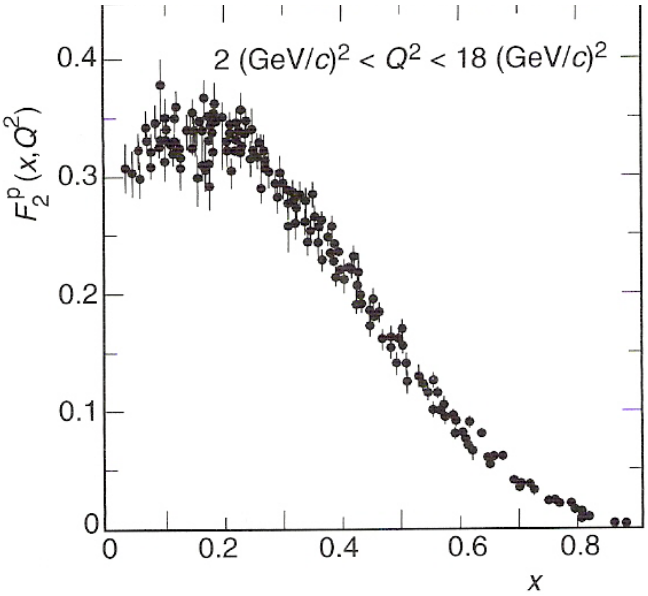
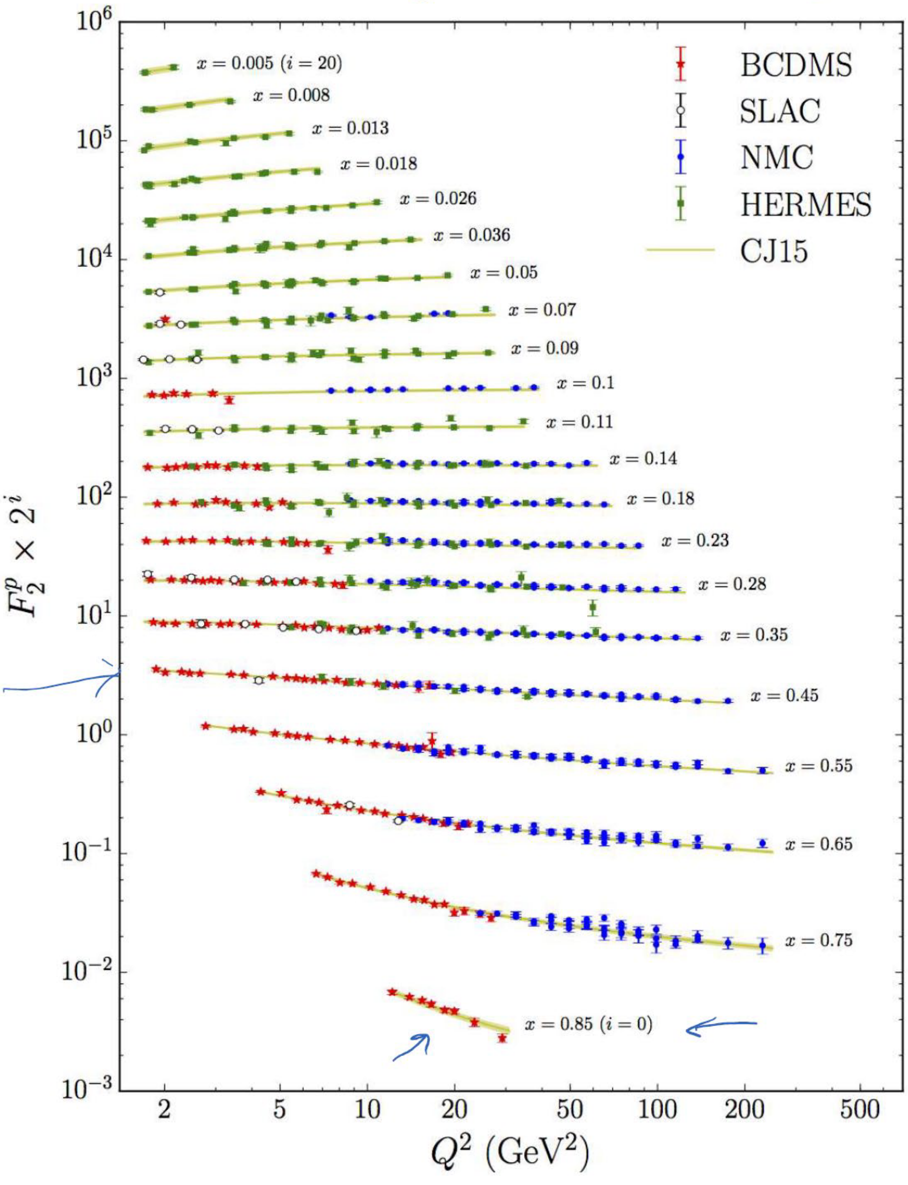


Data largely independent of Q^2 !!



