Ph203 L14 ①

QCD: from Perturb. to Non-Pert.

Last Time: discussed Confinement & Asymptotic Freedom

Compare QED & QCD $\vec{E}/\vec{B}$ fields

QCD vacuum prevents $\vec{E}_c/\vec{B}_c$ leakage

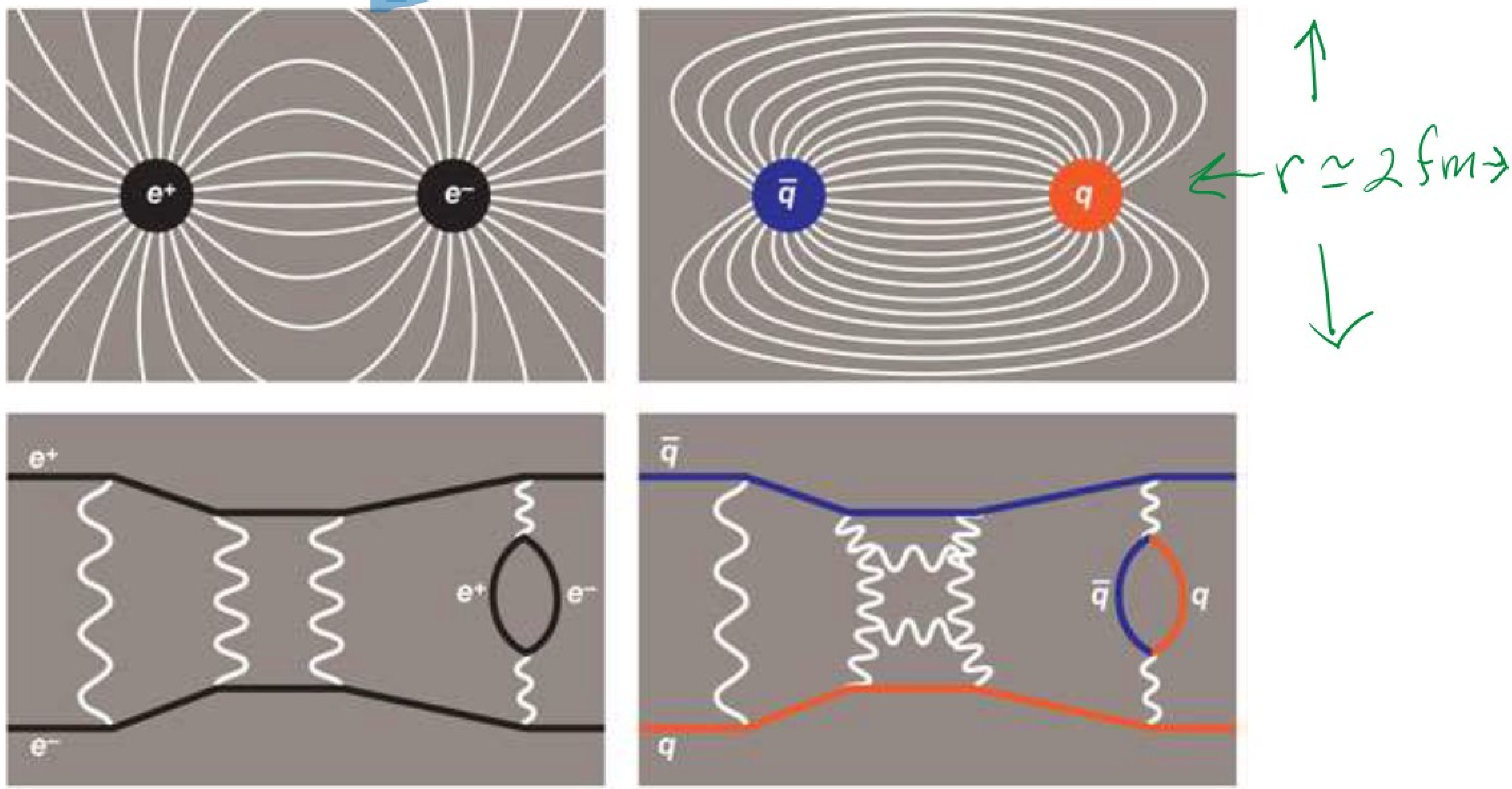
see pic

Approach to Non-Perturb.:

Start w QCD Lagrangian

see PDG → next page

QED vs QCD



E-field to $r = \infty$

E_c- field confined

9. Quantum Chromodynamics

Revised August 2023 by J. Huston (Michigan State U.), K. Rabbertz (KIT) and G. Zanderighi (MPI Munich).

quark-gluon interaction

gluon self-interaction

9.1 Basics

Quantum Chromodynamics (QCD), the gauge field theory that describes the strong interactions of colored quarks and gluons, is the SU(3) component of the SU(3)×SU(2)×U(1) Standard Model of Particle Physics. The Lagrangian of QCD is given by

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu}\,, \tag{9.1}$$

quarks

where repeated indices are summed over. The γ^μ are the Dirac γ-matrices. The $\psi_{q,a}$ are quark-field spinors for a quark of flavor q and mass m_q, with a color-index a that runs from $a = 1$ to $N_c = 3$, *i.e.* quarks come in three "colors." Quarks are said to be in the fundamental representation of the SU(3) color group.

