Ph 203 L11

Nucleon W.F. in NRCQM

w H_{SHO} $\psi_{spac} = \text{sym}$ w.r.t exchange of any pair

QCD says $\theta_c = \text{Antisym}$

$\therefore$ Spin x Isospin: $\chi_{spin}\phi_{iso} \Rightarrow \text{sym},$

$\hookrightarrow$ Both are $SU(2)$

then for 3 quarks $\Rightarrow [SU(2)]^3 = 4_s \oplus 2M_s + 2M_A$

but can consider $S = \frac{1}{2}, T = \frac{1}{2}$ as quadruplet $\begin{pmatrix} \uparrow \\ \downarrow \\ u \\ d \end{pmatrix}$

$SU(4) \Leftarrow$

& 3 quarks gives, from last lect. $[SU(4)]^3 = 4_A + 20M_S + 20M_A + 20_S$

$\hookrightarrow 4^3 = 64$ states

$\Rightarrow$ Must use 20_S, but what are 20_S states?

Recall for spin in $[SU(2)]^3$ got 4 sym states

w $S = \frac{3}{2}$ & 2 $S = \frac{1}{2}$ M_s & 2 $S = \frac{1}{2}$ M_A

$\therefore$ 4 sym $S = \frac{3}{2}$, $T = \frac{3}{2}$ gives 16 sym. states

What are these? $(2S+1)(2T+1)$

$\Leftrightarrow$ $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$ all in $|S = \frac{3}{2}\ M_S = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\rangle$

$\quad$ uuu, uud, udd, ddd

4 states left: $|p\,{\uparrow\atop\downarrow}\rangle, |n\,{\uparrow\atop\downarrow}\rangle$!

$\Leftrightarrow$ Must build from $2M_s$ & $2M_A$ states

Try (after staring...) $\rightarrow$ can check sym @ end

$$\chi_{spin}\phi_{iso} = \frac{1}{\sqrt{2}}\left(\chi_{M_S}\phi_{M_S} + \chi_{M_A}\phi_{M_A}\right)$$

then

$$|p\uparrow\rangle = |s_3 = +\tfrac{1}{2}, t_3 = +\tfrac{1}{2}\rangle$$

$$= \frac{1}{\sqrt{2}}\Big[\Big(\sqrt{\tfrac{2}{3}}|\uparrow\uparrow\downarrow\rangle - \tfrac{1}{\sqrt{6}}|\uparrow\downarrow\uparrow\rangle - \tfrac{1}{\sqrt{6}}|\downarrow\uparrow\uparrow\rangle\Big)\Big(\sqrt{\tfrac{2}{3}}|uud\rangle - \tfrac{1}{\sqrt{6}}|udu\rangle - \tfrac{1}{\sqrt{6}}|duu\rangle\Big)$$

$$+ \Big(\tfrac{1}{\sqrt{2}}|\uparrow\downarrow\uparrow\rangle - \tfrac{1}{\sqrt{2}}|\downarrow\uparrow\uparrow\rangle\Big)\Big(\tfrac{1}{\sqrt{2}}|udu\rangle - \tfrac{1}{\sqrt{2}}|duu\rangle\Big)\Big]$$

giving:

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\big(2|u\uparrow u\uparrow d\downarrow\rangle + 2|u\uparrow d\downarrow u\uparrow\rangle + 2|d\downarrow u\uparrow u\uparrow\rangle$$
$$- |u\downarrow u\uparrow d\uparrow\rangle - |u\uparrow d\uparrow u\downarrow\rangle - |d\uparrow u\downarrow u\uparrow\rangle$$
$$- |u\uparrow u\downarrow d\uparrow\rangle - |u\downarrow d\uparrow u\uparrow\rangle - |d\uparrow u\uparrow u\downarrow\rangle\big)$$

then for $|n\uparrow\rangle$ use $|p\uparrow\rangle$ w $u\to d$ & $d\to u$
assuming isospin symmetry

$\Rightarrow$ can use these W.F. to calculate observables
e.g. μ_n, μ_p, Q_n Q_p, $g_A \Rightarrow$ nucleon's axial vector coupling constant
measured in n β decay (see later)

Useful tricks:

① Since χ_{spin} & iso in gnd state are sym.,
arbitrary matrix elements over quarks
(e.g. operator $\hat{\Theta}_i$; $i = 1^{st}, 2^{nd}, 3^{rd}$ quark

$$\langle\hat{\Theta}\rangle = \langle \sum_{i\ \text{quarks}} \hat{\Theta}_i \rangle = 3 \times \langle \hat{\Theta}_{1^{st}} \rangle$$

quark operator
since for sym. W.F.

