

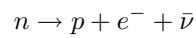
SP3

Consider a gas of N fermions ($N \gg 1$) in a volume Ω . Let Ω be so large that surface effects are negligible and let the number density ρ ($\rho = N/\Omega$) be large enough that the fermions are highly relativistic (eg. $E \sim pc$).

a) Assuming that only two fermions can occupy a given state \mathbf{p} , show that the Fermi energy ϵ_F is given by

$$\epsilon_F = cp_F = (3\pi^2)^{\frac{1}{3}} \hbar c \rho^{\frac{1}{3}}$$

b) Consider a neutron star core with mass density $> 10^{17}$ g/cm³. For such densities, the neutrons are in the extreme relativistic limit and the results from part a) are applicable. In such a gas, a certain fraction of protons, electrons, neutrons and heavier particles will be present. Consider only the protons and electrons appearing from the reaction



where the neutrinos escape without interacting. Noting that for equilibrium the Fermi energies must match, that is

$$\epsilon_F(n) = \epsilon_F(p) + \epsilon_F(e^-),$$

use your result from part a) and the fact that the gas is neutral to prove that the proton-to-neutron ratio is $N_p/N_n = \frac{1}{8}$