Physics 203

Homework 5

1.) Determine the non-relativistic elastic scattering form factor

$$F(|\mathbf{q}|) = \int d^3r \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

for the following two densities:

a) $\rho(r) = \rho_0$ =constant for $r \leq R$ and $\rho = 0$ for $r > R$,

b) $\rho(r) = \rho_0 e^{-r/R}$

2.) Beginning with the non-relativistic elastic scattering form factor

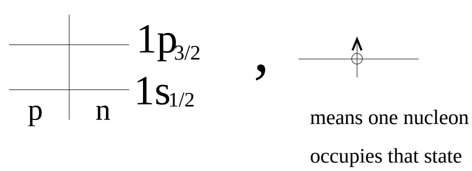
$$F(|\mathbf{q}|) = \int d^3r \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

show that the mean square radius of the charge distribution is related to the derivative of the form factor, i.e.:

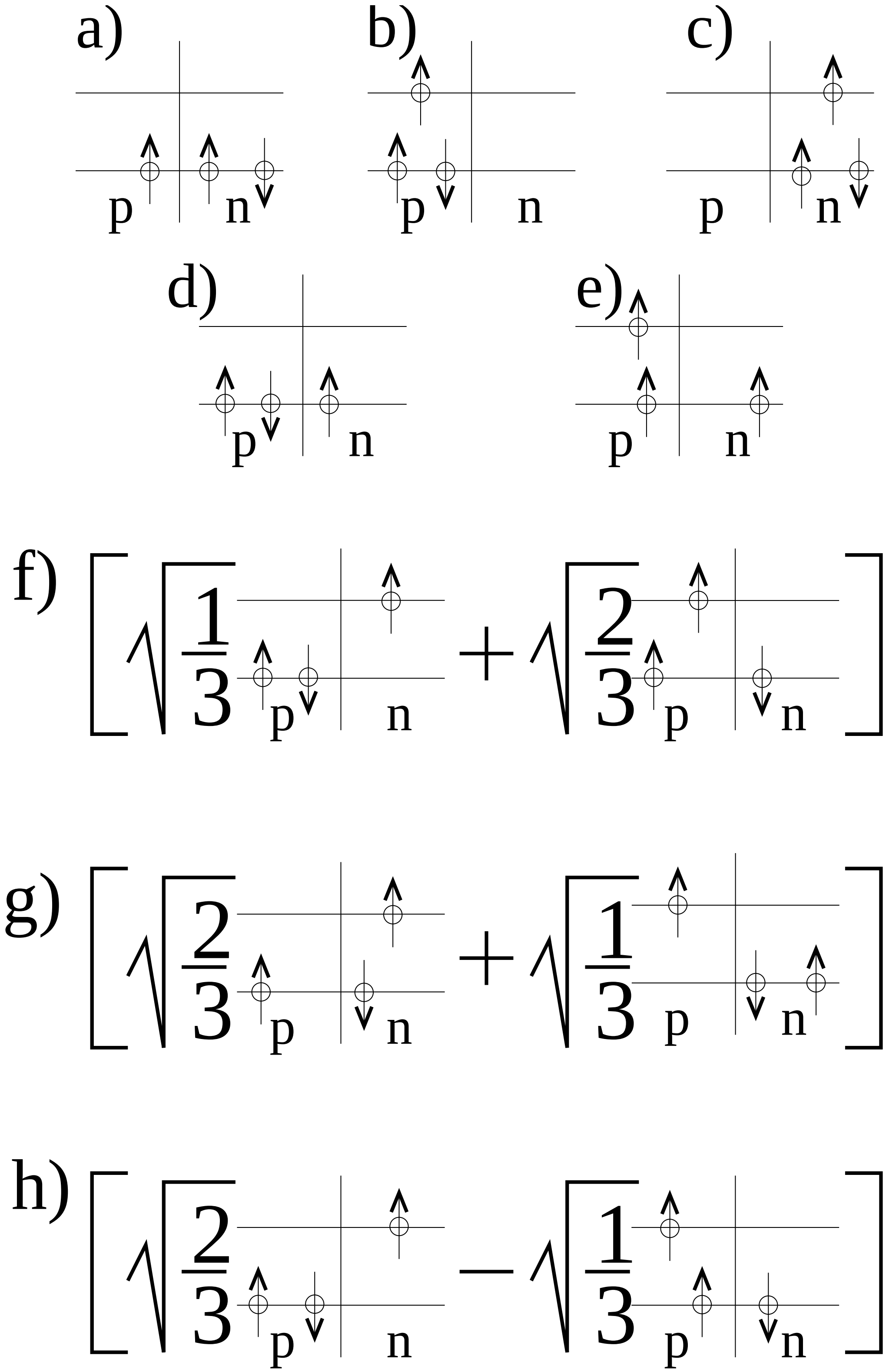
$$< r^2 >= \lim_{q \to 0} 6 \left| \frac{dF(|\mathbf{q}|)}{d|\mathbf{q}|^2} \right|$$

3.) What are the nuclei, spins, parities, isospins, and "3"- component of isospin for the diagrams [a) - h)], shown on the next page, representing states of the three nucleon system. Note that these diagrams are short-hand for the full states but are sufficient to identify the nuclei. Use the shell model as a guide, and assume the state of lowest excitation energy in each case. If some of the total spins are not unique, list the range of possible values. Group isospin multiplets together and indicate any that are missing (Hint: Find the state of maximum "3"-component of isospin first, and then identify other members of the multiplet). The notation of the diagrams is:

Notation:



Arrows denote relative orientation of nucleon spin 3−component in a state


a)
p
n
b)
p
n
c)
p
n
d)
p
n
e)
p
n
f)
√(1/3) + √(2/3)
p
n
g)
√(2/3) + √(1/3)
p
n
h)
√(2/3) − √(1/3)
p
n

Consider a gas of N fermions ($N >> 1$) in a volume Ω. Let Ω be so large that surface effects are negligible and let the number density ρ ($\rho = N/\Omega$) be large enough that the fermions are highly relativistic (eg. $E \sim pc$).

a) Assuming that only two fermions can occupy a given state $\mathbf{p}$, show that the Fermi energy ϵ_F is given by

$$\epsilon_F = cp_F = (3\pi^2)^{\frac{1}{3}}\hbar c\rho^{\frac{1}{3}}$$

b) Consider a neutron star core with mass density $> 10^{17}$ g/cm^3. For such densities, the neutrons are in the extreme relativistic limit and the results from part a) are applicable. In such a gas, a certain fraction of protons, electrons, neutrons and heavier particles will be present. Consider only the protons and electrons appearing from the reaction

$$n \rightarrow p + e^- + \bar{\nu}$$

where the neutrinos escape without interacting. Noting that for equilibrium the Fermi energies must match, that is

$$\epsilon_F(n) = \epsilon_F(p) + \epsilon_F(e^-),$$

use your result from part a) and the fact that the gas is neutral to prove that the proton-to-neutron ratio is $N_p/N_n = \frac{1}{8}$