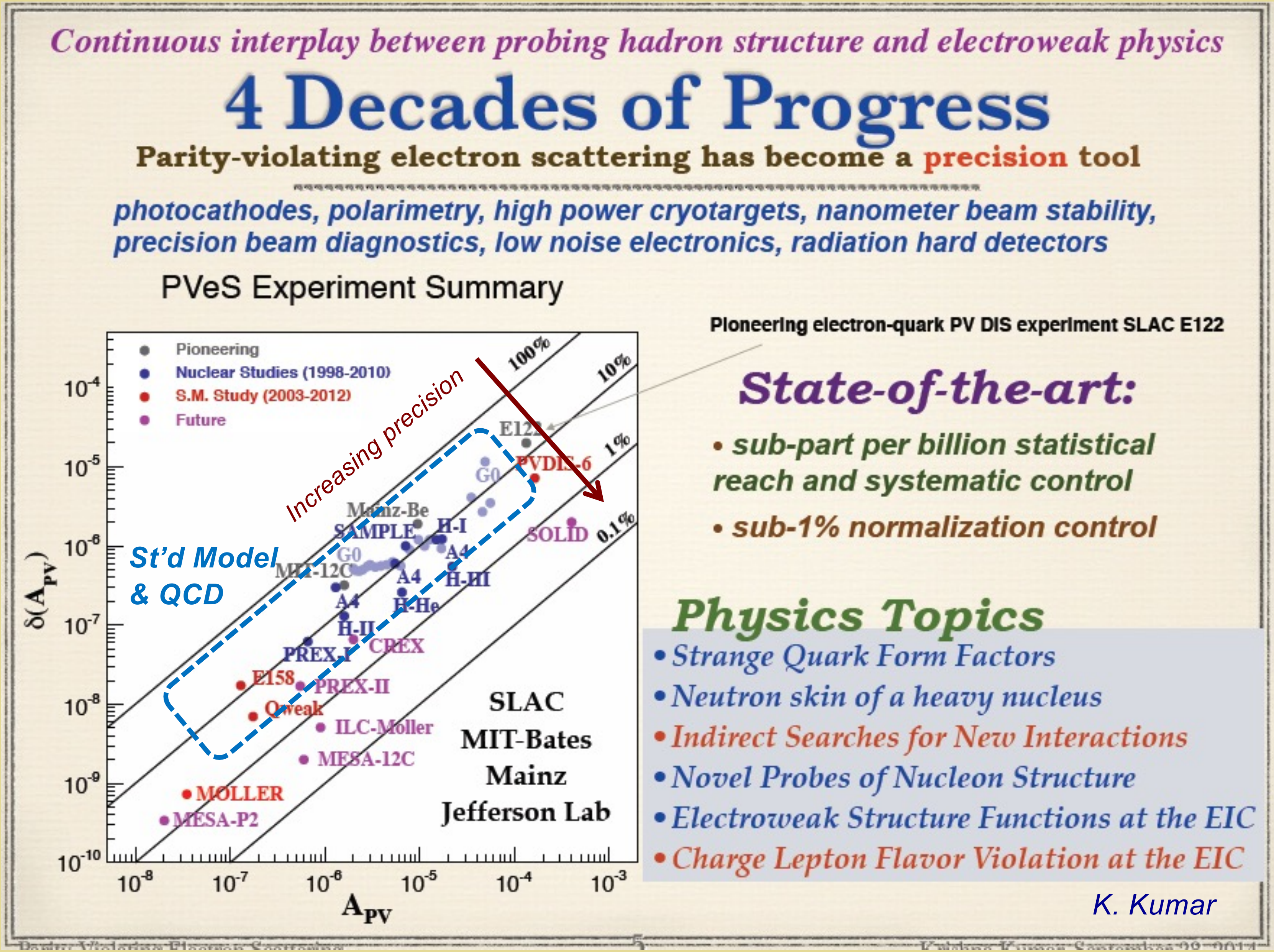
We have measured parity-violating asymmetries in the inelastic scattering of longitudinally polarized electrons from deuterium and hydrogen. For deuterium near $Q^2 = 1.6\ (\mathrm{GeV}/c)^2$ the asymmetry is $(-9.5 \times 10^{-5})Q^2$ with statistical and systematic uncertainties each about 10%.

Phys. Lett. B77, 347 (1977)



Phys. Lett. B84, 524 (1979)

PV Electron Scattering



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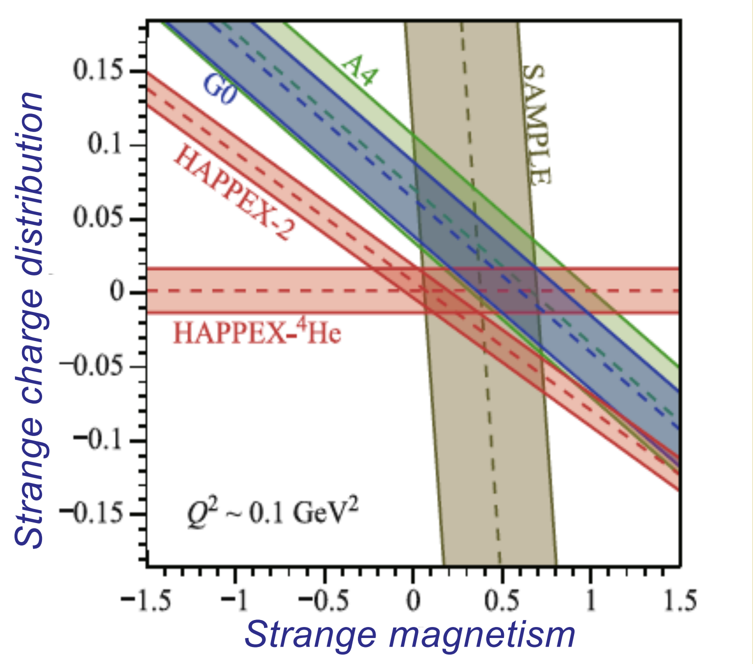
K. Kumar

Strange Quarks: Proton Magnetism & Charge Distribution

If strange quarks – not part of the quark model picture – give a sizeable contribution to the nucleon spin and mass, what about their effects on electromagnetic properties ?

