Proposal for value of y parameter

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0.1 Introduction

Third order perturbation theory states that the energy of the dressed states under a small constant magnetic field is

$$\frac{E}{\hbar} = \omega_0 J_0(\omega_1/\omega_{RF}) - \frac{1}{8}\frac{\omega_0^3}{\omega_{RF}^2} J_0(\omega_1/\omega_{RF}) \sum_{q=1}^{\infty} \frac{J_q^2(\omega_1/\omega_{RF})}{q^2}, \tag{1}$$

where ω_0, ω_1, and ω_{RF} are γB_0, γB_1, and the frequency of the dressing field, respectively. Hence, the difference in frequencies between the helium and neutrons will be, correct to third order,

$$\begin{aligned} \omega_{3n} \quad = \quad & \omega_0(J_0(\omega_1/\omega_{RF}) - \alpha J_0(\alpha\omega_1/\omega_{RF})) - \\ & \frac{1}{8}\frac{\omega_0^3}{\omega_{RF}^2}(J_0(\omega_1/\omega_{RF}) \sum_{q=1}^{\infty} \frac{J_q^2(\omega_1/\omega_{RF})}{q^2} - \\ & \alpha^3 J_0(\alpha\omega_1/\omega_{RF}) \sum_{q=1}^{\infty} \frac{J_q^2(\alpha\omega_1/\omega_{RF})}{q^2}). \end{aligned}$$

Under “perfect” dressing the first term is zero, leaving the relative precessing frequency between particles with a square dependence on the holding field B_0.

0.2 Sensitivity to experimental parameters

How sensitive is this frequency ω_{3n} to the holding field and to the amplitude and frequency of the holding field? We can estimate this with the derivatives of ω_{3n} with respect to the quantities of interest. Although we can also follow the path of extracting the sensitivity of ω_{3n} to the dimensionless parameters $y = \omega_0/\omega_{RF}$ and $x = \omega_1/\omega_{RF}$, in the experiment each of the parameters ω_0, ω_1, and ω_{RF} will fluctuate or drift independently and it is important to understand how their changes affect bias the measurement.

The total change of ω_{3n} will then be given by

$$\begin{aligned} \delta\omega_{3n} \quad &= \quad \frac{\partial\omega_{3n}}{\partial\omega_0}\delta\omega_0 + \frac{\partial\omega_{3n}}{\partial\omega_1}\delta\omega_1 + \frac{\partial\omega_{3n}}{\partial\omega_{RF}}\delta\omega_{RF}, \\ \delta\omega_{3n} \quad &= \quad \Delta\omega_0 + \Delta\omega_1 + \Delta\omega_{RF} \end{aligned}$$

Taking the partial derivatives is a tedious task better left for Mathematica. The results, assuming perfect dressing $x = x_c = 1.18899$ and leaving the derivates in terms of the dimensionless parameter y are

$$\begin{aligned}
\Delta\omega_0 &= \frac{\partial\omega_{3n}}{\partial\omega_0}\delta\omega_0 = 0.026y^2\delta\omega_0, && (2)\\
\Delta\omega_1 &= \frac{\partial\omega_{3n}}{\partial\omega_1}\delta\omega_1 = (0.156y - 0.008y^3)\delta\omega_1, && (3)\\
\Delta\omega_{RF} &= \frac{\partial\omega_{3n}}{\partial\omega_{RF}}\delta\omega_{RF} = (-0.185y - .007y^3)\delta\omega_{RF}. && (4)
\end{aligned}$$

Equation 3, 4, and 5 state the dependence of the fluctuations of each experimental parameter times some factor *i.e.* the RMS fluctuations of ω_{3n}. Let's assume $(\delta\omega_0, \delta\omega_1, \delta\omega_{RF}) = (10^{-6}, 10^{-4}, 10^{-4})$ rad/s and that the cubic terms are negligible. Hence

$$\begin{aligned}
\Delta\omega_0 &= 2.6 \times 10^{-8}y^2, && (5)\\
\Delta\omega_1 &= 1.56 \times 10^{-5}y, && (6)\\
\Delta\omega_{RF} &= 1.85 \times 10^{-5}y. && (7)
\end{aligned}$$

Eq. 5 shows the beauty of spin dressing. The measurement is highly insensitive to fluctuations in the holding field when the dressing is perfect. The amount of suppression of the other experimental parameters will be given by how small y is. If $y = 1/100$ then the expected fluctuation in the relative frequency between particles will be of the order of 10^{-7} rad/s!