Detailed optimization of $\pi/2$ pulse for UCN and ^{3}He atoms

Riccardo Schmid
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A simulation of the spin precession due to an oscillatory $\pi/2$ pulse was used to optimize the parameters of the pulse. The simulations implement a 5^{th} order Runge Kutta to numerically solve the spin equations of motion classically. The simulation uses a mean step size $\Delta t \sim 2 \times 10^{-5}\,\text{s}$, each step comprising of few intermediate steps from the numerical method. The gyroscopic ratios used in the simulations were $\gamma_n = -18.32471831\,\text{s}^{-1}\text{mG}^{-1}$[1] and $\gamma_3 = -20.37894\,\text{s}^{-1}\text{mG}^{-1}$[2].

A $\pi/2$ pulse is defined as an oscillatory magnetic field of the form

$$\boldsymbol{B}_{\pi/2} = 2\,B_1 \cos(\omega_{rf}\, t)\,\hat{\boldsymbol{y}}, \tag{1}$$

that exists for a period of time τ, that can be used, in the presence of a holding field $\boldsymbol{B}_0 = B_0\,\hat{\boldsymbol{x}}$, to rotate the spin polarization of UCN and ^{3}He to be perpendicular to $\boldsymbol{B}_0$. An optimal solution was found using spin precession simulations with running parameters (B_1,ω_{rf}) near $B_1 = 0.7\,\text{mG}$. Figure 1 shows the time evolution of the spin component aligned with $\boldsymbol{B}_0$. The parameters that optimize the effectiveness of the $\pi/2$ pulse are shown in Table I. The duration of the pulse τ was inferred from the simulation data.

	value	unit
B_0	10	mG
B_1	0.730563	mG
ω_{rf}	191.08115	rad s^{-1}
τ	123.31	ms

Table I: Optimal parameters for the $\pi/2$ pulse. The RF field is defined as $B_{\pi/2} = 2B_1\cos(\omega_{rf}t)$

The parameter search yielded a result for which both UCN and ^{3}He atoms were effectively rotated by $\pi/2$ with respect to the B_0 field to within $1\,\mu\text{rad}$, as shown in Fig. 2. The accumulated phase between UCN and ^{3}He spins was calculated to be $\phi_{n3} = 1.674679\,\text{rad}$.

A. Sensitivity to changes in parameters

Deviations from the optimal solution were introduced to calculate the sensitivity of the $\pi/2$ pulse. The results are shown in Table II.

x	$\Delta\theta_n/\Delta x$	$\Delta\theta_3/\Delta x$	$\Delta\phi_{n3}/\Delta x$	unit	frac. unc.
B_1	2.09	1.85	1.41	mrad$\cdot\mu$G^{-1}	1×10^{-3}
B_0	0.377	-0.749	0.114	mrad$\cdot\mu$G^{-1}	9×10^{-4}
ω_{rf}	-2.77×10^{-2}	3.21×10^{-2}	1.88×10^{-3}	s	2×10^{-3}
τ	~ 0	~ 0	20.5	rad$\cdot$s^{-1}	4×10^{-4}
α	-1.44×10^{-2}	-0.0118	0.0145	1	0.04

Table II: Sensitivity of $\pi/2$ pulse. The quantity α represents the angle between the $\boldsymbol{B}_1$ and $\boldsymbol{B}_0$ fields, away from orthogonality. Also shown is the required fractional uncertainty on the parameters to have the phase ϕ_{n3} reproducible at the mrad level. The sensitivity of the rotation angle $\theta_{n,3}$ due to changes in τ is very small as can bee seen in Fig 2.