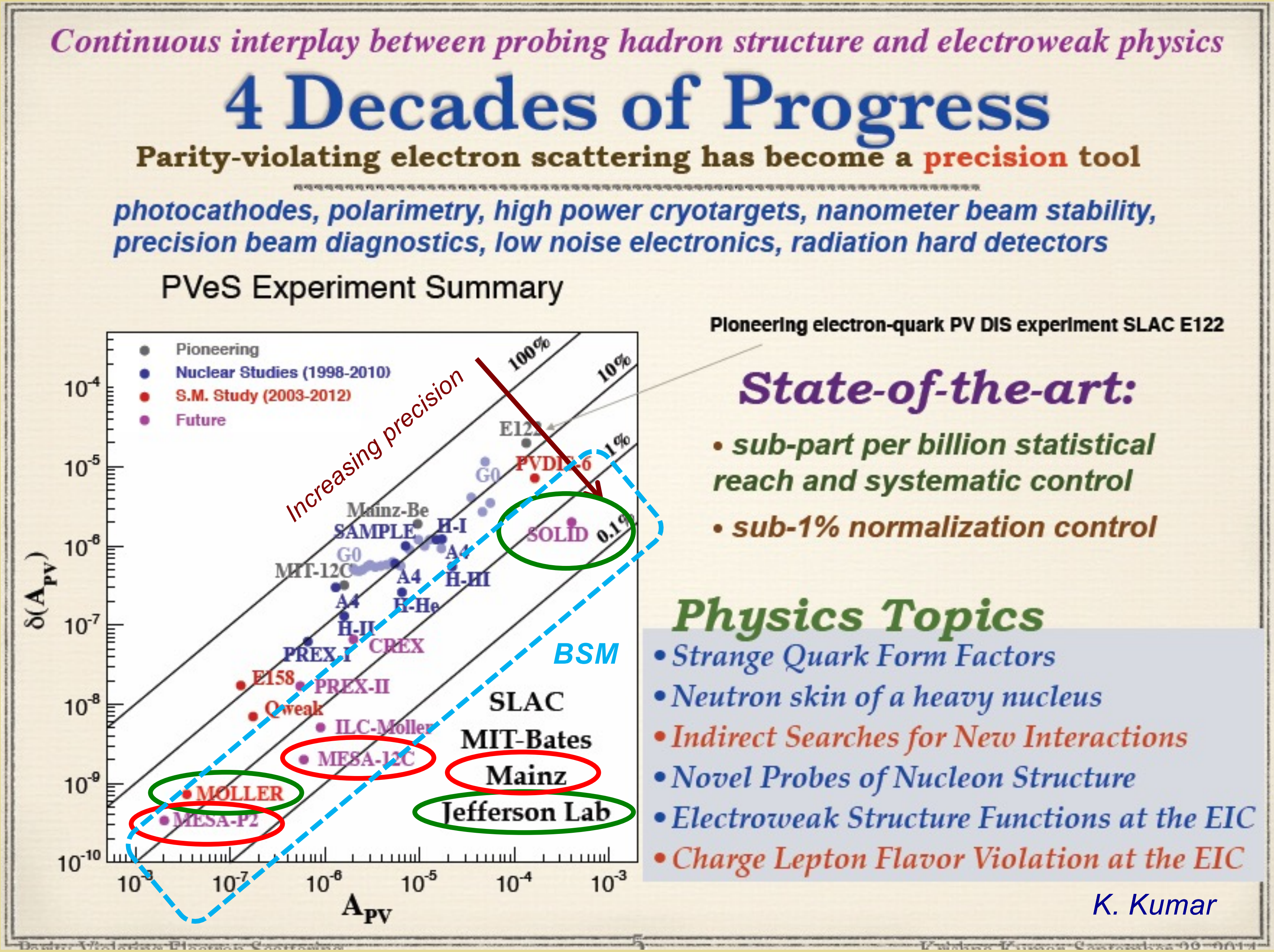


- *Small s-quark effects on E.M. properties*
- *We wouldn't have known this w/o enormous exp't effort and rigorous precision EW calculations & reliable statement of theoretical uncertainty*

P. Souder & K. Paschke, Front Phys 11 (2016) 111301

VI. PVES & BSM Physics

PV Electron Scattering



Continuous interplay between probing hadron structure and electroweak physics

4 Decades of Progress

Parity-violating electron scattering has become a precision tool

photocathodes, polarimetry, high power cryotargets, nanometer beam stability, precision beam diagnostics, low noise electronics, radiation hard detectors

PVeS Experiment Summary

Pioneering electron-quark PV DIS experiment SLAC E122

State-of-the-art:

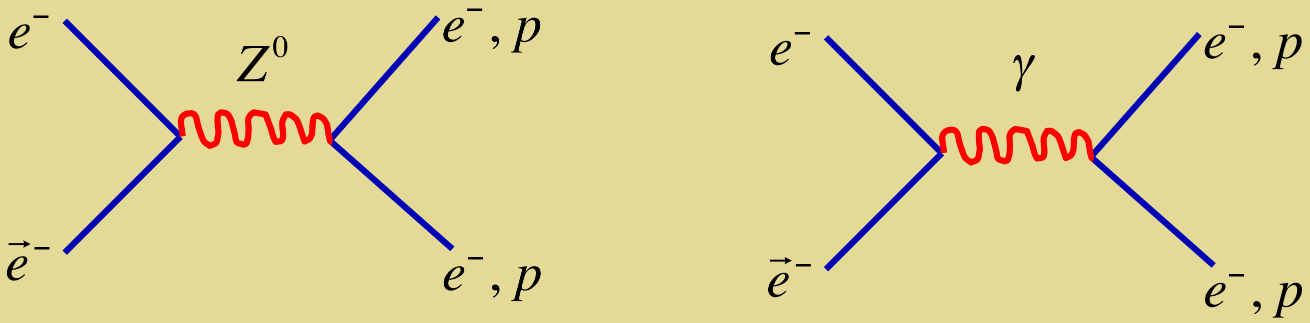
- *sub-part per billion statistical reach and systematic control*
- *sub-1% normalization control*

Physics Topics

- *Strange Quark Form Factors*
- *Neutron skin of a heavy nucleus*
- *Indirect Searches for New Interactions*
- *Novel Probes of Nucleon Structure*
- *Electroweak Structure Functions at the EIC*
- *Charge Lepton Flavor Violation at the EIC*

K. Kumar

Parity-Violation & Weak Charges



Parity-Violating electron scattering

$$A_{PV} = \frac{N_{\uparrow\uparrow} - N}{N_{\uparrow\uparrow} + N} \sim 10^{-6}\left(\frac{Q}{M_p}\right)^2 \left[Q_W + F(Q^2,\theta)\right]$$