$F_2 = \nu W_2$

$x = \dfrac{Q^2}{2M\nu}$

Bjorken x

$W > 2$ GeV

$W_p \simeq 0.94$ GeV

Ph 203: L10

Constituent Quark Model

⇒ Building nucleons & resonances (mesons also possible)

"Observation" of quarks via DIS showed quarks were more than Math. construct

⇔ Try non-rel. "shell" Model of 3 quarks in nucleon:

	Q	t_3
u quark	$+2/3$	$1/2$
d quark	$-1/3$	$-1/2$

Assume 1st only Constituent quarks w/ $m_q \simeq \frac{M_N}{3}$ in Non. Rel. potential (NRCQM)

⇔ will discuss "current" quarks ($m_q \sim 5$ MeV) later

but note $\langle p \rangle \simeq \Delta p \sim \frac{\hbar}{\Delta x} \simeq 250$ MeV/c for $\Delta x \sim 0.8$ fm

somewhat relativistic, but

Note: NRCQM works surprisingly well   will check later

↳ mass splittings, mag moments, couplings, ...

But 1st

Much of the model is built on the structure of Simple Unitary Groups ⇒ SU(N)

Combining SU(N) ⇒ Focus on 3 particles (quarks)

Want to build wave func. + identify states:

e.g. $\psi_{tot} = \psi_{space} \underbrace{\chi_{spin} \phi_{isospin} \theta_{other}}_{\text{need } SU(N) \Rightarrow \text{ e.g. spin } \frac{1}{2} = SU(2)}$

$\therefore$ 3 spin $\frac{1}{2}$ particles ⇒ $SU(2) \otimes SU(2) \otimes SU(2)$

$= [SU(2)]^3$ ← short hand

Note: In general, if combining 3 particles in SU(N), will find N^3 states w different exchange symmetries

$\Rightarrow \psi_{tot}$ must be antisym. for exch. of any pair

Simple Ex: 3 spin $\frac{1}{2} \Rightarrow N=2$ ∴ 8 states

Use brute force 1st, then learn tricks

Begin combining 2 part. $|JM\rangle = |\uparrow\uparrow\rangle$

$\left.\begin{array}{l} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |\downarrow\downarrow\rangle \end{array}\right\} J=1$

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$; $J=0$

Now add 3rd spin after above (use C.G. coeff.)

1. add 3rd $|\frac{1}{2}\rangle$ to $J=1$ to make $J=\frac{3}{2} \Rightarrow$ 4 states

$|\frac{3}{2}\frac{3}{2}\rangle = |\uparrow\uparrow\uparrow\rangle$

$|\frac{3}{2}\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}|\uparrow\uparrow\downarrow\rangle + \sqrt{\frac{2}{3}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)\frac{1}{\sqrt{2}}$

$|\frac{3}{2}\frac{-1}{2}\rangle = \frac{1}{\sqrt{3}}|\uparrow\downarrow\downarrow\rangle + \frac{1}{\sqrt{3}}|\downarrow\uparrow\downarrow\rangle + \frac{1}{\sqrt{3}}|\downarrow\downarrow\uparrow\rangle$

$|\frac{3}{2}\frac{-3}{2}\rangle = |\downarrow\downarrow\downarrow\rangle$

$\Leftrightarrow$ all are sym w.r.t. exch. of any pair (S)

2. add 3rd $|\frac{1}{2}\rangle$ to $J=1$ to make $J=\frac{1}{2} \Rightarrow$ 2 states

$|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|\uparrow\uparrow\downarrow\rangle - \frac{1}{\sqrt{6}}|\uparrow\downarrow\uparrow\rangle - \frac{1}{\sqrt{6}}|\downarrow\uparrow\uparrow\rangle$

$|\frac{1}{2}\frac{-1}{2}\rangle = \sqrt{\frac{1}{6}}|\uparrow\downarrow\downarrow\rangle + \frac{1}{\sqrt{6}}|\downarrow\uparrow\downarrow\rangle - \sqrt{\frac{2}{3}}|\downarrow\downarrow\uparrow\rangle$

$\Leftrightarrow$ No pure exchange sym. $\Rightarrow$ but sym w.r.t. 1st 2 part exch.

