Dressing Field Uniformity

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Inhomogeneities in a RF “dressing” field can contribute to the relaxation T_2 of a spin ensemble. The spin relaxation of ^{3}He atoms diffusing in superfluid He at temperatures near 500 mK is discussed in this note. We model the field produced by a pair of coaxial cylindrical coils and study the effect of inhomogeneities in the calculated field with a Monte Carlo simulation of spin precession. We consider the specific geometry of the fields in the nEDM measurement cells and note that the initial dressing coils design does not meet the uniformity requirements, leading to short relaxation times. An improved coil design, introducing an additional correction coil is considered, which satisfies the uniformity requirements for a long T_2 relaxation time.

1 Spin Dressing

The application of an oscillating “dressing” field B_1 can effectively change the gyromagnetic ratio of a particle with spin precessing about a small transverse static field B_0. If the oscillation frequency of the dressing field ω_{RF} is large with respect to the larmor frequency so that it obeys the condition

$$Y \equiv \frac{\gamma_0 B_0}{\omega_{RF}} \ll 1,$$

the effective gyromagnetic ratio with respect to B_0 can be calculated[2] to be

$$\gamma_{eff} = \gamma_0 \, J_0 \left(\frac{\gamma_0 \, B_1}{\omega_{RF}} \right), \tag{1}$$

where γ_0 is the “bare” gyromagnetic ratio of the particle.

For the case of UCN and ^{3}He atoms precessing about the same static field B_0, a combination of “critical” dressing parameters B_1 and ω_{RF} can be found so that the effective gyromagnetic ratios of the two spin species are matched. The bare gyromagnetic ratios of neutrons and ^{3}He atoms are $\gamma_n = -18.32472\,\mathrm{rad}\cdot\mathrm{s}^{-1}\cdot\mathrm{mGauss}^{-1}$ and $\gamma_3 = -20.37894\,\mathrm{rad}\cdot\mathrm{s}^{-1}\cdot\mathrm{mGauss}^{-1}$, respectively[1]. For convenience, $\alpha \equiv \gamma_3/\gamma_n \simeq 1.112$ can be defined. At critical dressing, $\gamma_n^{eff} = \gamma_3^{eff}$,

[1] NIST values

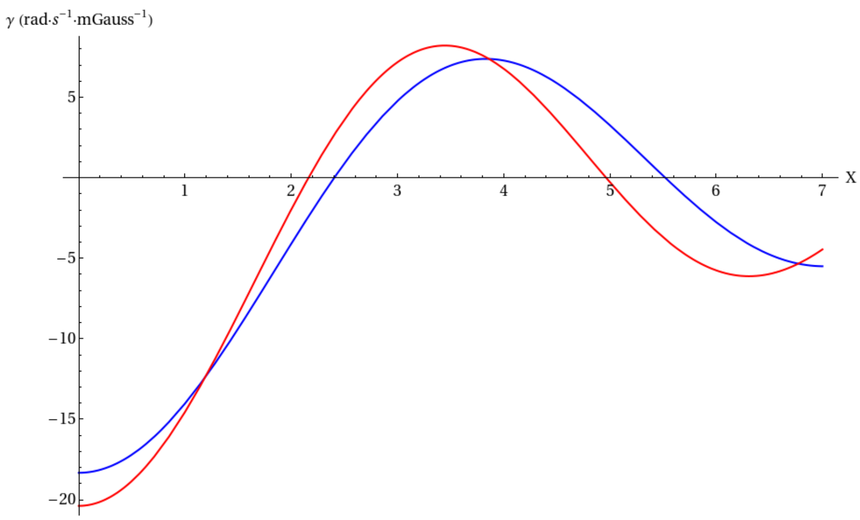

Fig. 1: Effective gyromagnetic ratios for neutrons (blue) and ^{3}He atoms (red) as a function of the dressing parameter $X = \gamma_n B_1/\omega_{RF}$. Solutions for the critical dressing parameter occur at $X = [1.19, 3.86, 6.77]$

and the dressing parameters can be calculated from

$$\gamma_n\, J_0\,(X) = \gamma_3\, J_0\left(\frac{\gamma_3}{\gamma_n}X\right), \tag{2}$$

where, for convenience, X has been defined as

$$X \equiv \frac{\gamma_n\, B_1}{\omega_{RF}}.$$

Fig. 1 shows the first solutions to Eq. 2. A solution for the critical dressing parameter can be found numerically to be $X_c = 1.1875$