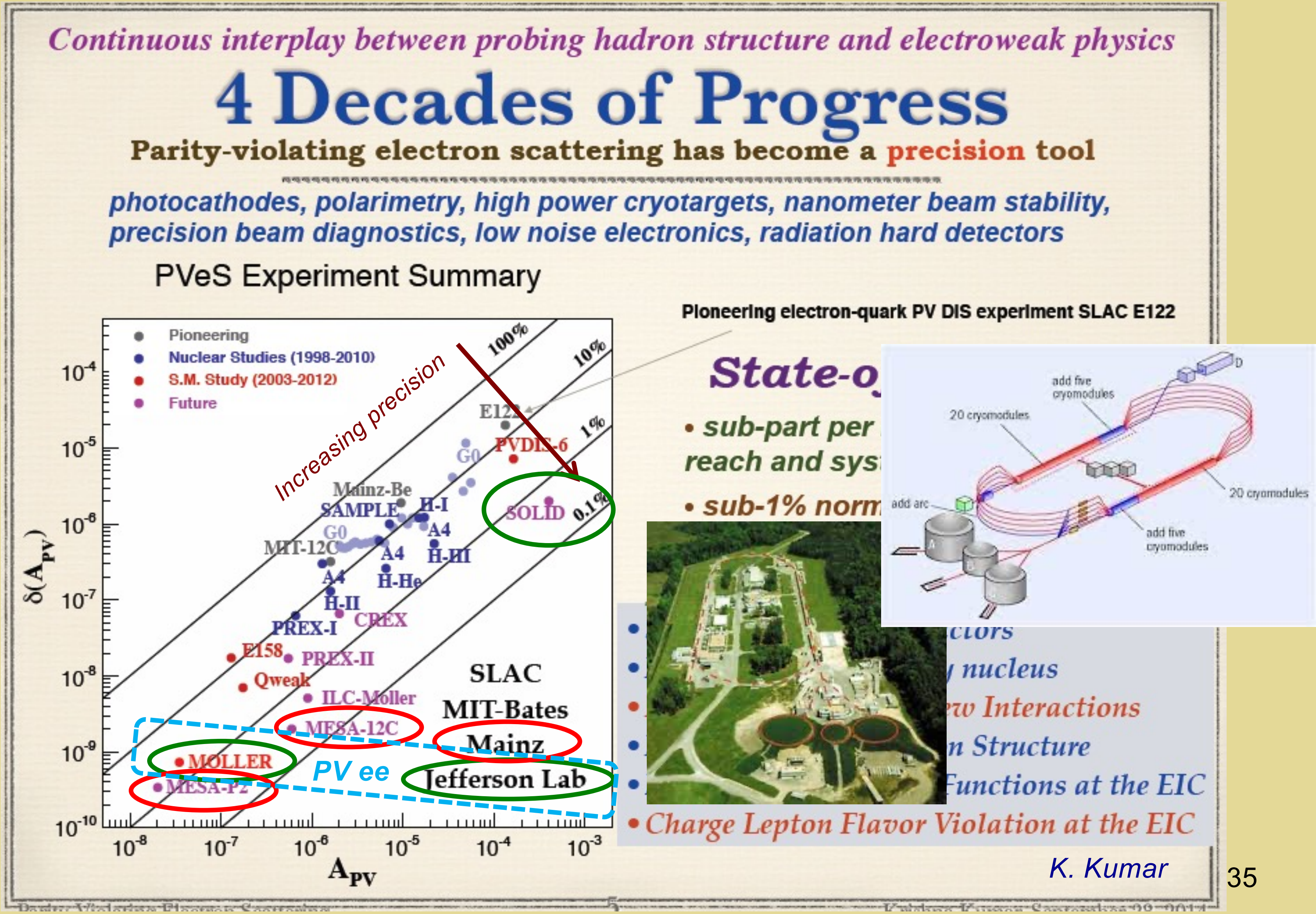
"Weak Charge" ~ 0.1 in SM

Enhanced transparency to BSM physics

Small QCD uncertainties (Marciano & Sirlin; Erler & R-M)

QCD effects (s-quarks): measured (MIT-Bates, Mainz, JLab)

PV Electron Scattering



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Pioneering electron-quark PV DIS experiment SLAC E122

State-o

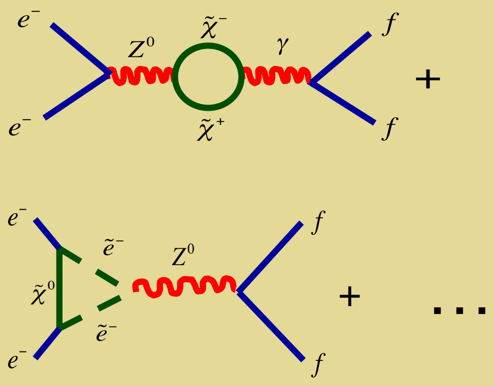
- sub-part per
reach and sys
- sub-1% norm

- ctors
- nucleus
- w Interactions
- n Structure
- Functions at the EIC
- Charge Lepton Flavor Violation at the EIC

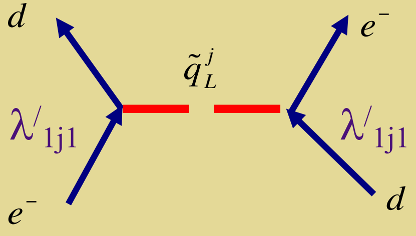
K. Kumar

Deviations: BSM "Footprints"

