HW 5 Solutions

Solution 1

Consider the magnetic moment operator of an odd nucleon

$$\hat{\mu} = a\hat{S}_3 + b\hat{L_3}$$

where $a = 2\mu_n$, $b = 0$ for an odd neutron and $a = 2\mu_p$, $b = \mu_N$ for an odd proton. Assume the odd neucleon is in a state $|l, m_l; s, m_s\rangle$ with $s = \frac{1}{2}$, which we need to express in $(l, m_l; s, m_s)$ basis. For a given l we have two distinct cases $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$.

(i) $j = l + \frac{1}{2}$

$$\begin{aligned}
&|j, m_j = j\rangle = \left| l, m_l = j - \frac{1}{2}; \frac{1}{2}, \frac{1}{2} \right\rangle \\
&\implies \langle j, m_j = j | \, \mu \, | j, m_j = j\rangle = \frac{a}{2} + b\left(j - \frac{1}{2}\right)
\end{aligned}$$

(ii) $j = l - \frac{1}{2}$

$$|j, m_j = j\rangle = -\sqrt{\frac{1}{2(j+1)}} \left| l, m_l = j - \frac{1}{2}; \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{2j+1}{2(j+1)}} \left| l, m_l = j + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2} \right\rangle$$

Therefore we have

$$\begin{aligned}
\langle j, j | \, \mu \, | j, j\rangle &= \frac{1}{2(j+1)}\left(\frac{a}{2} + b\left(j - \frac{1}{2}\right) + (2j+1)\big(-\frac{a}{2}\big) + (2j+1)b\left(j + \frac{1}{2}\right)\right) \\
&= -\left(\frac{j}{J+1}\right)\frac{a}{2} + \left(\frac{j(j+\frac{3}{2})}{j+1}\right)b
\end{aligned}$$

Solution 2

The 4 nuclei are ^{2}H, ^{6}Li, ^{10}B and ^{14}Li This is due to the pairing nature of the nucleons. They prefer to pair with like particles in the same energy orbital. For odd odd nuclei both a proton and a neutron remain unpaired making them highly susceptible to beta decay to produce an even even pairing. As the total number of nucleons increases the additional Coulomb load is too large to overcome. See also the semi-empirical mass formula.

Solution 3

Let the 4-momentum transfer be defined as $q = p_e - p'_e$. Then

$$\begin{aligned} q^2 &= (p_e - p'_e)^2 \\ &= p_e^2 + p_e'^2 - 2p_e \cdot p'_e \\ &= 2m_e^2 - 2(E_e E'_e - p_e \cdot p'_e) \end{aligned}$$

(a) For relativistic (E ¿¿ m) electrons have energy $E \approx |\vec{p}|$. We have

$$\begin{aligned} q^2 &\approx -2(E_e E'_e - p_e \cdot p'_e) \\ &\approx -2E_e E'_e (1 - \cos\theta) \\ &\approx -4E_e E'_e \sin^2 \frac{\theta}{2} \end{aligned}$$

for scattering angle θ

(b) Elastic scattering $e + P \to e' + P'$. From conservation of momentum $p_e + p = p'_e + p'$ we have

$$\begin{aligned} &p'^2 = (p + Q)^2 = p^2 + 2p \cdot q + q^2 \\ &M^2 = M^2 + 2p \cdot q + q^2 \\ &q^2 = -2p \cdot q \end{aligned}$$

in the lab frame where $p = (M, 0, 0, 0)$ this becomes

$$\begin{aligned} p \cdot q = M(E_e - E'_e) = M\nu \\ \implies x = \frac{Q}{2M\nu} = \frac{-q^2}{2p \cdot q} = 1 \end{aligned}$$

Solution 4

In the lab frame with the target proton at rest we have

$$W^2 = (p + q)^2 = p^2 + 2p \cdot q + q^2 = M^2 + 2M\nu - Q^2$$

Solution 5

The form factor is defined as

$$\frac{F_2^P(x)}{x} = \sum_q e_e^2 (q^P(x) + \bar{p}^P(x))$$

where $e_q = 2/3$, $e_d = 1/3$, $e_s = 1/3$ and

$$u(x) = u_v(x) + u_s(x), \quad \bar{u}(x) = u_s(x)$$
$$d(x) = d_v(x) + d_s(x), \quad \bar{d}(x) = d_s(x)$$
$$s(x) = s_s(x), \quad \bar{s}(x) = s_s(x)$$

Plugging these values into our expression for the form factor we find

$$\begin{aligned}\frac{F_2^P(x)}{x} &= \frac{4}{9}\Big(u_v(x) + u_s(x) + u_s(x)\Big) + \frac{1}{9}\Big(d_v(x) + d_s(x) + d_s(x)\Big) + \frac{1}{9}\Big(s_s(x) + s_s(x)\Big) \\ &= \frac{4}{9}u_v(x) + \frac{1}{9}d_v(x) + \frac{4}{3}S(x)\end{aligned}$$

In the limit where $x \to 0$, we see that

$$\lim_{x\to 0} F_2^P(x) = \lim_{x\to 0} x\left[\frac{4}{9}u_v(x) + \frac{1}{9}d_v(x) + \frac{4}{3}S(x)\right]$$

and so $S(x) \sim 1/x$ if $F_2^P(x)$ is indeed a constant as we approach 0.