2 ^{3}He spin relaxation T_2

Spin relaxation can occur due to inhomogeneities in the dressing field B_1. Deviations in the dressing field ΔB_1, result in deviations in the effective "dressed" gyromagnetic ratio of the ^{3}He atoms with respect to B_0, that can be calculated from eq. 1 to be

$$\Delta\gamma_{eff} = \frac{\gamma_3^2}{\omega_{RF}}\, J_1\left(\frac{\gamma_3\, B_1}{\omega_{RF}}\right)\, \Delta B_1,$$

where the relation $\partial J_0(x)/\partial x = -J_1(x)$ has been used. The deviation in precession frequency is then given by

$$\Delta\omega = \Delta\gamma_{eff}\, B_0$$

Spin relaxation comes from the dephasing of the spin ensemble over time.

The relaxation time T_2 can be calculated from

$$\frac{1}{T_2} = \left\langle \Delta\omega^2 \right\rangle \tau_D, \tag{3}$$

where τ_D is the diffusion time. The formalism in [3], describing spin relaxation due to static field gradients can be adapted[2] for the case of a non-uniform dressing field, by relating the deviations in the dressing field ΔB_1 to deviations in a static field

$$\Delta B_0 = \frac{\gamma_3 B_0}{\omega_{RF}} \frac{J_1(\alpha X)}{J_0(\alpha X)} \Delta B_1.$$

The relaxation time T_2 for a uniform gradient can be calculated to be

$$\frac{1}{T_2} \simeq \frac{\gamma_3^2 L_z^4}{120D} \frac{\gamma_3^2 B_0^2}{\omega_{RF}^2} J_1(\alpha X)^2 \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle, \tag{4}$$

where D is the diffusion coefficient, given by

$$D \simeq \frac{1.6}{T[K]^7}\, \mathrm{cm}^2 \cdot \mathrm{s}^{-1},$$

and L_z is the length of the measurement cell in the z direction (40 cm). Eq. 4 only includes the term that depends on a gradient in the z direction because it is likely to dominate due to the L_z^4 factor and the higher gradient $\partial_z B_1$ initially expected from the dressing coil geometry (see sec. 4).

The dressing field design planned for the nEDM experiment offers a better field profile than a uniform gradient, and Eq. 4 can be adapted to different field profiles by relating field deviations $\left\langle \Delta B_1^2 \right\rangle$, which directly affect the quantity of interest $\left\langle \Delta\omega^2 \right\rangle$ in eq. 3, to RMS gradients, so that

$$\left\langle \Delta B_1^2 \right\rangle = \frac{L_z^2}{15} \left\langle \frac{\partial B_1}{\partial z} \right\rangle^2 + \frac{L_z^2}{60} \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle, \tag{5}$$

for a field with up to quadratic dependence. For the case of a uniform gradient (i.e. $B_1 \propto z$), eq. 5 becomes

$$\left\langle \Delta B_1^2 \right\rangle = \frac{L_z^2}{12} \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle,$$

while for the case of a purely "quadratic" field profile (i.e. $B_1 \propto z^2$), in which the mean gradient $\langle \partial_z B_1 \rangle$ vanishes, the expression becomes

$$\left\langle \Delta B_1^2 \right\rangle = \frac{L_z^2}{60} \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle,$$

leading to a factor of 5 decrease in the relaxation rate of the latter, compared to the uniform gradient case with the same RMS gradient. In addition, the

[2] Clayton, S. (May2008 nEDM meeting)

symmetry of a quadratic field $B_1(-z) = B_1(z)$, reduces the effective diffusion time τ_D because only half of the measurement cell needs to be considered, so that

$$\tau_D \propto \frac{L_z^2}{D} \to \frac{1}{4}\tau_D.$$

The relaxation time T_2 for the planned dressing field of quadratic profile, becomes

$$\frac{1}{T_2} \simeq \frac{\gamma_3^2 L_z^4}{2400D} Y^2\, J_1(\alpha X)^2 \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle, \tag{6}$$

where the quantity $Y = \gamma_3 B_0/\omega_{RF} \simeq 0.01$ has been introduced for convenience.