SUSY

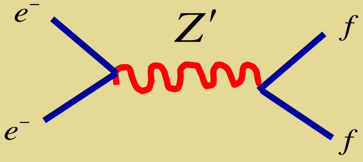


Radiative Corrections

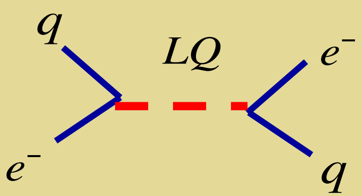


RPV

$Z^{/}$ Bosons



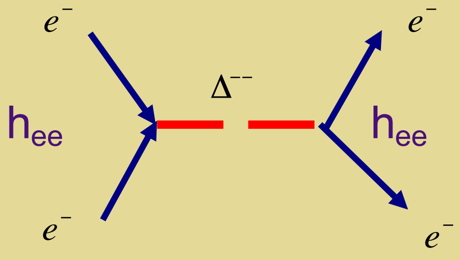
Leptoquarks



Semi-leptonic only

Doubly Charged Scalars

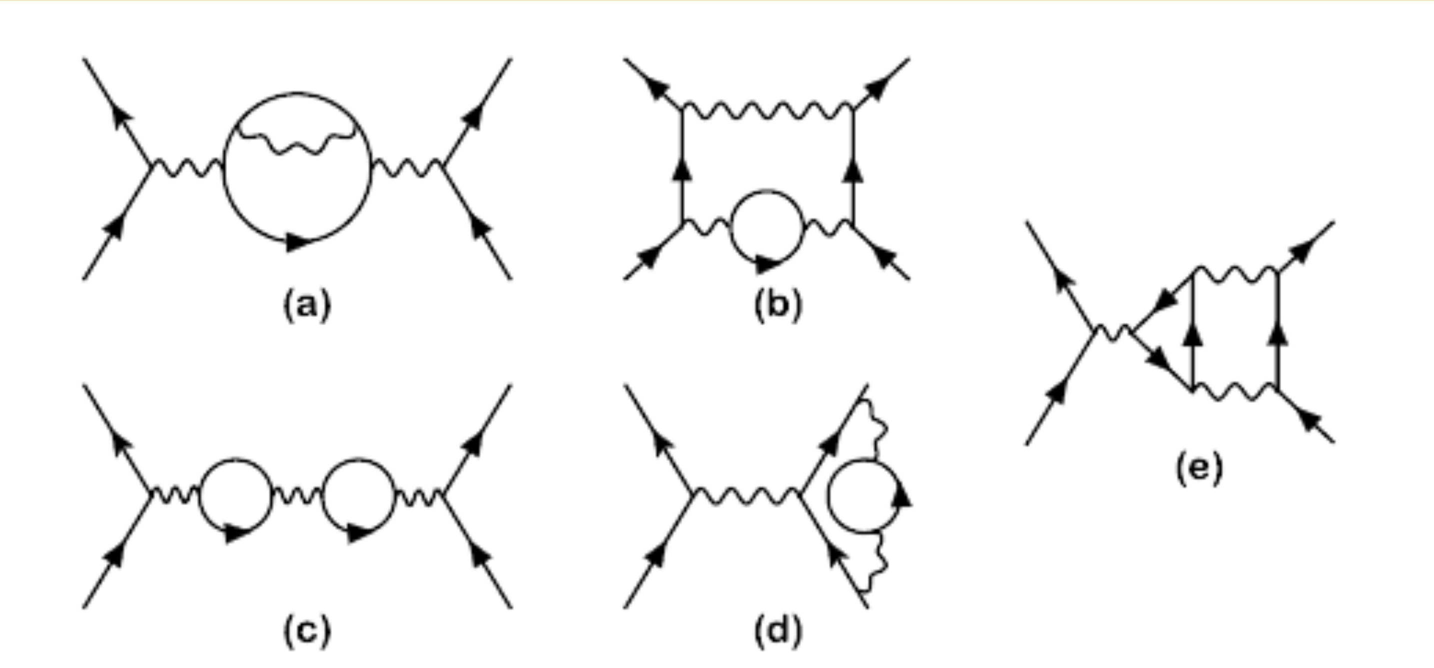
Moller only

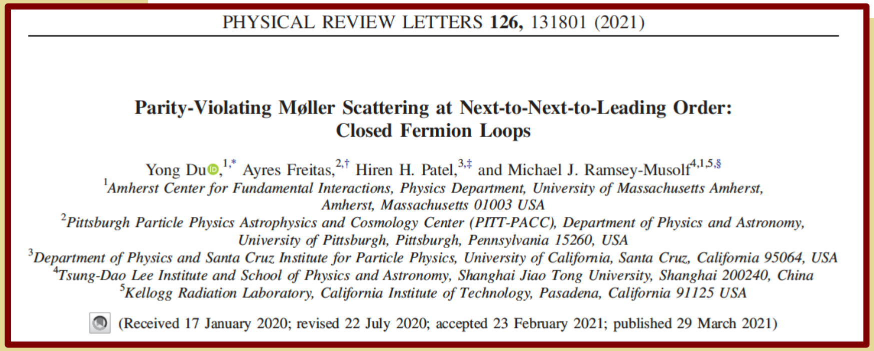


PV Moller scattering

Two-Loop EW Radiative Corrections

Closed fermion loops: gauge invariant




PHYSICAL REVIEW LETTERS **126,** 131801 (2021)

Parity-Violating Møller Scattering at Next-to-Next-to-Leading Order:
Closed Fermion Loops

Yong Du[1,*] Ayres Freitas,[2,†] Hiren H. Patel,[3,‡] and Michael J. Ramsey-Musolf[4,1,5,§]
[1]*Amherst Center for Fundamental Interactions, Physics Department, University of Massachusetts Amherst,*
Amherst, Massachusetts 01003 USA
[2]*Pittsburgh Particle Physics Astrophysics and Cosmology Center (PITT-PACC), Department of Physics and Astronomy,*
University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
[3]*Department of Physics and Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064, USA*
[4]*Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*
[5]*Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125 USA*