The $\mathcal{A}^C_\mu$ correspond to the gluon fields, with C running from 1 to $N_c^2 - 1 = 8$, *i.e.* there are eight kinds of gluon. Gluons transform under the adjoint representation of the SU(3) color group. The t^C_{ab} correspond to eight 3×3 matrices and are the generators of the SU(3) group (*cf.* the section on "SU(3) isoscalar factors and representation matrices" in this *Review*, with $t^C_{ab} \equiv \lambda^C_{ab}/2$). They encode the fact that a gluon's interaction with a quark rotates the quark's color in SU(3) space. The quantity g_s (or $\alpha_s = \frac{g_s^2}{4\pi}$) is the QCD coupling constant. Besides quark masses, which have electroweak origin, it is the only fundamental parameter of QCD. Finally, the field tensor $F^A_{\mu\nu}$ is given by

$$\begin{aligned} F^A_{\mu\nu} &= \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu\,, \\ [t^A, t^B] &= i f_{ABC} t^C\,, \end{aligned} \tag{9.2}$$

where the f_{ABC} are the structure constants of the SU(3) group.

Neither quarks nor gluons are observed as free particles. Hadrons are color-singlet (*i.e.* color-neutral) combinations of quarks, anti-quarks, and gluons.

Lattice Gauge Theory (LGT)

Refs:

QCD: Greiner, Schramm & Stein (GSS)

Intro LGT: U. Weise CFT + Path Integral + Stat Mech → LGT

Overview : Kronfeld 2012

Kronfeld Quote!

I Basics:

⇒ Path Integral:

Calc. Observables (e.g $\langle \hat{\theta} \rangle$) via

action

$$\langle \hat{\theta} \rangle = \frac{1}{Z} \int DA_\mu \, D\psi \, D\bar{\psi} \, \hat{\theta} \, e^{-S}$$

differentials for g & q

$$S \equiv \int d^4x \, \mathcal{L}_{QCD}(x)$$

$$Z = \int DA_\mu \ldots e^{-S} \quad \text{(without } \hat{\theta}\text{)}$$

to normalize $\langle \hat{\theta} \rangle$

Why Lattice Gauge Theory?

Kronfeld - 2012

the total "vacuum angle" $\theta = 0$; chiral symmetries emerge when two or more quark masses vanish (1, 2); and heavy-quark symmetries are revealed as one or more quark masses go to infinity (3, 4). More remarkable still are the phenomena that emerge at a dynamically generated energy scale Λ_{QCD}, the "typical scale of QCD." Much of what is known about QCD in this nonperturbative regime has long been based on belief. Evidence from high-energy scattering fostered the opinion that QCD explains the strong interactions and, therefore, the belief that QCD exhibits certain properties; otherwise, it would not be consistent with lower-energy observations. These emergent phenomena—such as chiral symmetry breaking, the generation of hadron masses that are much larger than the quark masses, and the thermodynamic phase structure—are the most profound phenomena of gauge theories. The primary aim of this review is to survey how lattice QCD has enabled us to replace beliefs with knowledge. To do so, we cover results that are interesting in their own right, influential in a wider arena, qualitatively noteworthy, and/or quantitatively impressive.

The rest of this article is organized as follows. Section 2 introduces the QCD

$\Lambda_{QCD} \sim 150\,\text{MeV}$

$E \gg \Lambda_{QCD}$

Perturbative

$E \lesssim \Lambda_{QCD}$

Non-Perturb.

⇒ Euclidean Space-Time to start

x_μ ; $\mu = 1-4$: $x_4 \equiv \tau = it$

↳ convenient for M.C. integration

so that QM time propagation via Hamiltonian :

$$\psi(t) = e^{-iHt}\psi(0)$$

becomes: $\psi(\tau) = e^{-H\tau}\psi(0)$

w τ = Boltzman factor

$$\tau = \frac{1}{kT}$$

(II) How to code it ?

① Discretize space-time $\Delta x_i \simeq 0.05$ fm, Δt variable
$i = 1-3$

try $\underset{x_i}{64^3}, \underset{\tau}{\times 128}$ w $a = \Delta x_i$

⟺ τ dependence imp. to stabilize non-t dep. observables m_q, g_A, α_s, ...

② Discretize Action ⇒ $\int \mathcal{L}\, d^4x \equiv \sum_{x_i, \tau} \mathcal{L}_{\text{Discrete}}$