3 Simulations

A Monte Carlo simulation has been used to calculate the relaxation time T_2 due to dressing field inhomogeneities. The simulation method uses a 5^{th} order Runge Kutta numerical integration to solve the Bloch equations

$$\frac{\partial \boldsymbol{S}}{\partial t} = \gamma\, \boldsymbol{S} \times \boldsymbol{B}$$

The mean time step used in the simulations is $\langle \Delta t \rangle \simeq 5 \times 10^{-7}$ s. Spins precess about a constant uniform holding field B_0 of 10 mGauss and a transverse "dressing" field B_1 of magnitude 1.3 Gauss, oscillating at $\omega_{RF} = 2 \times 10^4\,\mathrm{rad} \cdot \mathrm{Hz}$. The expression for the dressing field is calculated from the dressing coils model (see sec. 4)

The relaxation time T_2 is extracted from the dephasing of the spin ensemble over time. T_2 is the time constant in the exponential fit to the decay of the transverse spin polarization

$$\langle \cos \Delta\phi \rangle \simeq 1 - \frac{\Delta\phi^2}{2}.$$

Fig. 2 shows the effect of dephasing on the transverse spin polarization. Table 1 shows the relaxation times, obtained from the simulation, due to a particularly large dressing field gradient.

T (mK)	T_2^{sim} (s)	T_2^{th} (s)
400	976(41)	820
450		360
500	218(8)	170

Tab. 1: Simulation results for the relaxation time of ^{3}He atoms in a non-uniform dressing field. The simulation values are compared to the values obtained from eq. 6

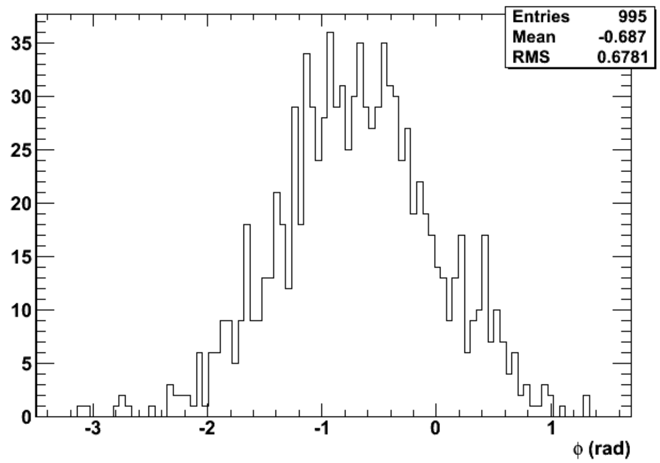

Fig. 2: Angular distribution of spins for ^{3}He at 500 mK after 50 s of precession. A dressing field with gradient of $\partial B_1/\partial z_{RMS} = 3.22\times10^{-4}\,\text{Gauss}\cdot\text{cm}^{-1}$ was extracted from the initial dressing coil model and used in the simulation.

The requirements for a relaxation time of $T_2 > 10000\,\text{s}$ (at $T = 450\,\text{mK}$) imply that the RMS gradient of the dressing field be

$$\frac{\partial B_1}{\partial z} < 6 \times 10^{-5}\,\text{Gauss}\cdot\text{cm}^{-1} \tag{7}$$

assuming the same field symmetry and quadratic profile (i.e. $B_1 \propto z^2$) and with the other components of the gradient negligible.

4 Dressing Coils

A pair of dressing coils is used to generate the field required for critical dressing. The dressing field has a magnitude of $B_{RF} \simeq 1.3\,\text{Gauss}$ and is transverse to the axis of the cylindrical magnet package (i.e. points in the y direction in nEDM coordinates). The pair of dressing coils is comprised of two co-axial cylindrical coils of radii $R_1 = 50\,\text{cm}$ and $R_2 = 59\,\text{cm}$ respectively with a maximum length given by the length of the magnet package. The pair of coils are designed to reduce the eddy current resistive heating on the B_0 ferromagnetic metglas shield, a cylindrical shell of radius $R_{FM} = 67\,\text{cm}$ and length $L_{FM} = 4.16\,\text{m}$ also coaxial to the dressing coils.

The design of the dressing coils is not finalized, but solutions have been found which provide a reduction in heating $P_{shielded}/P_{unshielded} \sim 60$. The solutions assume a $\cos\theta$ inner dressing coil with an outer "active shield" coil designed to mimic the current distribution on a superconducting cylinder (see