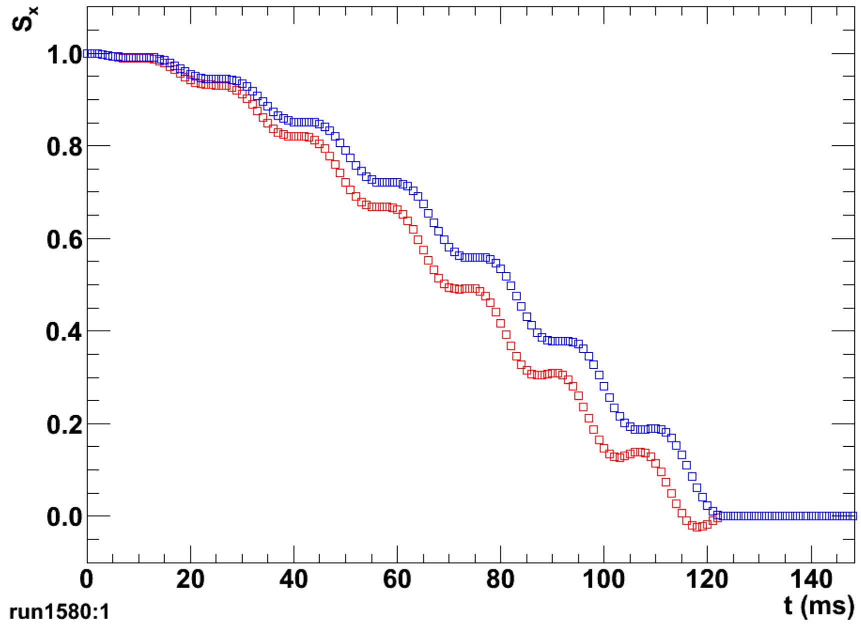

Figure 1: Projection of the spin component aligned with $\boldsymbol{B}_0$ as a function of time, for UCN (blue) and ^{3}He, under the influence of a $\pi/2$ pulse. The pulse starts at $t = 0$ and continues for 123.31 ms, when both spins are perpendicular to $\boldsymbol{B}_0$.

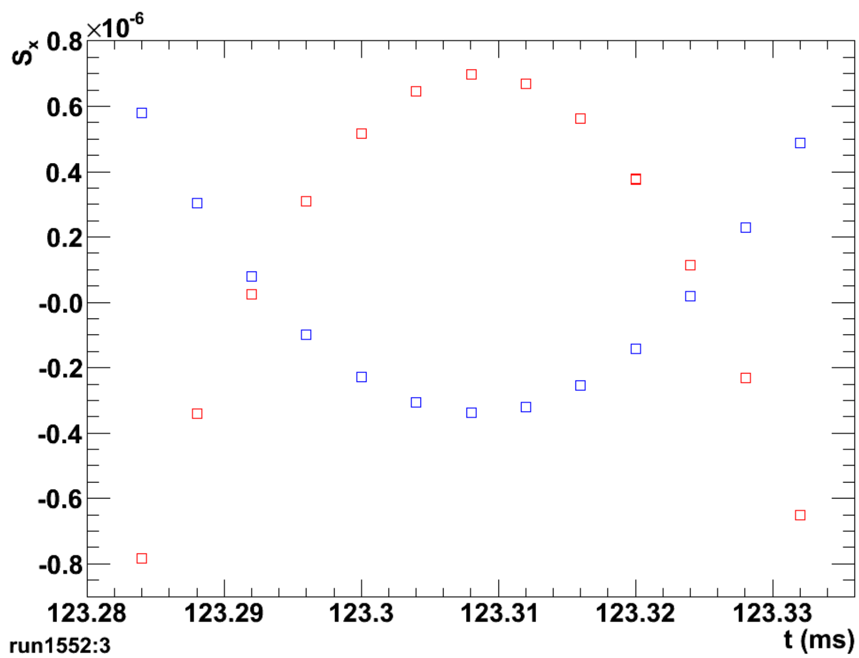

Figure 2: Optimization of the $\pi/2$ pulse. The pulse duration τ was chosen to lie in the middle of the time period in which both species spin components were below the μrad level.

B. Previous calculations

We analyzed the parameters constrained by previous calculations in [3] using spin precession simulations with the parameters needed for the $\pi/2$ pulse being

B_0	10	mG
B_1	0.7	mG
ω_{rf}	189.426	rad·s^{-1}
τ	132.3	ms

The new simulations, which were run using the gyromagnetic ratios found in [3] showed that the parameters do not offer an optimal solution for the $\pi/2$ pulse for which both species are rotated within a mrad of $\pi/2$. The rotation angles for UCN and ^{3}He spins were calculated to be, respectively, $\theta_n - \pi/2 = -0.80\,\mathrm{mrad}$ and $\theta_3 - \pi/2 = 21.7\,\mathrm{mrad}$, the accumulated phase $\phi_{3n} = 1.8046\,\mathrm{rad}$. The results of the sensitivity studies on these parameters show values relevant to phase sensitivity of

$$\Delta\phi_{n3}/\Delta B_1 = 1.58\,\mathrm{mrad}\cdot\mu\mathrm{G}^{-1},$$

$$\Delta\phi_{n3}/\Delta B_0 = 0.0607\,\mathrm{mrad}\cdot\mu\mathrm{G}^{-1},$$

$$\Delta\phi_{n3}/\Delta\omega_{rf} = 5.25\times10^{-3}\,\mathrm{s}$$

The sensitivity to B_1 differs from [3] by an order of magnitude.

[1] C. A. et al., (Particle Data Group) Physics Letters **B667, 1** (2008).
[2] J. L. Flowers, B. W. Petley, and M. G. Richards, Metrologia **30**, 75 (1993), URL `http://stacks.iop.org/0026-1394/30/75`.
[3] M. Cooper (2009), nEDM Technical Note.