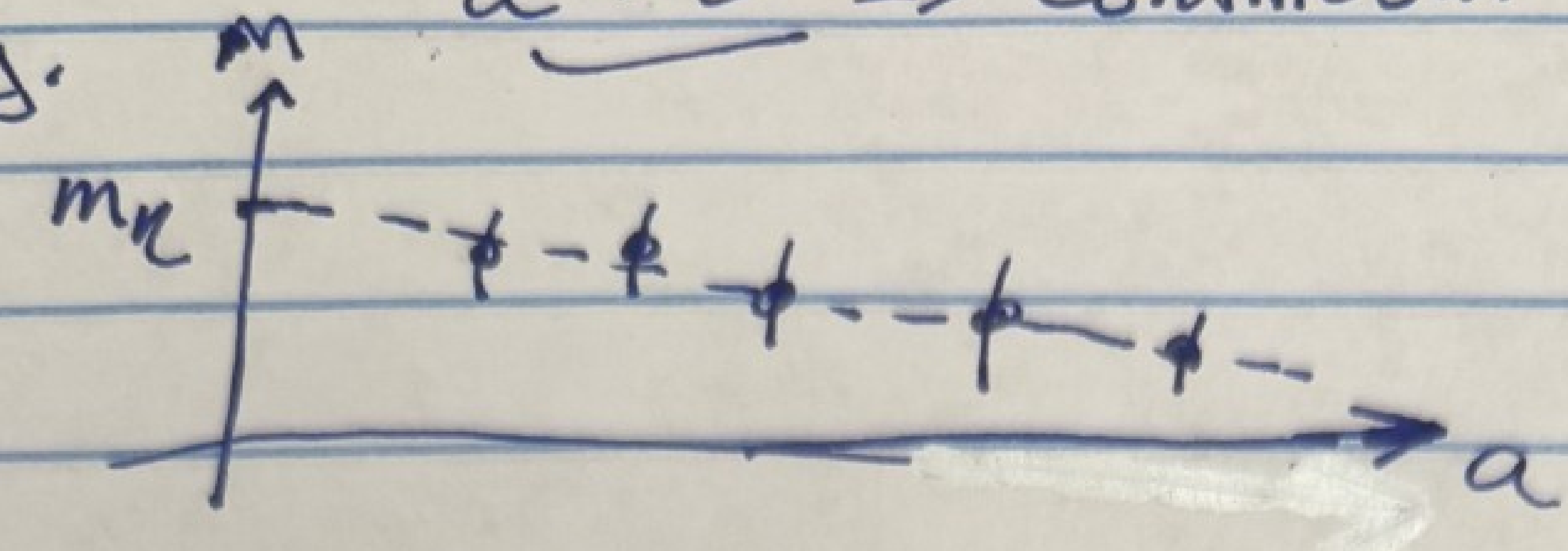
③ Do Monte Carlo Integration

$$Z \;\{\; \int DA_\mu \ldots \hat{O}\, e^{-S}$$

④ Fit one (or a few) masses to set scale ⟺ $M_{exp.}$

⑤ Redo Calc at diff a & extrapolate to
$a \to 0$ ⇒ continuum limit

e.g.



III Challenges:

MC integration (Metropolis) requires:

⇒ For "Quenched" (only valence quarks) need T flops
10^{12} floating points ops/s

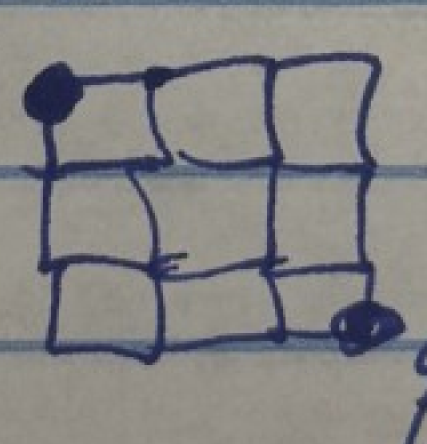
Unquenched calcs (includes sea quarks) need Exaflops
10^{15} Flops

⇒ Challenging to maintain Gauge Symmetry (gluons)
& Chiral Symmetry (quarks) while discretizing
Action

① Gluon Action S^G

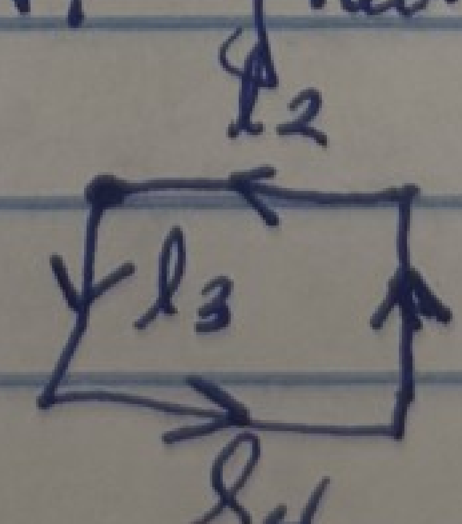
Discretize Space-time 4D hypercube

e.g. in 2D q



q quarks @ vertices
gluons "link" quarks

For gauge-inv. gluon "propagation" use
plaquette



w $S^G_{QCD} = \frac{1}{4}\int d^4x \, F^c_{\mu c} F_c^{\mu\nu}$

@ each link get link variable:

$$U_\mu(x) = 1 + ia A_\mu(x)$$

& Trace around Plaquette gives Action via

$$W_\square \equiv \mathrm{Tr}\left[U(l_1) U(l_2) U(l_3) U(l_4)\right]$$

$$\Leftrightarrow S^G_{LGT} = \sum_\square \frac{2}{g^2}(3 - W_\square) + \mathcal{O}(a^5)$$

↳ see GSS

All is OK as long as

$$\lim_{a \to 0} S^G_{LGT} = S^G_{QCD}$$

② Quark Action

$$S^q_{QCD} = \int d^4x \left[i\bar{\psi}\gamma^\mu \partial_\mu \psi + m\bar{\psi}\psi \right]$$

For LGT try

$$S^q_{LGT} = \sum_{n,\mu} \left[i\bar{\psi}_n \gamma^\mu \frac{(\psi_{n,+\mu} - \psi_{n,-\mu})}{2a} + m\bar{\psi}_n\psi_n \right]$$