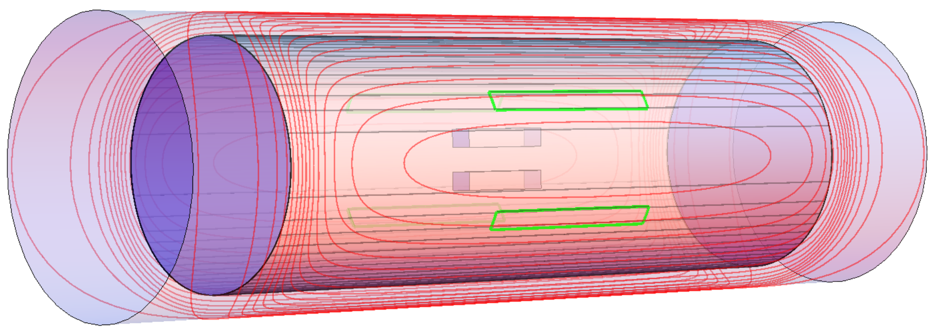

Fig. 3: Model of the dressing coils. The inner coil is a $\cos\theta$ coil producing a transverse field. The outer coil is an "active shield" designed to constrict the field to regions within it. The thick (green) coils represent correction coils, running in series with the inner coil and located on the same cylinder (R_1), introduced to reduce field deviations in the measurement cells (shown as rectangular prisms).

fig. 3), calculated analytically using the methods described in [1], in order to minimize the induced field at larger radii. The shielded design negatively affects the uniformity of the dressing field, leading to deviations

$$\Delta B_1 = \sqrt{\langle \Delta B_1^2 \rangle} = 1.67 \times 10^{-3}\,\text{Gauss}$$

in the field with (largest) RMS gradient of

$$\partial B_1/\partial z = 3.22 \times 10^{-4}\,\text{Gauss}\cdot\text{cm}^{-1}$$

This particular field has been used to calculate relaxation times in the simulations of sec. 3, and produces a relaxation time $T_2 \sim 400\,\text{s}$ at 450 mK, much shorter than required by nEDM.

Correction coils have been designed in order to satisfy the requirements on the dressed field uniformity of eq. 7. These coils, also shown in fig. 3, do not contribute a significant amount of eddy current heating, but improve the uniformity of the dressing field, resulting in deviations

$$\Delta B_1 = 5.05 \times 10^{-4}\,\text{Gauss}$$

in the field with non-negligible gradients of

$$\begin{aligned} \langle \partial B_1/\partial x \rangle &= 2.09 \times 10^{-4}\,\text{G}\cdot\text{cm}^{-1} \\ \partial B_1/\partial x_{RMS} &= 2.29 \times 10^{-4}\,\text{G}\cdot\text{cm}^{-1} \\ \partial B_1/\partial z_{RMS} &= 2.74 \times 10^{-5}\,\text{G}\cdot\text{cm}^{-1} \end{aligned} \tag{8}$$

Simulations of spin precession of ^{3}He atom spins in the corrected dressing field, result in relaxation times $T_2 > 10^4\,\text{s}$ at 400 mK.

APPENDIX

We attempt to estimate the relaxation time T_2 for the field with gradients described by eqs. 8. This field has a quadratic profile in z and non-negligible $\langle \partial B_1/\partial x \rangle$. The contribution to the field deviation for the components of the gradients in the x and z direction can be calculated using the relation in eq. 5, so that

$$\begin{aligned} \left\langle \Delta B_1^2 \right\rangle_x &\simeq \frac{L_z^2}{15} \left\langle \frac{\partial B_1}{\partial x} \right\rangle^2 + \frac{L_z^2}{60} \left\langle \frac{\partial B_1}{\partial x}^2 \right\rangle \simeq 2.19 \times 10^{-7}\,\mathrm{Gauss}^2 \\ \left\langle \Delta B_1^2 \right\rangle_z &\simeq \frac{L_z^2}{60} \left\langle \frac{\partial B_1}{\partial z}^2 \right\rangle \simeq 2.01 \times 10^{-8}\,\mathrm{Gauss}^2 \end{aligned}$$

Eq. 4 can be adapted for the individual contributions to the relaxation, so that

$$\frac{1}{T_2} = \frac{\gamma_3^2\, Y^2\, J_1^2}{10D} \left[\frac{1}{4} L_z^2 \left\langle \Delta B_1^2 \right\rangle_z + L_x^2 \left\langle \Delta B_1^2 \right\rangle_x \right]$$

where the factor of 1/4 in the z term comes from the symmetry of the field, present in z, but not x, leading to $\tau_D \rightarrow \tau_D/4$, as described in sec. 2. This leads to an estimated relaxation time $T_2 = 1.9 \times 10^4$ s for He at 450 mK. The simulation results yield $T_2 = 1.7(4) \times 10^4$ s.

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