$\Leftrightarrow$ called Mixed sym symmetric $= M_S$

3. add 3rd $|\frac{1}{2}\rangle$ to $J=0$ to make $J=\frac{1}{2} \Rightarrow$ 2 states

$|\frac{1}{2}\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\uparrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\uparrow\rangle$

$|\frac{1}{2}\frac{-1}{2}\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\downarrow\rangle$

Antisym. w.r.t exch. of only 1st 2 part.

$\Leftrightarrow$ called Mixed sym antisym $= M_A$

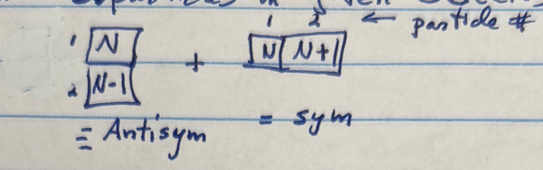
Bottom line = $[SU(2)]^3 = 4_S \oplus 2_{M_S} \oplus 2_{M_A}$

⇒ How to do this for arbitrary $[SU(N)]^3$?

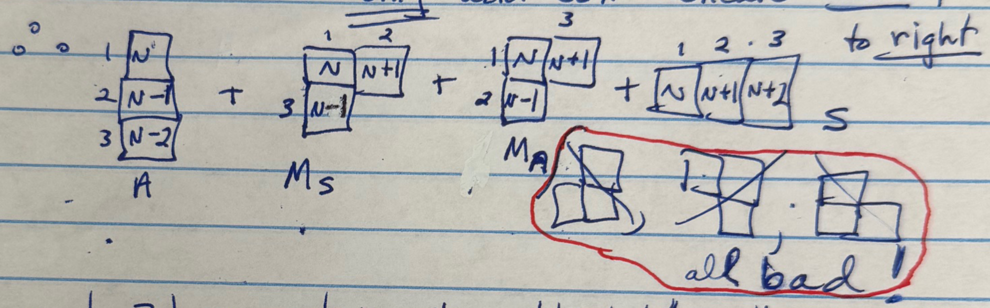
⇒ Use Young Tableau tricks/rules:

Each $SU(N)$ (e.g. spin $\frac{1}{2}$, isospin $\frac{1}{2}$, ...) for 1 particle = □ (box!)

∴ to combine 2 particles in given $SU(N)$ use 2 boxes



& for 3 particles can only add box "concave" down & to right



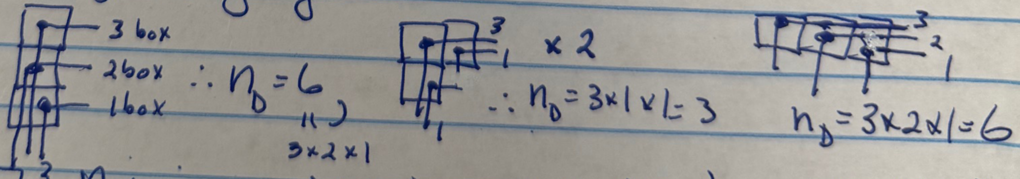
& Now each 3 box set is a "multiplet" with specific exch. sym (S, M_S, M_A, A) & # of states given by

$\frac{n_N}{n_D}$ ← num. ← Den ⇒ where $n_N \equiv$ product of box contents [e.g. $N(N-1)\cdots$]

$n_D \equiv$ product of "hook" numbers

Define hooks via how many boxes are crossed starting from right & going down

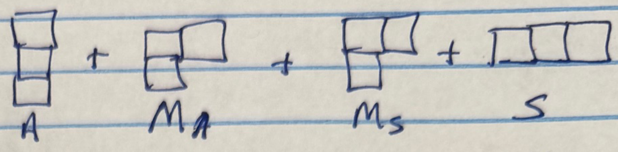
e.g.



$$\therefore [SU(N)]^3 = \frac{n_N}{n_D} = \frac{N(N-1)(N-2)}{6} + \frac{N(N+1)(N-1)}{3} + \frac{N(N-1)(N+1)}{3} + \frac{N(N+1)(N+2)}{6}$$

& exch. sym of state given by box shapes:

e.g. $[SU(2)]^3 =$



$$\frac{n_N}{n_D}: 0 + 2_{M_A} + 2M_S + 4S$$

↑ num = 0

& For future Reference:

$$[SU(3)]^3 = 1_A + 8_{M_A} + 8_{M_S} + 10_S$$
$$[SU(4)]^3 = 4_A + 20M_A + 20M_S + 20_S$$
$$[SU(6)]^3 = 20_A + 70M_A + 70M_S + 56_S$$

Note: M_A & M_S are not useless (see later)

e.g. given 3 spin $\frac{1}{2}$ particles: $[SU(2)]^3 = 4_S + 2M_S + 2M_A$

but combining w/ 3 isospin $\frac{1}{2}$ gives $[SU(4)]^3$

since $SU(2) \otimes SU(2) = SU(4)$

but $[SU(4)]^3$ gives

$$4_A + 20_{M_A} + 20_{M_S} + 20_S \quad \text{(above)}$$

↑ How to get 20_S?

who cares?

see later ⇒

We need more info ⇒ θ_{other}?