Lattice sites ↗

However, due to periodic structure of lattice quark propagator:

$$\frac{1}{\frac{i}{a}\gamma^\mu \sin(ap^\mu) - m} = \frac{-i\frac{\gamma^\mu}{a}\sin(ap^\mu) - m}{\frac{1}{a^2}\sum_\mu \sin^2(p^\mu a) + m^2}$$

w pole @ Physical quark mass $p_0 = -iE$:

@ $(p_x, p_y, p_z) = (0,0,0) \Rightarrow E = m$

<u>but</u> get extra fermions when

$$(ap_x, ap_y, ap_z) = (n_x\pi, n_y\pi, n_z\pi)$$

$\Rightarrow$ Called Fermion Doubling Problem
& they don't vanish as $a \to 0$
(ouch!)

K. Wilson fixed this "Wilson fermion"
by adding term so that $m_{doub.} = m + \frac{\delta}{a}$
& $m_{doublers} \to \infty$ as $a \to 0$

but this broke chiral Symmetry

$\Rightarrow$ Domain Wall (add 5^{th} dimension w size L_5)
& trap chiral fermions on Wall
then as $L_G \to \infty$, $m_{double} \sim \frac{1}{a}$
& Chiral Sym. OK

Now including $\sum'_{n,\mu} \bar{\psi}_n \gamma_\mu U_{n\mu} \psi_{n+\mu} + \ldots$
gives g-g interactions...

Leading to

IV Results

$\to$ see PICS $\to$

Results from LGT:

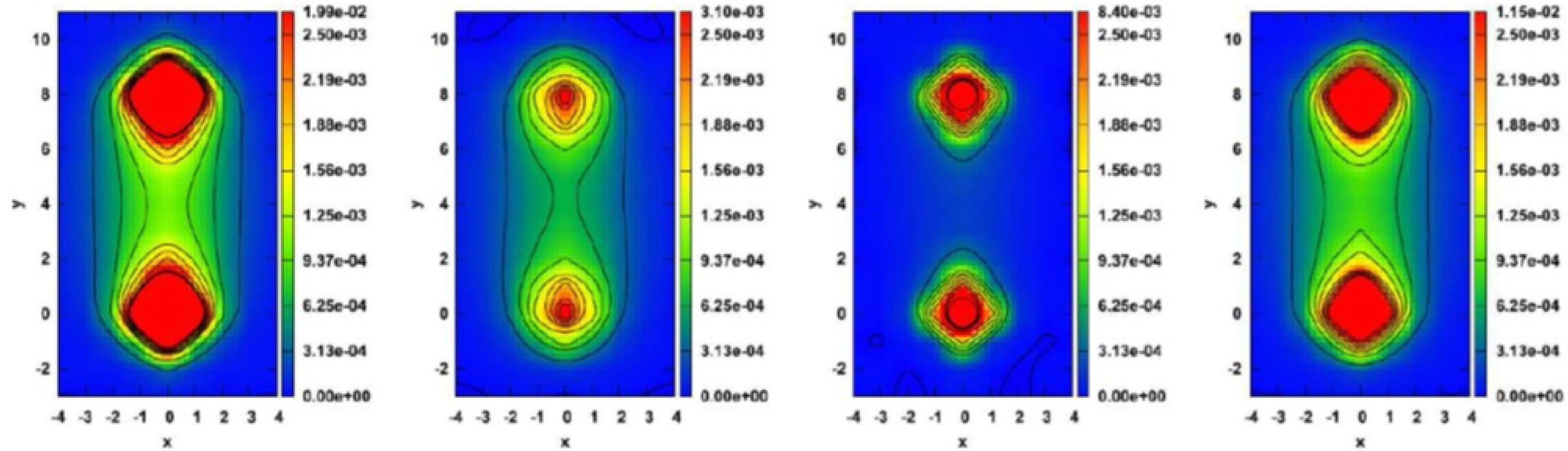
"Real" Confinement!

"♫ More than a feeling…"

q

$\bar{q}$

M. CARDOSO, N. CARDOSO, AND P. BICUDO PHYSICAL REVIEW D **81**, 034504 (2010)



(a) $\langle E^2 \rangle$ (b) $-\langle B^2 \rangle$ (c) Energy Density (d) Lagrangian Density

FIG. 5 (color online). Results for the static quark-antiquark system. To make the comparison between the values of different fields easier we have chosen a common scale for values between 0 and $2.5e-3$, but in each figure the deepest red represents the maximum value of the represented field. Because of this choice the flux tube, responsible for the string tension, that should appear in the energy density plot (c) is less visible. The axes and results are in lattice spacing units.

Results from LGT:

6 Standard Model Parameters

The Standard Model (with nonzero neutrino masses and mixing angles) has 28 free parameters:

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (M_W/v)^2/\pi$;
- Quark sector: $m_u e^{i\bar{\theta}}$, m_d, m_s, m_c, m_b, m_t; $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, δ_{KM};
- Lepton sector: m_{ν_1}, m_{ν_2}, m_{ν_3}, m_e, m_μ, m_τ; θ_{12}, θ_{23}, θ_{13}, δ_{PMNS}, α_{21}, α_{31};
- Standard electroweak symmetry breaking: $v = 246$ GeV, $\lambda = (M_H/v)^2/2$.