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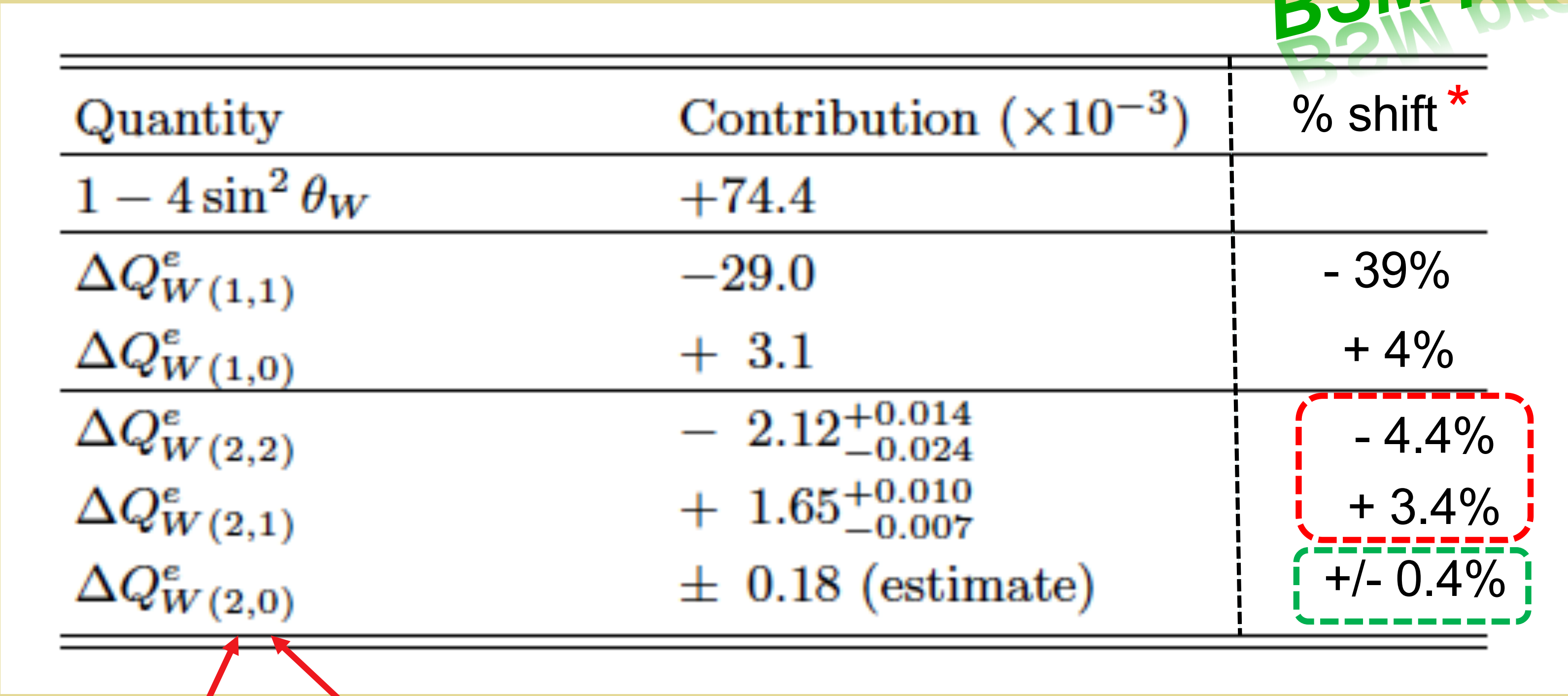
Two-Loop EW Radiative Corrections

$\delta(Q^e_W) = \pm 2.1\ \%\ (\text{stat.}) \pm 1.1\ \%\ (\text{syst.})$

Exp't precision (goal)

BSM probe !



Quantity	Contribution $(\times 10^{-3})$	% shift *
$1 - 4\sin^2\theta_W$	$+74.4$	
$\Delta Q^e_{W(1,1)}$	-29.0	- 39%
$\Delta Q^e_{W(1,0)}$	$+\ 3.1$	+ 4%
$\Delta Q^e_{W(2,2)}$	$-\ 2.12^{+0.014}_{-0.024}$	- 4.4%
$\Delta Q^e_{W(2,1)}$	$+\ 1.65^{+0.010}_{-0.007}$	+ 3.4%
$\Delta Q^e_{W(2,0)}$	$\pm\ 0.18$ (estimate)	+/- 0.4%

Must !

Safe !

Loop order

# of closed fermion loops

** Relative to preceding order*

Du, Freitas, Patel, MJRM PRL 126 (2021) 131801 [1912.08220]

谢谢

where $\int d^3x\, \rho(\vec{x}) = 1 = F(0)$

VII.C.2. Elastic ep Scattering

In many ways, elastic ep scattering is like eμ scattering. Recall the result (7.44)

$$\left.\frac{d\sigma}{d\Omega}\right|_{Lab}^{e\mu} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{t}{2m_\mu^2}\sin^2\frac{\theta}{2}\right] \quad (7.44)$$

where $E' = E/f_{rec}$

$$f_{rec} = 1 + \frac{2E}{m_\mu}\sin^2\frac{\theta}{2}$$

For scattering from a pointlike proton, we'd arrive at the same result w/ $m_\mu \to M_p \equiv M$. Experimentally, however, this result does not obtain. By studying $d\sigma/d\Omega|_{ep\ elastic}$ as a function of θ, it is found that the coefficients of the

$$\cos^2\theta/2 \quad \& \quad -\frac{t}{2M_p^2}\sin^2\frac{\theta}{2} \qquad \text{terms}$$

are not unity but functions of q^2. Doing so is a procedure known as a "Rosenbluth Separation." The result is that these two terms come w/ their own form factors.

The result can be accommodated by making the following replacement-

$$\bar{u}_p(p')\,\gamma^\mu\, u_p(p)\Big|_{\text{pointlike}} \longrightarrow \bar{u}_p(p')\,\Gamma^\mu\, u_p(p) \qquad [7.93_a)$$

$$\Gamma^\mu = F_1(q^2)\,\gamma^\mu + \frac{i\,\kappa_p}{2M}\,F_2(q^2)\,\sigma_{\mu\nu}\,q^\nu \qquad [7.93b]$$

$F_1 =$ "Dirac FF"

$F_2 =$ "Pauli FF"

Repeating the $e\mu$ calculation but w/ $m_\mu \to M$, and (7.93) $\Rightarrow$

$$\frac{d\sigma}{d\Omega}\bigg|^{ep\to ep}_{Lab} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\left(F_1 + \kappa F_2\right)^2\sin^2\frac{\theta}{2}\right\} \qquad [7.94)$$

Now it is convenient to define

$$\tau \equiv -q^2/4M^2 \qquad [7.95]$$

and two new form factors:

"Electric": $G_E = F_1 - \tau \kappa F_2$ (7.96)

"Magnetic": $G_M = F_1 + \kappa F_2$

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{Lab} = \left(\frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right\} \quad (7.97)$$

"Rosenblum formula".

Experimentally, we have $\kappa_p = 1.79$
$\kappa_n = -1.91$

and $G_E^p(0) = 1$
$G_E^n(0) = 0$

Some plots, comments on G_E^p / G_M^p, ...

VII.C.3 INELASTIC ep SCATTERING

We now generalize the foregoing discussion to cases where the final state hadron ≠ p but either

(a) A specific state, such as Δ(1232) "Exclusive"
(b) A sum over final states of hadrons "Inclusive"

If we write, for this general case

$$\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT} W^{(EM)} \qquad [7.98]$$

↖ [7.92]

$W^{(EM)}$ is called the "Response Function" (EM)

