SP3

Consider a gas of N fermions ($N >> 1$) in a volume Ω. Let Ω be so large that surface effects are negligible and let the number density ρ ($\rho = N/\Omega$) be large enough that the fermions are highly relativistic (eg. $E \sim pc$).

a) Assuming that only two fermions can occupy a given state $\mathbf{p}$, show that the Fermi energy ϵ_F is given by

$$\epsilon_F = cp_F = (3\pi^2)^{\frac{1}{3}} \hbar c \rho^{\frac{1}{3}}$$

b) Consider a neutron star core with mass density $> 10^{17}$ g/cm^3. For such densities, the neutrons are in the extreme relativistic limit and the results from part a) are applicable. In such a gas, a certain fraction of protons, electrons, neutrons and heavier particles will be present. Consider only the protons and electrons appearing from the reaction

$$n \rightarrow p + e^- + \bar{\nu}$$

where the neutrinos escape without interacting. Noting that for equilibrium the Fermi energies must match, that is

$$\epsilon_F(n) = \epsilon_F(p) + \epsilon_F(e^-),$$

use your result from part a) and the fact that the gas is neutral to prove that the proton-to-neutron ratio is $N_p/N_n = \frac{1}{8}$