CKM

Lattice QCD is essential or important in determining the values of eleven parameters (the first under gauge couplings and all but m_t under quark sector).

Table 2: Quark masses from lattice QCD converted to the $\overline{\mathrm{MS}}$ scheme and run to the scale indicated. Entries are in MeV.

Flavor (scale)	Ref. (28)	Ref. (53)	Ref. (54)	Ref. (55)	Ref. (56)
$\bar{m}_u(2\ \mathrm{GeV})$	1.9 ± 0.2	2.01 ± 0.14	2.24 ± 0.35	2.15 ± 0.11	
$\bar{m}_d(2\ \mathrm{GeV})$	4.6 ± 0.3	4.79 ± 0.16	4.65 ± 0.35	4.79 ± 0.14	
$\bar{m}_s(2\ \mathrm{GeV})$	88 ± 5	92.4 ± 1.5	97.7 ± 6.2	95.5 ± 1.9	
$\bar{m}_c(3\ \mathrm{GeV})$					986 ± 10
$\bar{m}_b(10\ \mathrm{GeV})$					3617 ± 25

MeV

Note

$M_{q\bar{q}}$

140 MeV

1.0 GeV

3.1 GeV

9.5 GeV

$M_{q\bar{q}} > 2 m_q$!

Different LGT calcs.

QCD coupling constant from LGT:

the errors on almost all determinations are dominated by the perturbative truncation error. Instead, the error on the pre-range for α_s from the step-scaling method is taken, since perturbative truncation errors are sub-dominant in this method. The final FLAG 2021 average (rounded to four digits) is

$$\alpha_s(m_Z^2) = 0.1184 \pm 0.0008 \quad \text{(FLAG 2021 average)}, \quad \text{LGT} \tag{9.23}$$

which is fully compatible with the FLAG 2019 result of $\alpha_s(m_Z^2) = 0.1182 \pm 0.0008$.

We believe that this result expresses to a large extent the consensus of the lattice community and that the imposed criteria and the rigorous assessment of systematic uncertainties qualify for a direct inclusion of this FLAG average here. As in the previous review, we therefore adopt the FLAG average with its uncertainty as our value of α_s for the lattice category. Moreover, this lattice result will not be directly combined with any other sub-field average, but with our non-lattice average to give our final world average value for α_s.

9.4.8 *Determination of the world average value of $\alpha_s(m_Z^2)$:*
Obtaining a world average value for $\alpha_s(m_Z^2)$ is a non-trivial exercise. A certain arbitrariness and subjective component is inevitable because of the choice of measurements to be included in the average, the treatment of (non-Gaussian) systematic uncertainties of mostly theoretical nature, as well as the treatment of correlations among the various inputs, of theoretical as well as experimental origin.

We have chosen to determine pre-averages for sub-fields of measurements that are considered to exhibit a maximum degree of independence among each other, considering experimental as well as theoretical issues. The seven pre-averages, illustrated also in Fig. 9.2, are listed in column two of Table 9.1. We recall that these are exclusively obtained from extractions that are based on (at least) NNLO QCD predictions, and are published in peer-reviewed journals at the time of completing this *Review*. To obtain our final world average, we first combine six pre-averages, excluding the lattice result, using a χ^2 *averaging method*. This gives

— a.k.a. Experiment

$$\alpha_s(m_Z^2) = 0.1175 \pm 0.0010 \quad \text{(PDG 2023 without lattice)}. \tag{9.24}$$

This result is fully compatible with the lattice pre-average Eq. (9.23) and has a comparable error. To avoid a possible over-reduction, we combine these two numbers using an unweighted average and take as an uncertainty the average between these two uncertainties. This gives our final world average value

$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009 \quad \text{(PDG 2023 average)}. \tag{9.25}$$

Now has equal footing to experiment!
→ took 45 yrs + ExaCPU

If for the sub-field of hadron colliders we are more restrictive and instead only accept results from a simultaneous fit of PDFs, we arrive at 0.1157 ± 0.0021 for this sub-field leading to 0.1172 ± 0.0010

Hadron Masses:

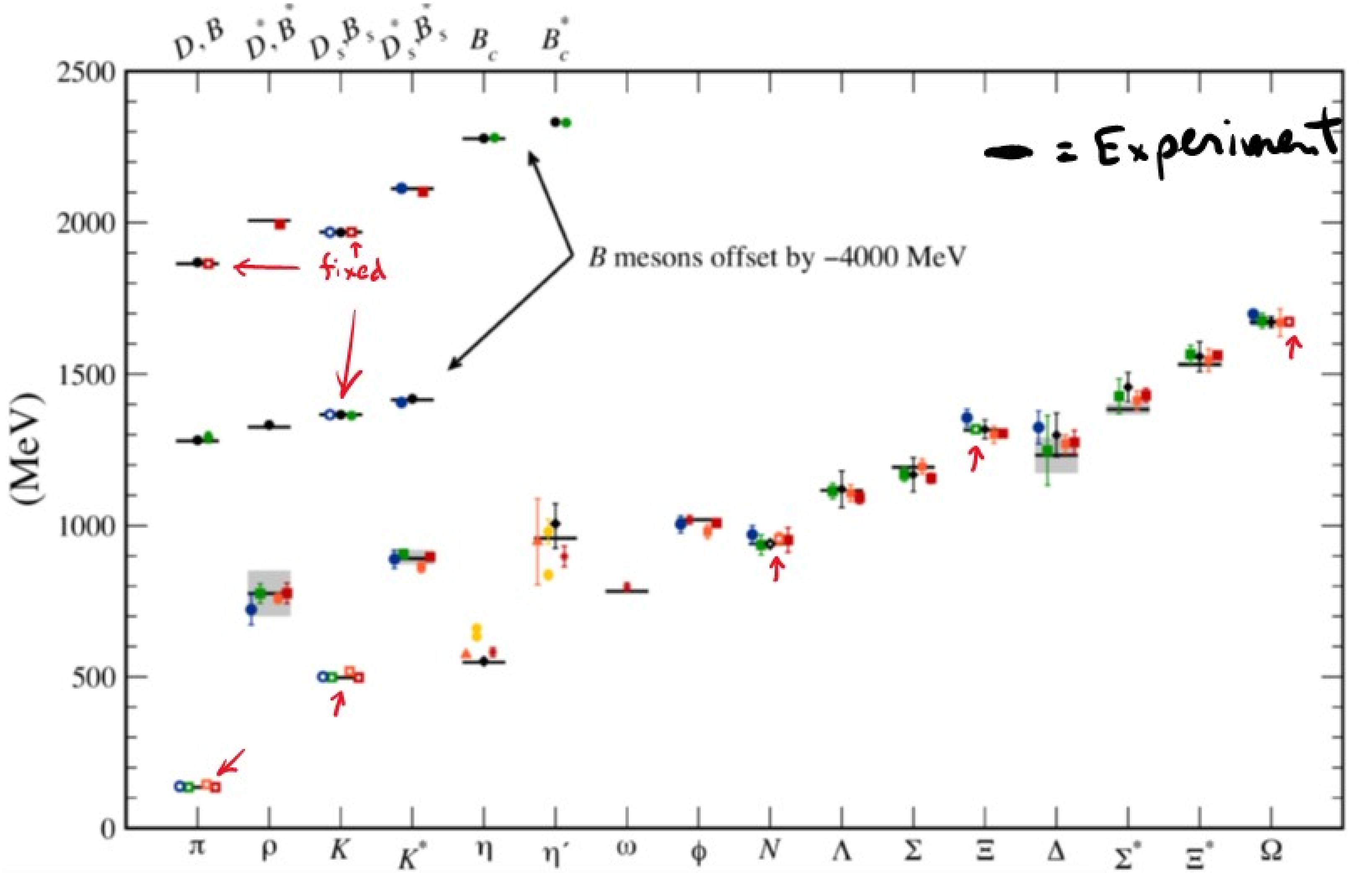


Figure 15.9: Hadron spectrum from lattice QCD. Comprehensive results for mesons and baryons are from MILC [111,112], PACS-CS [113], BMW [114], QCDSF [115], and ETM [116]. Results for η and η' are from RBC & UKQCD [21], Hadron Spectrum [117] (also the only ω mass), UKQCD [118], and Michael, Ottnad, and Urbach [119]. Results for heavy-light hadrons from Fermilab-MILC [120], HPQCD [121,122], and Mohler and Woloshyn [123]. Circles, squares, diamonds, and triangles stand for staggered, Wilson, twisted-mass Wilson, and chiral sea quarks, respectively. Asterisks represent anisotropic lattices. Open symbols denote the masses used to fix parameters. Filled symbols (and asterisks) denote results. Red, orange, yellow, green, and blue stand for increasing numbers of ensembles (i.e., lattice spacing and sea quark mass) Black symbols stand for results with 2+1+1 flavors of sea quarks. Horizontal bars (gray boxes) denote experimentally measured masses (widths). b-flavored meson masses are offset by -4000 MeV.

Glueball Masses (?!?):

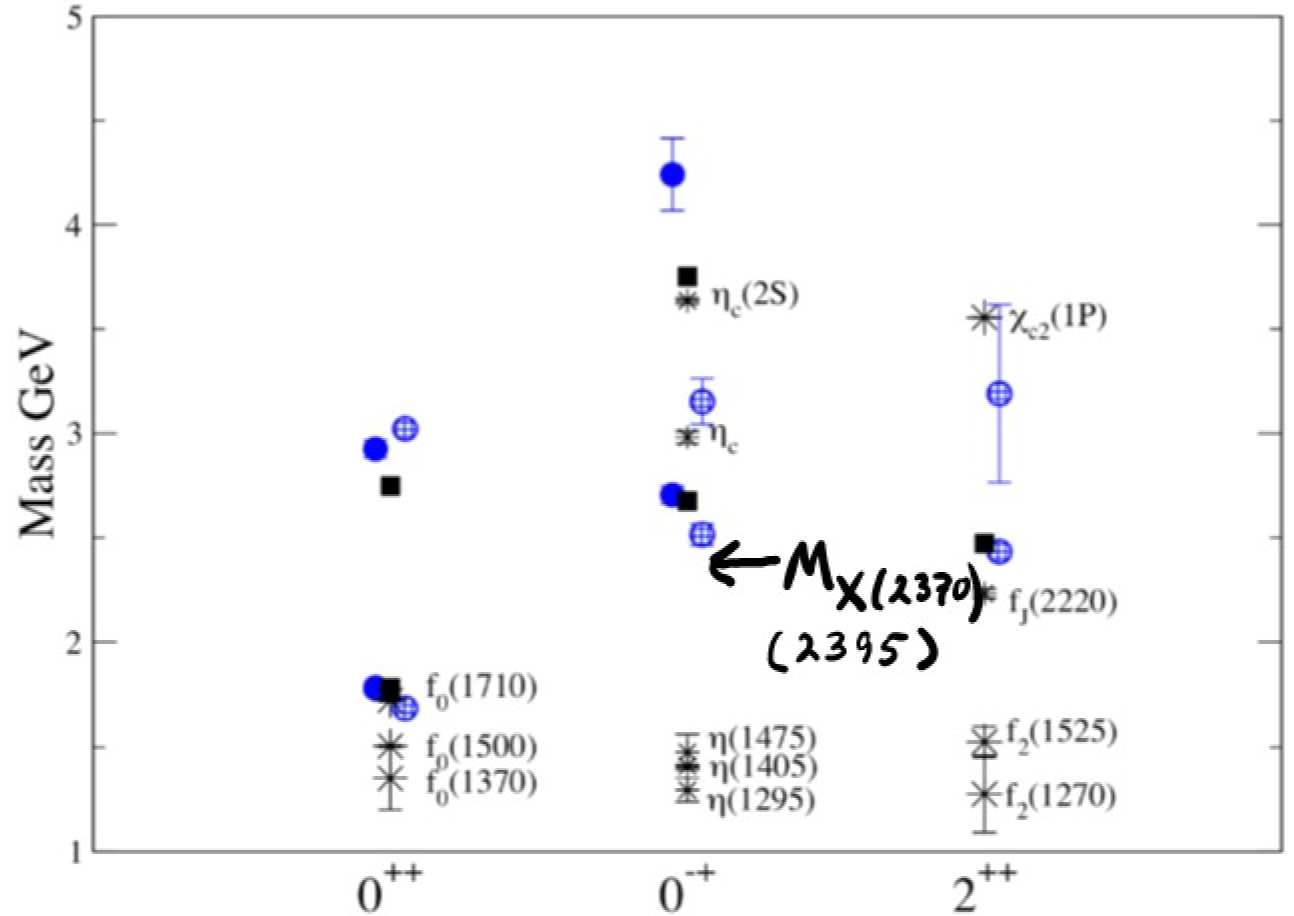


Figure 15.15: Lattice QCD predictions for glueball masses. The open and closed circles are the larger and smaller lattice spacing data of the full QCD calculation of glueball masses of Ref. [140], at pion masses of 280 and 360 MeV. Squares are the quenched data for glueball masses of Ref. [33]. The bursts labeled by particle names are experimental states with the appropriate quantum numbers.

$n \to p e^- \overline{\nu}_e$ DECAY PARAMETERS

See the above "Note on Baryon Decay Parameters." For discussions of recent results, see the references cited at the beginning of the section on the neutron mean life. For discussions of the values of the weak coupling constants g_A and g_V obtained using the neutron lifetime and asymmetry parameter A, comparisons with other methods of obtaining these constants, and implications for particle physics and for astrophysics, see DUBBERS 91 and WOOLCOCK 91. For tests of the $V-A$ theory of neutron decay, see EROZOLIMSKII 91B, MOSTOVOI 96, NICO 05, SEVERIJNS 06, and ABELE 08.

$\lambda \equiv g_A / g_V$

VALUE	DOCUMENT ID		TECN	COMMENT
−1.2754 ±0.0013 OUR AVERAGE	Error includes scale factor of 2.7. See the ideogram below.			
−1.2796 ±0.0062	[1] HASSAN	21	SPEC	Proton recoil spectrum
−1.2677 ±0.0028	[2] BECK	20	SPEC	Proton recoil spectrum
−1.27641 ±0.00045 ±0.00033	[3] MAERKISCH	19	SPEC	pulsed cold n, polarized
−1.2772 ±0.0020	[4] BROWN	18	UCNA	Ultracold n, polarized
−1.2748 ±0.0008 $^{+0.0010}_{-0.0011}$	[5] MUND	13	SPEC	Cold n, polarized
−1.275 ±0.006 ±0.015	SCHUMANN	08	CNTR	Cold n, polarized
−1.2686 ±0.0046 ±0.0007	[6] MOSTOVOI	01	CNTR	A and B × polarizations

PoS PROCEEDINGS OF SCIENCE

Lattice QCD Determination of g_A

André Walker-Loud*, Lawrence Berkeley National Laboratory
Evan Berkowitz, University of Maryland
David A. Brantley, Arjun Gambhir, Pavlos Vranas, Lawrence Livermore National Laboratory
Chris Bouchard, University of Glasgow
Chia Cheng Chang, RIKEN-iTHEMS
M.A. Clark, NVIDIA Corporation
Nicolas Garron, Liverpool Hope University
Bálint Joó, Thomas Jefferson National Accelerator Facility
Thorsten Kurth NERSC, Lawrence Berkeley National Laboratory
Henry Monge-Camacho, Amy Nicholson, University of North Carolina Chapel Hill
Christopher J Monahan, Kostas Orginos, The College of William & Mary
Enrico Rinaldi, Arithmer Inc. & RIKEN-iTHEMS

The nucleon axial coupling, g_A, is a fundamental property of protons and neutrons, dictating the strength with which the weak axial current of the Standard Model couples to nucleons, and hence, the lifetime of a free neutron. The prominence of g_A in nuclear physics has made it a benchmark quantity with which to calibrate lattice QCD calculations of nucleon structure and more complex calculations of electroweak matrix elements in one and few nucleon systems. There were a number of significant challenges in determining g_A, notably the notorious exponentially-bad signal-to-noise problem and the requirement for hundreds of thousands of stochastic samples, that rendered this goal more difficult to obtain than originally thought.

2. A percent-level determination of g_A from QCD

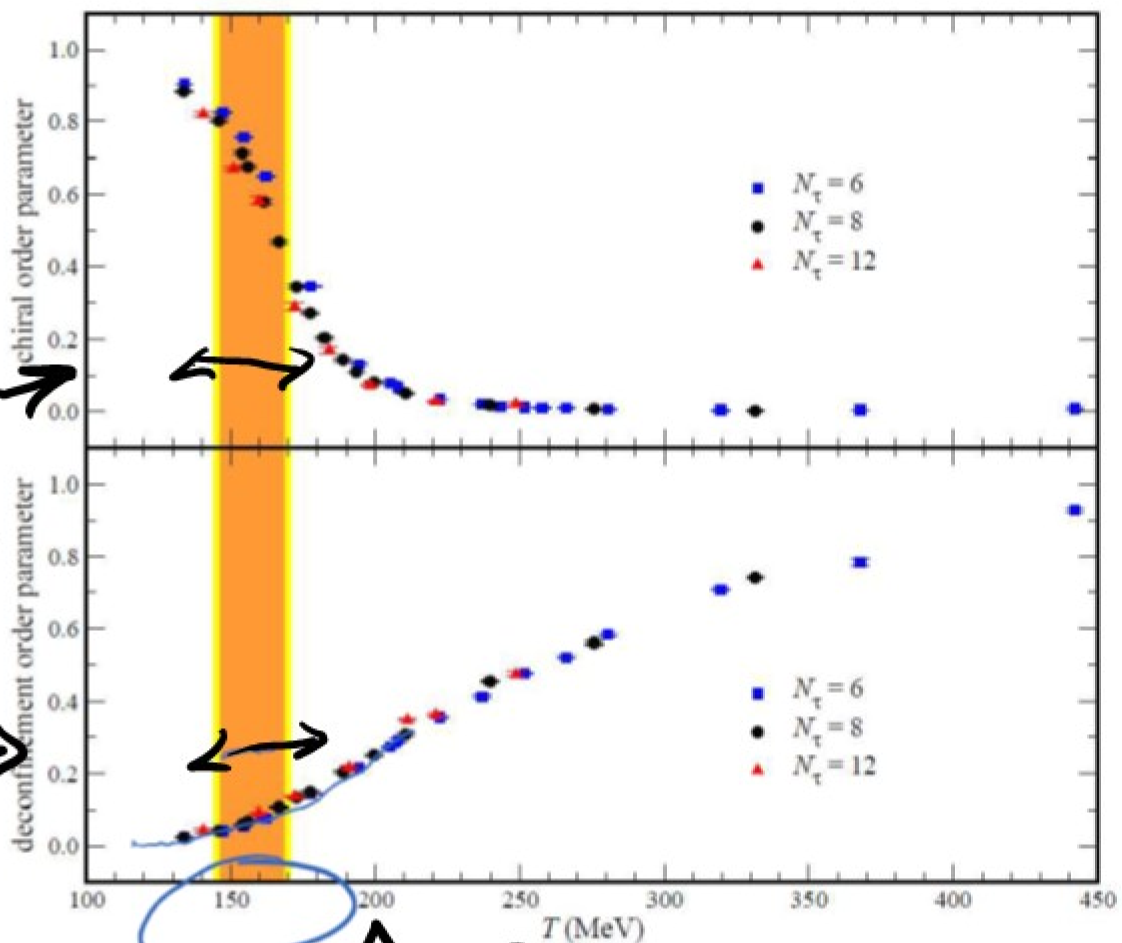
We have recently determined g_A with an unprecedented percent-level of uncertainty [5]

$$g_A = 1.2711(103)^s(39)^\chi(15)^a(04)^V(55)^M\,. \tag{2.1}$$

The sources of uncertainty are statistical (s), extrapolation to the physical pion mass (χ), continuum extrapolation (a), infinite volume extrapolation (V) and a model average uncertainty (M). Prior to this result, it was estimated that a 2% uncertainty could be achieved with near-exascale computing (such as Summit at OLCF) by 2020 [6]. There were several key features of our calculation that enabled a determination with 1% uncertainty with the previous generation of supercomputers:

LGT @ Finite Temperature:
QCD Phase Transition:

Soft Phase Transition



Quark Gluon Plasma

LGT calc.

Λ_{QCD}

Figure 5: Order parameters for deconfinement (*bottom*) and chiral symmetry restoration (*top*), as a function of temperature. The physical temperature $T = (N_\tau a)^{-1}$, where a is the lattice spacing and $N_\tau = N_4$. Agreement for several values of N_τ thus indicates that discretization effects from the lattice are under control. Data are from Reference 126.