In general, $W^{(EM)}$ depends on two variables

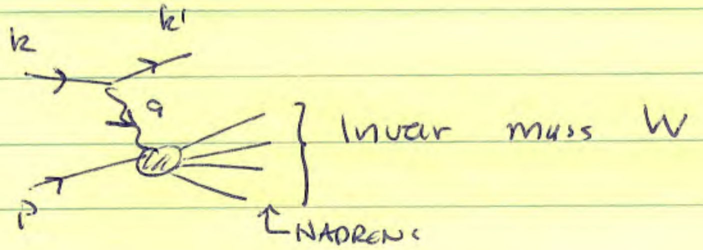
$$q^2 \qquad [7.94a]$$

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$$

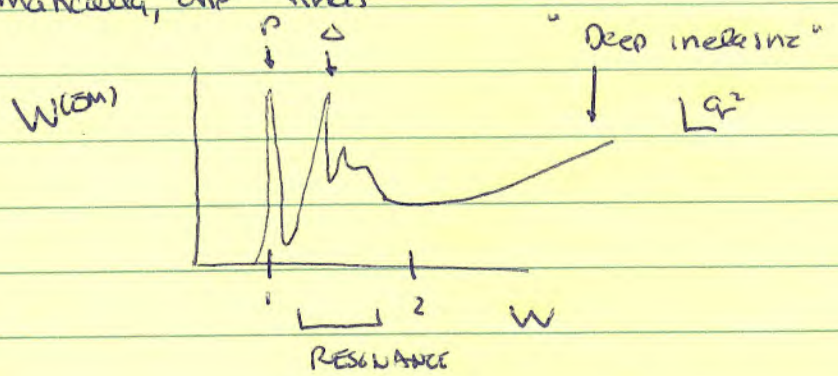
$$\nu = p \cdot q / M \qquad [7.94b]$$

$$W^{(EM)} = W^{(EM)}(q^2, W^2) \quad \text{or} \quad W^{(EM)}(q^2, \nu).$$

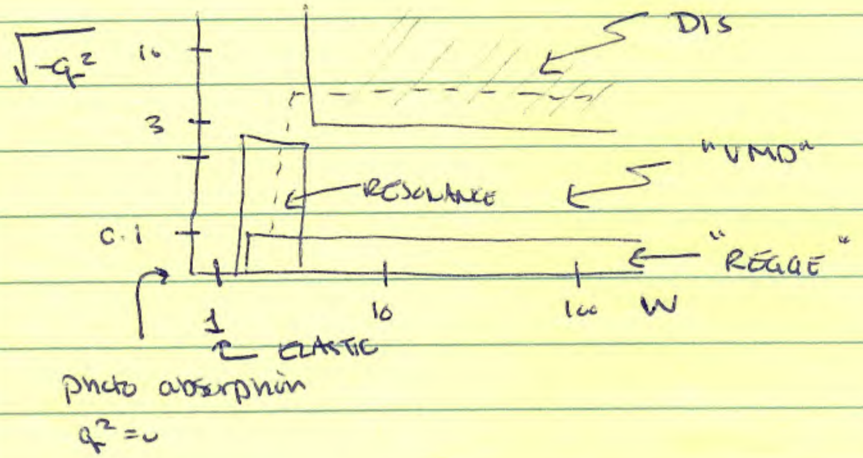
Diagrammatically:



Schematically, one finds



We can also characterize the response in the two variable kinematic place.



We have thus far concentrated on the elastic region; will soon focus on DIS. First, some general features of $W^{(EM)}$

Going back to our general expression for $d\sigma$;

$$d\sigma = \left(\frac{1}{2S_\ell+1}\right)\underset{\substack{\uparrow\\ \text{Target}}}{\left(\frac{1}{2S_T+1}\right)}\ \frac{1}{F}\sum_{\substack{s_\ell, s_\ell'\\ S_T}}\ \underset{\substack{\uparrow\\ \text{Final State}\\ \text{hadron Phase sphase}\\ \text{\& spin.}}}{\sum_{X}}\left(\frac{d^3k'}{(2\pi)^3\, 2k_0'}\right) \longrightarrow$$

$$\times \quad (2\pi)^4 \delta(p+\overset{k-k'}{\overset{''}{q}}-P_x)\, |\bar{M}|^2 \qquad (7.100)$$

Now

$$iM = (ie)^2 (Q_e Q_x)\, \bar{u}(k' s_e')\gamma^\lambda u(k, s_e)$$

$$\times \quad \frac{-i g_{\lambda\rho}}{q^2} \langle X | J^\rho | p, s_T \rangle \qquad (7.101)$$

where we have replaced

$$\bar{u}_p(p' s_p')\Gamma^\rho u_p(p, s_p) \rightarrow \langle X | \underset{\uparrow \, \equiv \, J^\rho(q)}{J^\rho} | p, S_T \rangle \qquad (7.102)$$

With $F = 4Mk$

(7.100) & using $S_T = 1/2$ for a proton target

$$\frac{k_0' \, d\sigma}{d^3 k'} \overset{\text{Lab}}{=} \frac{1}{32(2\pi)^3 Mk} \sum_{\substack{s_e, s_e' \\ s_T, X}} (2\pi)^4 \delta^{(4)}(p+q-P_x)\, |\bar{M}|^2$$

$$= \frac{2}{k}\left(\frac{\alpha Q_e}{q^2}\right)^2 L^{\mu\nu} W_{\mu\nu} \qquad (7.103)$$

where $W_{\mu\nu}$ is the "Hadron Tensor" with

$$W_{\mu\nu} = \left(\frac{1}{4\pi M}\right)\left(\frac{1}{2S_T+1}\right)\sum_{X, S_T} (2\pi)^4 \delta^{(4)}(p+q-p_X)$$

$$\times \langle p, S_T | j_\mu^\dagger | X \rangle \langle X | j_\nu | p, S_T \rangle \qquad [7.104]$$

Now, let's work out the general structure of $W_{\mu\nu}$. It can be built from $g^{\mu\nu}$, p^μ, q^μ; in general

$$W_{\mu\nu} = -W_1 g_{\mu\nu} + \frac{W_2}{M^2} p_\mu p_\nu + \frac{W_3}{M^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta$$

$$+ \frac{W_4}{M^2} q_\mu q_\nu + \frac{W_5}{M^2}(p_\mu q_\nu + p_\nu q_\mu) + \frac{W_6}{M^2}(p_\mu q_\nu - p_\nu q_\mu) \qquad [7.105]$$

Since $L^{\mu\nu}$ is symmetric in (μ,ν) we have that the W_3 and W_6 terms do not contribute to $L^{\mu\nu} W_{\mu\nu}$. Now, the conservation of j^μ has impt consequences for the remaining terms:

Maxwell $\quad \partial_\nu F^{\mu\nu} = -e j^\mu$

$$\underbrace{\partial_\mu \partial_\nu}_{\text{sym}} \underbrace{F^{\mu\nu}}_{\text{antisym}} = 0 = -e\, \partial \cdot j \Rightarrow \partial \cdot j = 0 \qquad [7.106]$$