<u>Building Nucleon from</u> 3 u/d constituent quarks

$$\Psi_{tot} = \Psi_{sp}\, \chi_{spin}\, \phi_{isospin}\, \theta_{color}$$

χ_{spin}: $SU(2)$

$\phi_{isospin}$: $SU(2)$ or $SU(3)$ for uds "flavor"

θ_{color}: ↳ $\begin{pmatrix} R \\ G \\ B \end{pmatrix} \Rightarrow SU(3)$

↳ quarks possess color charge but free quarks are unobserved ∴ form "colorless" state

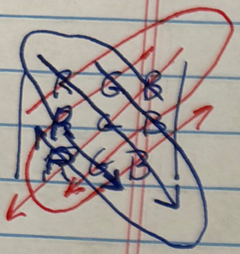
How? => In analogy w Spin $\frac{1}{2}$ for 2 particles

Spin 0 => $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

assume Θ_c => Anti sym. states are colorless

Θ_c from $[SU(3)_c]^3 = 1_A + 8_{M_A} + 8_{M_S} + 10_S$

↑ check → $\left\{ \begin{array}{|c|} \hline N \\ \hline N-1 \\ \hline N-2 \\ \hline \end{array} \right.$ $\frac{3\times2\times1}{3\times2\times1} = \frac{n_N}{h_D} = 1$ ✓



∴ only possible state (slater Det.)

$$\Theta_c = \frac{1}{\sqrt{6}}(RGB + GBR + BRG - BGR - RBG - GRB)$$

& ∴ must have $\psi_{sp}\,\chi_{spin}\,\Phi_{isospin}$ = Sym. wrt exchange (diff. from nucleus) e.g. ^3He

Check ψ_{sp} gndstate is sym via S.H. Oscill. (see notes)

$$H_{SHO} = \frac{1}{2m_q}(\vec{P}_1^{\,2} + \vec{P}_2^{\,2} + \vec{P}_3^{\,2}) + \frac{k}{2}\sum_{i\neq j}(\vec{r}_i - \vec{r}_j)^2$$

← spring const.

↪ 3-body problem => above causes CM motion to produce spurious states

↪ Use Jacobi coords:

$$\vec{R}_{cm} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3};\quad \vec{\rho} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}};\quad \vec{\lambda} = \frac{(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)}{\sqrt{6}};\quad M = 3m_q$$

then $\vec{P}_\rho = m_q\dot{\vec{\rho}}$, $\vec{P}_\lambda = m_q\dot{\vec{\lambda}}$; $P_{cm} = M\dot{\vec{R}}_{cm}$

$$\& \; H_{SHO} = \left(\frac{\vec{P}_\rho^{\,2}}{2m_q} + \frac{3}{2}k\rho^2\right) + \left(\frac{\vec{P}_\lambda^{\,2}}{2m_q} + \frac{3}{2}k\lambda^2\right) + \frac{P_{cm}^2}{2M}$$

⟺ 2 indep. 3D SHO plus CM motion

⟺ ignoring CM motion kills spurious states

Gives: $\psi_{space} = R_{n_\rho \ell_\rho}(\rho) Y_{\ell_\rho m_\rho} R_{n_\lambda \ell_\lambda}(\lambda) Y_{\ell_\lambda m_\lambda}$

& $E_N = (N + \frac{3}{2})\hbar\omega, \quad \omega = \sqrt{\frac{3k}{m_q}}$

$N = 2n_\rho + \ell_\rho + 2n_\lambda + \ell_\lambda \longrightarrow \alpha_0^4 = 3m_q k$

gnd. state & W.F.

$\Rightarrow \psi_{sp}^{g.s}(\rho, \lambda) = 4\pi \left(\frac{\alpha_0^{3/2}}{\pi^{3/4}}\right)^2 e^{-\alpha_0^2(\rho^2+\lambda^2)/2} \left(Y_{00}\right)^2$

& $\rho^2 + \lambda^2 = \frac{(\vec{r}_1 - \vec{r}_2)^2}{3} + \frac{(\vec{r}_2 - \vec{r}_3)^2}{3} + \frac{(\vec{r}_1 - \vec{r}_3)^2}{3}$

$\therefore$ for $N = 0$; $\psi_{sp}^{g.s.}$ is sym.

$\therefore$ must have $\chi_{spin}\, \phi_{iso} \Rightarrow$ sym.

W.F. next time