② Alternatively can simplify $|p\uparrow\rangle$ to

$$|p\uparrow\rangle = \frac{1}{\sqrt{6}}\big(2|u\uparrow u\uparrow d\downarrow\rangle - |u\downarrow u\uparrow d\uparrow\rangle - |u\uparrow u\downarrow d\uparrow\rangle\big)$$

$\Leftrightarrow$ suppressing the permutations

but now

$$\langle\hat{\Theta}\rangle = \langle \sum_{i\ \text{quarks}} \hat{\Theta}_i \rangle \Rightarrow$$

Bhaduri uses ①
I'll use ②: lets evaluate $\langle\hat{Q}\rangle$ where

$$\hat{Q}u = \left(\tfrac{2e}{3}\right)u,\quad \hat{Q}d = \left(-\tfrac{e}{3}\right)d$$

then:

$$\langle Q_p \rangle = \langle \sum_i Q_i \rangle = \sum_i \langle p\uparrow | \hat{Q}_i p\uparrow \rangle$$

$$= \frac{1}{6}\Big[\overset{4(e)}{4\left(\tfrac{2e}{3}+\tfrac{2e}{3}-\tfrac{e}{3}\right)} + \overset{(e)}{\left(\tfrac{2e}{3}+\tfrac{2e}{3}-\tfrac{e}{3}\right)} + \overset{(e)}{\left(\tfrac{2e}{3}+\tfrac{2e}{3}-\tfrac{e}{3}\right)}\Big]$$

$$= \underline{\underline{e}} \checkmark$$

$$\langle Q_n \rangle = \frac{1}{6}\Big[\overset{4(0)}{4\left(-\tfrac{e}{3}-\tfrac{e}{3}+\tfrac{2e}{3}\right)} + \overset{0}{\left(-\tfrac{e}{3}-\tfrac{e}{3}+\tfrac{2e}{3}\right)} + \overset{0}{0}\Big] = 0 \checkmark$$

W.F. also does reasonably well for μ_n, μ_p (H.W.)
(5-7%)
where, e.g.

$$\mu_p \equiv \langle \hat{\mu} \rangle = \langle p\uparrow | \sum_i \mu_i \hat{\sigma}_{3_i} p\uparrow \rangle$$

recall $\hat{\mu}$ always measured in "stretched" state $\leftrightarrow$ max M_J

Note: $\mu_u = \frac{2e/3}{2m_q}$, $\mu_d = \frac{-e/3}{2m_q}$ ← Dirac Moments ($\hbar = 1$)

also

$$\hat{\sigma}_3 |q\uparrow\rangle = |q\uparrow\rangle, \quad \hat{\sigma}_3 |q\downarrow\rangle = -|q\downarrow\rangle$$

How about nucleon's axial vector coupling constant?

g_A => part of weak-interaction
=> measured in neutron β-decay or ν-N scattering

UCNA: $g_A = 1.2772(20)$
Kellogg grad student thesis
world's best at time

non-rel. approx.

$$g_A \equiv \langle p\uparrow | \sum_i \hat{\sigma}_{3_i} \hat{\tau}_{3_i} p\uparrow \rangle; \quad \hat{\tau}_3 |u\rangle = |u\rangle, \quad \hat{\tau}_3 |d\rangle = -|d\rangle$$

$$\therefore g_A = \frac{1}{6}\left[4(1+1+1) + (-1+1-1) + (1-1-1)\right] = \frac{1}{6}(12-2)$$

$$= 5/3 = 1.66\ldots \text{ off by } \underline{\underline{30\%}} \text{ (see later)}$$

Can use W.F. to consider additional terms beyond H_{SHO} ...

e.g., $\hat{S}_i \cdot \hat{S}_j$ can explain why $M_\Delta > M_N$

particle #

see PDG tables

Nucleon “Excitations” (a.k.a. Resonances)

“Just a bunch of messy resonances”

Particle	J^P	overall	Status as seen in $N\gamma$	$N\pi$	$\Delta\pi$	$N\sigma$	$N\eta$	ΛK	ΣK	$N\rho$	$N\omega$	$N\eta\prime$
N	$1/2^+$	****										
$N(1440)$	$1/2^+$	****	****	****	****	***						
$N(1520)$	$3/2^-$	****	****	****	****	**	****					
$N(1535)$	$1/2^-$	****	****	****	***	*	****					
$N(1650)$	$1/2^-$	****	****	****	***	*	****	*				
$N(1675)$	$5/2^-$	****	****	****	****	***	*	*	*			
$N(1680)$	$5/2^+$	****	****	****	****	***	*	*	*			
$N(1700)$	$3/2^-$	***	**	***	***	*	*			*		
$N(1710)$	$1/2^+$	****	****	****	*		***	**	*	*	*	
$N(1720)$	$3/2^+$	****	****	****	***	*	*	****	*	*	*	
$N(1860)$	$5/2^+$	**	*	**		*	*					
$N(1875)$	$3/2^-$	***	**	**	*	**	*	*	*	*	*	
$N(1880)$	$1/2^+$	***	**	*	**	*	*	**	**		**	
$N(1895)$	$1/2^-$	****	****	*	*	*	****	**	**	*	*	****
$N(1900)$	$3/2^+$	****	****	**	**	*	*	**	**		*	**
$N(1990)$	$7/2^+$	**	**	**			*	*	*			

* * **	Existence is certain.
* * *	Existence is very likely.
**	Evidence of existence is fair.
*	Evidence of existence is poor.

Particle	J^P	overall	Status as seen in $N\gamma$	$N\pi$	$\Delta\pi$	ΣK	$N\rho$	$\Delta\eta$
$\Delta(1232)$	$3/2^+$	****	****	****				
$\Delta(1600)$	$3/2^+$	****	****	***	****			
$\Delta(1620)$	$1/2^-$	****	****	****	****			
$\Delta(1700)$	$3/2^-$	****	****	****	****	*	*	
$\Delta(1750)$	$1/2^+$	*	*	*		*		
$\Delta(1900)$	$1/2^-$	***	***	***	*	**	*	
$\Delta(1905)$	$5/2^+$	****	****	****	**	*	*	**
$\Delta(1910)$	$1/2^+$	****	***	****	**	**		*
$\Delta(1920)$	$3/2^+$	***	***	***	***	**		**
$\Delta(1930)$	$5/2^-$	***	*	***	*	*		
$\Delta(1940)$	$3/2^-$	**	*	**	*			*
$\Delta(1950)$	$7/2^+$	****	****	****	**	***		

Relativistic Quark Model of Nucleon

using "current" quarks [$m_u \simeq 2.5$ MeV, $m_d \simeq 2 \times m_u$]

w $m_{u,d} << \Delta p = 250$ MeV

↳ Dirac Eq.

$$i \gamma_\mu \partial^\mu \psi = m \psi \quad \text{(free particle)}$$

w $x^\mu = (t, \vec{r})$; $\mu = 0 \to 3$

$$p^\mu = \left(i \frac{\partial}{\partial t}, -i \vec{\nabla}\right) = i \frac{\partial}{\partial x_\mu} \equiv i \partial^\mu$$

define:

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} \underset{\sim}{1} & \underset{\sim}{0} \\ \underset{\sim}{0} & -\underset{\sim}{1} \end{pmatrix}$$ where $\underset{\sim}{1}, \underset{\sim}{0}, \ldots$ are 2x2 matrix

$$\gamma^{1,2,3} = \begin{pmatrix} \underset{\sim}{0} & \underset{\sim}{\sigma}_{1,2,3} \\ \underset{\sim}{\sigma}_{1,2,3} & \underset{\sim}{0} \end{pmatrix}$$ ← Pauli Matrices

&

$$\gamma^5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} \underset{\sim}{0} & \underset{\sim}{1} \\ \underset{\sim}{1} & \underset{\sim}{0} \end{pmatrix}$$

∴ can write W.F. as 4-component spinor: $\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$ ← sorry, these are Not spin/isospin!

↳ each 2-comp. spinor

& Probab. current density includes:

Vector Current: $j_V^\mu = \bar{\psi} \gamma^\mu \psi$

↳ $\bar{\psi} \equiv \psi^\dagger \gamma^0$

Axial Vector Current: $j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$

Now consider quarks confined by central, scalar potential via

$$\left\{ \gamma^\mu p_\mu - [m_q + U(r)] \right\} \psi = 0$$

↳ simple spherical well:

$U(r) = 0, \; r < R$

$U(r) = U_0 \; r \geq R$

Note: $U_0 \to \infty$ = confinement

And

assume quarks are non-interacting but bound by $U(r)$

$\hookrightarrow$ this is opposite of NRCQM where quarks interact via springs

$\hookrightarrow$ We'll Solve Dirac Eq. next time...