Take FT:

$$\int d^4x\, e^{-iq\cdot x}\, \partial\cdot j(x) = 0 \overset{\text{parts}}{=} i q^\mu \int d^4x\, e^{-iq\cdot x} j_\mu(x)$$

$$= i\, q\cdot j(q) \Rightarrow \qquad q\cdot j(q) = 0 \qquad [7.107]$$

Since $W_{\mu\nu} \sim \langle p s_i | j^\dagger_\mu(q) | X\rangle\langle X | j_\nu(q) | p, s_i\rangle$
we must have

$$q^\mu W_{\mu\nu} = 0 = q^\nu W_{\mu\nu} \qquad [7.108]$$

From this it is straight forward to relate W_4 & W_5 to $W_{1,2}$ $\Rightarrow$

$$W_{\mu\nu} = W_1\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + \frac{W_2}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) \qquad [7.109]$$

The W_j are called "Structure Functions"

I now will leave it as a HW exercise to show that [7.109] + [7.103] + [7.98] $\Rightarrow$

$$W^{(EM)} = W_2 + 2\tan^2\frac{\theta}{2}\, W_1 \qquad (7.110)$$

For the case of elastic scattering we see that

$$W_1 = \frac{1}{f_{REC}} \times \tau \times G_M^2 \times \delta(\nu + q^2/2M) \leftarrow \text{see } [7.36] \text{ ff} \qquad [7.111]$$

$$W_2 = \frac{1}{f_{REC}}\, \frac{G_E^2 + \tau G_M^2}{1+\tau} \times \delta(\nu + q^2/2M)$$

VII. C. 4. QUARK PARTON MODEL

Experimentally, it was observed that for inclusive scattering in the regime of very large $|q|^2$ and energy transfer, structure became independent of $|q|^2$ and ν separately and depend only on their ratio. Defining

$$Q^2 = -q^2 \qquad [7.112]$$

we characterize this ratio in terms of "Bjorken x"

$$x_B = Q^2/2M\nu = -q^2/2p\cdot q \qquad [7.113]$$

The interpretation of this observation can be easily seen from [7.111]. It is conventional to factor the 1/frec out of the defn of the elastic structure function. In this case

$$f_{rec}(2M)W_1^{el} = \frac{Q^2}{2M} G_M^2(Q^2)\,\delta(\nu - Q^2/2M)$$

$$= \frac{Q^2}{2M\nu} G_M(Q^2)\,\delta(1 - Q^2/2M\nu) \qquad [7.114a]$$

$$f_{rec}(\nu)W_2^{el} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\right)\delta(1 - Q^2/2M\nu) \qquad [7.114b]$$

For structureless targets, $G_E = G_M = 1$ & we'll get

$$\text{frec}\left(2MW_1^{el}\right)_{\text{Point-like}} = \frac{Q^2}{2M\nu}\,\delta\left(1 - Q^2/2M\nu\right) \qquad [7.115a]$$

$$\text{frec}\left(\nu W_2^{el}\right)_{\text{Point}} = \delta\left(1 - Q^2/2M\nu\right) \qquad [7.115b]$$

now define

$$F_1(\nu, Q^2) = \text{frec}\left(MW_1\right) \qquad [7.116a]$$

$$F_2(\nu, Q^2) = \text{frec}\left(\nu W_2\right) \qquad [7.116b]$$

Then we see that for pointlike scattering

$$F_2(\nu, Q^2) \rightarrow F_2(x_B) = 2x_B F_1(x_B) \qquad [7.117]$$

"<u>CALLAN GROSS RELATION</u>"

This observation provides the motivation for the quark-parton model, which says the structure functions in the <u>scaling limit</u>

$Q^2 \rightarrow$ LARGE
$\nu \rightarrow$ LARGE
x_B FIXED

involve an incoherent sum of contributions from pointlike constituents = the partons:

$$F_2(x_B) = \sum_i q_i^2 \, x_B \, f_i(x_B) \qquad [7.118a]$$

$$F_1(x_B) = \frac{1}{2} \sum_i q_i^2 \, f_i(x_B) \qquad [7.118b]$$

where "i" denotes any of the partons of charge q_i and

$f_i(x_B)$ = probability of finding a parton "i" at the particular value of x_B.
= "PARTON DISTRIBUTION FUNCTIONS"

Now, let's interpret x_B. Introduce another variable X = the fraction of energy & momentum of the target carried by a parton in a frame where the target is moving at ≈ "infinite momentum" (called the "infinite momentum frame", IMF):

In this frame, protons & partons have no transverse momentum:

	Proton	Parton
Energy	E	xE
Momentum	$P_T = 0$	0
	p_L	$x p_L$
Mass	M	$\left[(xE)^2 - (xp_L)^2\right]^{1/2} = xM \equiv m$
		(an effective mass)

Now, the structure function for an individual parton is

$$F_1^{(i)}(\nu,Q^2) = \frac{Q^2}{4m\nu}\,\delta(1-Q^2/2m\nu)$$

$$= \frac{1}{x}\,\frac{Q^2}{4M\nu}\,\delta(1-Q^2/2Mx\nu)$$

$$= (x_B/2x)\,\delta(1-x_B/x) \qquad [7.119a]$$

$$F_2^{(i)} = \delta(1-x_B/x) \qquad [7.119b]$$

IE, x_B = fraction of proton's p^μ in IMF carried by the parton!

As your text discusses, the kinematics in the above table only make sense in the IMF. In this frame, relativistic time dilation slows down the rate for parton-parton interactions, so that we can think of the scattering as resulting from an incoherent sum of scattering from individual partons. In this case, the differential cross section is

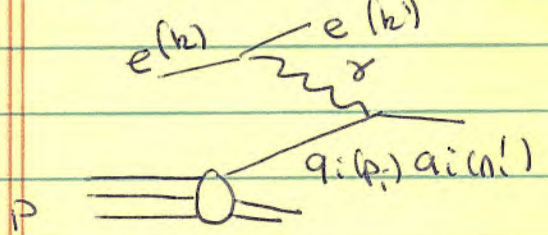
$$d\sigma(ep\to eX) = \sum_i \int dx_B\, f_i(x_B)\, d\hat{\sigma}(eq_i\to eq_i) \qquad [7.120]$$

where q_i denotes parton q, ith species (suggestively "q" for quark) and where we integrate over x_B w/ the constraint that

$$\sum_i \int dx_B \, f_i(x_B) \, x_B = 1 \qquad [7.121]$$

gives the total fraction of the proton momentum carried by all the partons (charged or neutral).

In the future, it will be useful to consider "partonic subprocesses": $e q_i \to e q_i$



It is helpful to relate the Mandelstaam variables (s, t, u) to those for the subprocess:

Neglecting masses:

$$\left.\begin{aligned} \hat{s} &= 2k\cdot p_i = 2x_B\, k\cdot P = x_B\, s \\ \hat{u} &= -2k'\cdot p_i = -2x_B\, k'\cdot P = x_B\, u \\ \hat{t} &= -2k\cdot k' = t \end{aligned}\right\} \quad [7.122]$$

VII. C. 4. A. QUARKS, GLUONS & PARTONS

The quark-parton model must be consistent w/ the Q.M. picture of the nucleon, that is:

$$\int_0^1 dx_B \left[u(x_B) - \bar{u}(x_B) \right] = 2$$

$$\int_0^1 dx_B \left[d(x_B) - \bar{d}(x_B) \right] = 1 \qquad [7.123]$$

$$\int_0^1 dx_B \left[s(x_B) - \bar{s}(x_B) \right] = 0 \qquad \text{etc.}$$

IE, the proton has two "valence" up quarks & one "valence" down quark, and no valence sea quarks. We call

$$u_v(x) \equiv u(x) - \bar{u}(x) \qquad [7.124]$$

$$d_v(x) \equiv d(x) - \bar{d}(x)$$

etc

the valence quark distributions, so that

$$u(x) = u_v(x) + \bar{u}(x) \qquad [7.125]$$

with $\bar{u}(x)$ denoting "sea quarks". Intuitively, the picture is

$\Rightarrow u_{sea}(x) = \bar{u}_{sea}(x)$

It turns out the probability for finding sea quarks grows at low x. One can isolate the valence quarks by considering the combination

$$F_2^{ep}(x) - F_2^{en}(x)$$

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2\left[u^p(x) + \bar{u}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[d^p(x) + \bar{d}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[s^p(x) + \bar{s}^p(x)\right] + \cdots \qquad [7.126a]$$

$$\frac{1}{x}F_2^{en} = \left(\frac{2}{3}\right)^2\left[u^n + \bar{u}^n\right] + \left(\frac{1}{3}\right)^2\left[d^n(x) + \bar{d}^n(x)\right] + \left(\frac{1}{3}\right)^2\left[s^n(x) + \bar{s}^n(x)\right] \qquad [7.126b]$$

If we now assume charge symmetry

$$\begin{aligned} u^p(x) &= d^n(x) & \bar{u}^p &= \bar{d}^n \\ d^p(x) &= u^n(x) & \bar{d}^p &= \bar{u}^n \qquad [7.127] \\ s^p(x) &= s^n(x) \cdots \end{aligned}$$

$$\frac{1}{x}F_2^{en} = \left(\frac{2}{3}\right)^2\left[d^p + \bar{d}^n\right] + \left(\frac{1}{3}\right)^2\left[u^p + \bar{u}^n\right] + \left(\frac{1}{3}\right)^2\left[s^n + \bar{s}^n\right] + \cdots$$

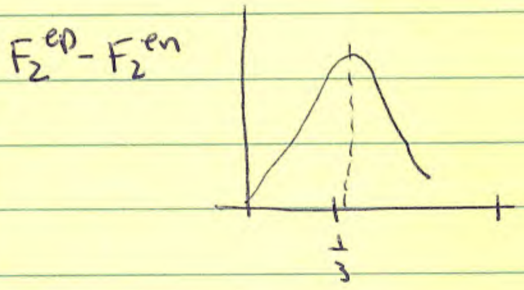
$$\Rightarrow \frac{1}{x}\left[F_2^{en} - F_2^{en}\right] = \frac{1}{3}\left[u^n + \bar{u}^p - \bar{d}^p - d^p\right]$$

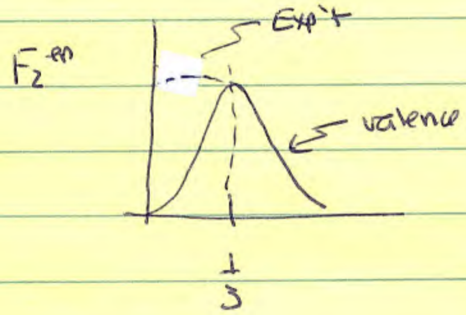
$$= \frac{1}{3}\left[u_v^p - d_v^p\right] + \frac{2}{3}\left[u_{sea}^n - d_{sea}^n\right] \qquad [7.128]$$

And assuming the light quark sea is flavor independent symmetric, $u^p_{sea} - d^p_{sea} \cong 0 \Rightarrow$

$$F_2^{ep} - F_2^{en} \cong \frac{1}{3} x \left[u_v^p - d_v^p \right] \qquad [7.129]$$

Experimentally, one finds that this difference is peaked around $x_B \cong 1/3$. (see Fig 9.8).





whereas for F_2^{ep}, one sees growth at low-x_B → due to the sea.

Now consider how much each species of parton contributes to the total proton momentum.

Let

$$\varepsilon_q = \int_0^1 dx\, x \,(q + \bar{q}) \qquad [7.130)$$

$$\int_0^1 dx\, F_2^{ep}(x) = \frac{4}{9}\varepsilon_u + \frac{1}{9}\varepsilon_d + \frac{1}{9}\varepsilon_s + \cdots = 0.18 \qquad [7.131a]$$

$$\int_0^1 dx\, F_2^{en}(x) = \frac{1}{9}\varepsilon_u + \frac{1}{9}\varepsilon_d + \frac{1}{9}\varepsilon_s + \cdots = 0.12 \qquad [7.131b]$$

Other expts (eg, $\nu p \to \mu X$) tells us ε_s, ε_c are very small $\Rightarrow$

We can solve for $\mathcal{E}_{u,d}$ ⇒

$$\mathcal{E}_u = 0.36$$
$$\mathcal{E}_d = 0.18$$

∴ $\mathcal{E}_{quark} \cong 0.54$, 54% of proton momentum carried by quarks; hence gluons carry the other 46%

$$\mathcal{E}_g \doteq 1 - \mathcal{E}_u - \mathcal{E}_d \approx 0.46 \qquad [7.132)$$

⇒ SOME PLOTS

SCALING, $\nu N \to \mu X$ & s-quarks...

VII. D. HADRON-HADRON SCATTERING

Thus far, we have concentrated on lepton-lepton & lepton-hadron scattering. For the high-energy physics at the LHC (and earlier the Tevatron), the parton model gives us a way to predict & interpret cross sections. It is based on the idea of incoherent scattering of partons: consider

$p\,p \to A + B + X$: